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Title	An Investigation into the Structure of Numerical Cognition
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AN INVESTIGATION INTO THE STRUCTURE OF NUMERICAL COGNITION

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ABSTRACT

This thesis reports work relating to theoretical frameworks in the area of numerical cognition that have been developed by McCloskey, Caramazza & Basili (1985), Clark & Campbell (1991), Dehaene (1992) and Noel & Seron (1992). The associations between numerical cognition and memory processes in relation to the working memory model of Baddeley (1986) were investigated. The first study used the factor analytic method to elucidate the factor structure of the processes that underlie numerical cognition, and to investigate the various components of the working memory model in relation to arithmetic. A battery of 21 tests was administered to 100 participants. The contribution of the factor analytic study to the structure of numerical cognition is discussed. An examination of the factors (labelled 'access to representations' and 'working memory') identified specific aspects of numerical cognition that were investigated further using experimental methods. The data on magnitude comparisons of numbers and animals that have been found to load onto Factor 1 were reanalysed. Similar patterns were found with the two types of stimuli in some cases. This suggested that Dehaene's notion of a 'number line' might not be specific to numbers. To build on the investigation of magnitude comparisons two experiments were carried out using the dual task paradigm. The results confirmed that magnitude judgements are represented at the level of semantic processing and may not be specific to numbers.

The subitizing circles test was also found to load onto Factor 1. This raised a question about the common processes that may be involved both in this test and in other tests loading on that factor. A dual task experiment was used to investigate that possibility. It appeared from the results that the verbally presented tasks in the control and experimental groups produced interference with the subitizing task. This result lent support for the view that subitizing is an early pre-lexical perceptual process, possibly based on canonical representations of the stimuli.

Complex addition and multiplication loaded onto Factor 2, 'working memory' and a further dual task experiment was conducted to investigate the speculative view held by Aschraft (1995), that the visuo-spatial sketchpad may play a role in arithmetic problem solving. The results lent support for the view held by Aschraft (1995) of the involvement of the visual-spatial component of working memory in the calculation of multi-digit addition problems. Thus the research reported in this thesis has used a range of investigative techniques and data analysis, with the aim of clarifying the scope and the limitations of major recent models of numerical cognition and the role of working memory in numerical processing. The results of the research programme supported those models which link numerical cognition with other forms of mental processing by identifying specific ways in which diverse numerical processes such as magnitude comparison, subitizing and the calculation of multi-digit problems draw on forms of processing associated with other types of stimuli.

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PREFACE

Parts of the work reported in this thesis were presented at conferences held by the British Psychological Society, Roberts & Ward (1998a;b, 1999, 2003).

DECLARATION

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of Luton. It has not been submitted before for any degree or examination in any other University.

Signed P. Roberts

7th day of May 2004

Chapter 1 – Introduction to numerical cognition

1.1 Overview of the thesis

The aim of this thesis is to extend knowledge of the structure of numerical cognition. An in depth investigation into models of numerical cognition and experimental work designed to elaborate on existing knowledge of these models is discussed. Associations between numerical cognition and working memory are examined in detail. Chapters 1-3 provide an introduction to the development of themes and models that are later elaborated on. Chapters 4-6, reports an extensive factor analytic study and explains the results and Chapter 7 gives a summary of the factor analytic study. Chapters 8-11, develop the findings from the factor analytic study. These chapters aim to elaborate on the structure of numerical cognition drawing on new experimental work. Chapter 12 provides the overview and conclusions of the research carried out in this thesis.

1.2 Introduction

This chapter presents a review of influential studies that have given recent research a firm foundation on which to build. This early work provides a background to key themes that are investigated in greater detail later in the thesis.

Numerical cognition is of great interest to many researchers in a range of areas, for example, developmental psychology, neuropsychology and cognitive psychology. Systematic research in numerical cognition is relatively recent but the fascination with how humans process and calculate numerical problems has existed for centuries (Jerons 1871). Psychologists and neuropsychologists have, through empirical research and with the use of advanced technological equipment, developed theories and models which allow investigations into the organisation and manipulation of numerical processing. Numerical cognition, however, cannot be studied in isolation or to the exclusion of a variety of other cognitive activities. To be able to calculate a simple addition problem, for example, 2 + 3, the problem has to be attended to, read, memorised, calculated and the answer retrieved and produced. Therefore, investigations into numerical cognition involve a range of cognitive activities with each activity, whether it is attention, memory processes or visual spatial processing playing a specific part in the production of an answer to a numerical problem. The question then arises as to what cognitive activities are involved when calculating the answer to a particular problem, bearing in mind the large variety of numerical problems that exist with each one carrying a degree of difficulty. As a result of research conducted over the past 20 years theories and models have arisen which assist in answering this question. However, the theories and models are diverse in their make-up with their composition reflecting different ideas about how cognitive functioning is activated within the brain. Dehaene (1992) viewed adult human numerical cognition 'as a layered modular architecture, the preverbal representation of approximate numerical magnitudes supporting the progressive emergence of language-dependent abilities such as verbal counting, number transcoding, and symbolic calculation' (p.35).

1.3 Historical Background

Cognitive psychology as a science emerged in the latter part of the 20th century providing extensive research material based on frameworks, theories, and models which allow investigations into the organisation and function of mental activities. James (1890) and Tolman (1932) contributed important research material towards its development. James (1890) considered that mental processes were activities and the mind was not an entity but a functional activity of the organism. His contribution to the study of attention and memory was substantial, drawing a distinction between primary memory and secondary memory later to be referred to as short-term and long-term memory. This concept returned to the forefront of psychological research during the 1960s when psychologists began to study short and long-term memory in a systematic way. During the early part of the century psychologists formed interpretations from their observations of animal behaviour. Tolman (1932) observed that rats were able to run and swim through a maze. This research suggested that rats had not only been able to learn certain specific motor responses but they had also formed a cognitive map or internal representation of the maze. The results were interpreted by suggesting that learning in rats could be understood only by focusing on internal structures and processes rather than on observable motor responses. This was an innovative concept which further enhanced the development of cognitive psychology.

1.3.1 Early studies in numerical cognition

Under the broad heading of cognitive psychology there are many sub-divisions, for example, psychobiology, neuropsychology and mathematical psychology. Mathematical psychology can be further subdivided into problem solving techniques, transference of knowledge during problem solving, the psychology of mathematical anxiety, and numerical cognition. The development of numerical cognition has strong links with much earlier research, for example, memory, attention visual imagery and the internal structures and process that are involved in the manipulation of information. The following sections introduce concepts which are investigated further in the thesis, although their roots stem much further back.

1.3.2 Numerical discrimination (subitizing)

Jerons (1871) investigated numerical discrimination, otherwise known today as subitizing. His experiment, carried out upon himself, investigated the estimating of any large number of objects without counting them successively. He thought that a small number in a group could be determined by using a 'single act of mental attention'. Jerons noted that earlier authors had offered a range of estimates of between 4 – 6 objects. Jerons suggested that the topic deserved more systematic investigation as it was one of the few areas in psychology that could be subjected to experimental work.

The experiment consisted of a round, paper box standing a quarter of an inch high, which was lined with white paper and contained black beans. A random number of the beans was picked up and thrown towards the box so that it would be uncertain as to how many fell into the box. Immediately the beans fell into the box their number was estimated and recorded together with the real number by counting the beans. The number of beans landing in the box ranged from 3 to 15 and 1027 trials were conducted.

The results produced no errors if the number in the box was made up of 3 or 4 beans but Jerons was surprised to find that errors were made when 5 were in a group. Based on the calculations of his results he concluded that the ability to estimate 5 was flawed and that the limit of accuracy was neither 4 nor 5 but fell half-way between them. Jerons suggested that the number 5 was beyond the limit of accurate estimation.

Warren (1897) took the same concept as Jerons but used a different research method to investigate the concept. Warren investigated individuals' knowledge of the number of items in a group by the use of reaction time experiments using a chronoscope (an early piece of equipment used to record reaction time). The apparatus used for the experiments comprised paper circles of 14 mm. in diameter placed on pieces of card 16.5 cm. square. The number of paper circles on each card did not exceed eight. Four participants took part in the experiment and the length of time taken to identify the number of circles on a piece of card was recorded together with error scores.

The results of Warren's experiment differed from those found by Jerons, with the findings suggesting that an individual's immediate response without counting the number of items in a group was limited to one, two and three and in some cases as great as four. Warren made a distinction between "perceptive counting" and "progressive counting". Perceptive counting according to Warren did not involve the counting together of the individual items in a group and this applied to his conclusions that a maximum of four items could be judged without the conscious counting-on process. However, the term progressive counting was applied when the number in the group exceeded the ability to use perceptive counting, in this case four.

Anecdotal evidence was provided by Galton (1880) and Bertillon (1881) on how numbers might be visualised. These reports, although not considered as experimental evidence, do provide interesting background to the account of the development of current theories of numerical cognition to be given in Chapter 2.

According to Galton, (1880) individuals' perception of numbers is precise and formed regular arrangements or schemes. The case studies reported indicated that numbers had their own particular place on a line. The line was not always described by individuals in the same way as it may be straight or curved. However, reports suggested that the line disappeared into the distance. One common feature that appeared to arise was that low numbers, particularly 1 to 12, were visualised as close together, whereas the larger numbers were distributed with a greater distance between them and may increase in increments of 5 or 10 or more. It seemed that the lower numbers had a precise place on this line, yet when higher numbers were involved in a calculation their place on the line became less clear.

Bertillon (1881) reported the case study of a gentleman who visualised numbers in the same configuration as reported in Galton's case studies with the low numbers very prominent on the scale and the high numbers less clear. His suggested explanation was that during simple mental arithmetic calculations higher numbers were less frequently used.

The above case studies and early experimental work highlight issues that are extensively studied today in relation to numerical cognition. They provide illustrative examples of the methods used and theories developed by philosophers and early experimental psychologists during the 19th Century.

1.3.3 Early 20th Century Research

Browne (1906) conducted an experimental study with eight university students. The students were required to solve addition, subtraction, multiplication and division problems. In one set of experiments the students were required to produce the answers verbally, (without the use of paper and pencil) to a series of addition, subtraction, multiplication and division problems; in another set of experiments they were required to write down their answers. The length of time taken to verbally produce the answers or write down the answers and error scores was recorded. The answers to the addition problems were given verbally and 'considered as a process of four stages:

1 A distinct consciousness of a number to which another is to be added
2 The recognition of this other number
3 The associative process leading to the sum of the two
4 The distinct consciousness of that sum' (p.4)

According to Browne (1906) 'distinct consciousness' related to attention and the associative stage related to the subconscious. However, Browne referred to the work of Ebbinghaus (1864) and his study of the memory for nonsense syllables. Ebbinghaus suggested that the likelihood of any series of ideas being remembered was dependent upon repetition and the formation of associations or bonds between any items in a given series. How well the series was committed to memory depended upon the strength of the associations formed between the items. According to Ebbinghaus, items in a series that were distant from each other formed weaker associations than items that were closer together. Browne applied Ebbinghaus's results to his study suggesting that the difficulty of adding two digits together was directly proportional to the difference between the two digits. Thus, in the example 9 + 2 = 11, the associative bond between 9 and 11 is relatively strong but if the addition problem is turned around as 2 + 9 = 11 the association between 2 and 11 is weak. If the difference between the two digits is small and where there is only a difference of 1, for example 6 + 5 = the adding of the digits is considered to be easy. Similarly if the distance is very large, for example, 7 + 2 =, this was also found to be easy. The most difficult digits to combine were found to be digits falling between the two extremes where the difference is neither very large nor very small, for example 7 + 5 and 9 + 5. Browne concluded that the solving of single addition problems involved the recognition of the digits to be added, and the recognition of the association between the digits but not the recognition of the digits in isolation from each other such as 6 or 9. From the results it appeared that, for all participants, even numbers were easier to add together than odd numbers.

The results of the subtraction problems indicated that Ebbinghaus's law of serial association operated in the calculation of subtraction problems. However, whereas in the solving of addition problems the association between the numbers was strong as the counting was forward, in subtraction problems the association between the digits was weaker as the necessary counting operations were performed backwards.

With regard to multiplication Browne (1906) placed greater emphasis on imagery which he described as motor, auditory and visual imagery. Browne appears to have used motor and auditory imagery to refer to the participant repeating the digit or digits to him or herself. These methods of imagery related to carrying operations involved in the multiplying of two digits together, such as 8 x 9.

Division, according to Browne, depended upon multiplication in much the same way that subtraction depended upon addition. The main findings suggested that the difficulty of solving division problems increased proportionally with the size of the divisor.

This research highlighted interesting phenomena that continue to be studied today. For example Ebbinghaus's law of associations has influenced the contemporary connectionist approach to the study of numerical cognition and cognitive psychology as a whole. The use of visual imagery in carrying operations is a much discussed topic in numerical cognition, with researchers differing in their conclusions about the degree of involvement of visual imagery.

Work on individuals' ability to estimate the numbers of items in a group was extended by Taves (1941), Saltzman & Garner (1948) and Kaufman, Lord, Reese and Volkmann (1949). These pieces of research developed the subject further and introduced new concepts. Taves (1941) presented 113 participants with a series of dots. The number of dots in each presentation ranged from 2, 4, 6, 8, 10, 20, 50, 100 and 180. The participants were instructed to report the number of dots in each display and also to report how confident they were with their response on a scale of 0 to 5. Zero indicated no confidence and five meant complete certainty. From the results Taves suggested participants' responses were accurate up to and including the presentations of 6 dots. Between 6 and 8 dots the reports became inaccurate and participants' confidence in their responses decreased as the size of the dot displays increased. Taves concluded that there were two 'mechanisms for the discrimination of visual numerousness', (p.499). Numerousness is the ability to discriminate the number of objects in a group without counting them. The first mechanism was 'adequate perception of number'. This was defined as a response that accurately reflected the number of objects in a display the majority of the time. The second mechanism was the 'inadequate perception of number', (p.500) which reflected increased errors and reduced confidence in the responses made by the participants.

Saltzman & Garner (1948) used accuracy and reaction time data to investigate the ability to discriminate the number of objects in a group. Firstly, the tachistoscopic method in which the trials were shown for half a second and in which participants' accuracy in identifying the number of circles displayed was recorded. Secondly, the time taken for participants to make their response was recorded. In the experiment the stimuli were concentric circles with the number of circles presented varying from 2 to 10. They found that no more than 3 circles were correctly identified 100% of the time and that reaction time increased with the number of circles presented.

To summarise the outcomes of the above studies Taves (1941) suggested that people achieved complete accuracy in judging up to 5 stimuli objects but Saltzman & Garner (1948) suggested that 3 was the maximum number an individual was able to judge accurately.

The experiment conducted by Kaufman, et al. (1949) used groups of dots (solid black circles drawn on white paper) projected on a screen. The number of dots in the first set of trials increased by one from 1 to 15. Further trials included arrangements of dots ranging from groups of 17 to 210. Reaction time and accuracy data were recorded. The results indicated that when participants were shown between 1 and 5 dots they usually responded with the correct number and from 6 dots and above accuracy decreased. Reaction time results were similar to those found by Saltzman & Garner (1948) whereby reaction time increased with the size of the group.

Kaufman et al. (1949) explained their findings in terms of how individuals discriminate the number of objects by estimating, or subitizing or counting. They applied the term 'estimating' to the discrimination of stimulus numbers greater than 6. For numbers below 6 they proposed the introduction of a new term – 'subitizing'. The word is derived from the classical Latin adjective 'subitus', meaning sudden and the Medieval Latin verb 'subitare', meaning to arrive suddenly. Counting, according to Kaufman, et al. (1949), is the process where the individual counts the number of items in a group when the number is too large to subitize or estimate. This distinction between estimating, subitizing and counting influenced later research.

This research was an important development in the area of visual identification of members of a group and stimulated a fertile new line of research in this field that is discussed in more detail in Chapter 10.

1.4 Information processing approach to the study of numerical cognition

The current interest in numerical cognition could be viewed as a relatively recent branch of mainstream cognitive psychology. The recent revival of research activity stems from Groen & Parkman's (1972) empirically based studies. This research is interpreted in terms of the information processing approach to the mental processes that contribute to the manipulation of numerical information. Groen & Parkman (1972) used simple addition problems and timed children as they gave answers to addition problems, for example, 4 + 2 =. All sums given totalled to less than or equal to 9. The results indicated that as the problem increased in difficulty the time taken to produce the answer increased as did the number of errors made (problem size effect). Aschraft (1992) considered this research to be the first study to provide an extensive examination of the problem size effect and to provide explanations in terms of mental processes. The research by Groen & Parkman is described in greater detail below.

The identification of the problem size effect may seem to be a small contribution to the study of numerical cognition but it indicated an important transformation from the relative lack of interest in the subject area to the formation of a constructive interest in mental processes that manipulate, comprehend, calculate and produce arithmetic and numerical knowledge. However, despite the apparent dormant nature of this area of psychology in the early part of this century, interest has been restored with prolific theoretical frameworks being proposed.

From the above research it is suggested that adults are able to calculate two single digit arithmetic problems with speed and accuracy. When given any two single digits from 0 to 9 adults are able to calculate quickly either by addition, subtraction, multiplication and magnitude comparisons. However, while these operations may appear simple, there was still great uncertainty as to the representations and mental processes that provided the basis for calculation procedures.

Groen & Parkman (1972) described calculation procedures in terms of repeated internal and external counting. They suggested that addition problems were solved by counting up to the sum of the problem from the larger of the two figures to be added. This concept was supported from the findings of their research with children. The mean age of the children was 6 years and 10 months. The stimuli consisted of 55 addition problems with the answer less than or equal to 9. The children made their response by pressing a button from a row of buttons ranging from 0 to 9. The results showed that reaction time was positively correlated with the size of the smaller figure. A further aspect investigated was to determine whether or not tied problems such as 4 + 4 would be solved quicker than 7 + 2. The results showed that the reaction time for tied problems was quicker than for non-tied problems. It was suggested by Parkman & Groen (1971) that the answer for tied problems was retrieved directly from memory with very little or no active processing necessary whereas problems that were not tied might require a greater amount of processing and calculation. A further finding from Groen & Parkman's (1972) study suggested that if the larger digit was positioned on the left of the sum participants found the sum easier to process than if it was positioned on the right.

Research by Svenson (1975) supported the findings of Groen & Parkman (1972). Svenson (1975) conducted a partial replication study of the work by Groen & Parkman (1972). Svenson increased the stimulus range to include all additions with an answer equal to or smaller than 13. The age of the children tested was between 10 and 11 years. The children gave oral answers and the experimenter pressed a button to record the reaction time. The results showed that the quickest responses were for stimuli containing a zero. The average response time was 1.38 seconds. The time taken to provide answers to tied problems was on average 1.58 seconds. It was assumed that this time included the retrieval of the answer from long-term memory. From the results Svenson suggested that if the stimulus did not contain a zero or a tie there was the possibility that children might perform a mental transformation of the figures presented, for example, 3 + 5 was turned around to 5 + 3 before commencing the counting process. If the presentation is 5 + 3 then no transformation was necessary. Svenson (1975) considered that the time taken to perform a mental transformation took approximately 0.1 of a second and that short-term memory played an active part in this process. Svenson (1975) concluded that this research was an important starting point for further investigations to attempt to identify the cognitive processes involved in children, solving addition problems as this would be of benefit to children who had difficulties with arithmetic.

Groen & Parkman (1972) proposed the counting model or 'min' (for minimum addend) model. According to this model, addition problems are calculated by setting an internal counter to the larger of the two addends and then increasing this value by one at a time for the number of times specified by the smaller addend. The assumption was that the counter setting time should be constant and therefore increases in reaction time should be proportional to the number of increments added. Following Groen & Parkman's research the conclusion reached was that the 'min' model may not generalise to adult problem solving for the following reasons. Firstly, the results from reaction time data showed that young school children took 400 milliseconds to add the smaller figure on to the larger figure in a two digit addition sum, whereas it took 20 milliseconds for adults. It seemed unlikely that adults were counting. Secondly, the reaction time for tied problems showed that in both adults and children there was very little difference in time taken over sums of different sizes. They concluded that within the range of sums studied the problem size effect did not apply to tied problems.

To sum up they considered that addition was associated with a counting process and proposed the counting or 'min' model leading to the digital theory of calculation. A parallel strand of numerical cognition is the study of magnitude comparison described below.

1.5 Studies of magnitude comparisons

The study of magnitude comparisons is an important element of numerical cognition and early research by Moyer & Landauer (1967) influenced later research. This section provides a review of early research that is expanded upon later in the thesis.

Moyer & Landauer (1967) tested comparison judgements between digits, for example, indicating which is the larger of two digits, 2 or 6. According to Moyer & Landauer (1967) when participants were shown a pair of digits and asked to indicate which was the larger by pressing one of two buttons, reaction times were quicker as the difference between the specified number increased. For example, reaction times were quicker for numbers that were more distant in magnitude such as 9 and 3 than for numbers that were close in magnitude such as 9 and 8. It also appeared that reaction times decreased the larger the numbers were. These two effects were generally known as the distance effect and the minimum effect. Moyer & Landauer further suggested that numbers were converted to analogue representations in order to compare them. The assumption was that numbers were converted to representations of magnitude that were analogueous to representations of such physical dimensions as size, length or brightness.

Banks, Fujii & Kayra-Stuart (1976) proposed an alternative model to the analogue representation theory suggesting a semantic coding model where each digit is encoded as large or small. These semantic codes are compared and a response is given quickly if the codes differ. If the codes for two digits are the same then the digits are recorded to discriminate between a large and a very large number or a small and very small number. It was suggested that this model was able to account for the distance effect and the minimum effect by assuming that the large/small boundary was closer to the small numbers.

Parkman (1971) replicated Moyer and Landauer's (1967) research finding that reaction time for determining the larger of two digits was positively correlated with the size of the smaller digit. The interpretation of the results given by Parkman (1971) was of a process in which participants began with the number one and continued in increments of one until the smallest of the two figures was found and they responded with the number.

Interest in magnitude comparison has not been restricted to numbers but has extended to the comparison of objects and animal size. Research by Moyer (1973) and Paivio (1975) compared the size of named animals. Subjects were visually presented with the names of two animals such as frog-wolf and were required to respond by throwing a switch under the name of the larger animal. The results indicated that the reaction time increased as the difference in animal size became smaller. The conclusion made was that animal names were converted to analogue representations that preserved animal size. It would seem, therefore, that small differences between number size and animal size were represented as smaller differences between the internal analogue scales, which resulted in a decreased ability to discriminate between two digits or animals and this was then reflected in increased reaction times. However, the nature of the postulated analogue representations was not specified but it was speculated that the items, whether digits, animals or objects, were positioned along an imagined spatial dimension. Whatever the structure of the analogue scale, it was considered that a primary function was the preservation of information in relation to size.

It appears that single digit addition problems and the magnitude comparison of numbers can be accounted for within the counting model. However, simple multiplication is not so easily explained by this theory. Multiplication, for adults, requires the retrieval of previously learned facts from memory with very little conscious effort required. Parkman (1972) investigated reaction times for 100 single digit multiplication problems derived from the possible combination of the single digits from 0 to 9. Six participants took part in the verification task. The participants were required to indicate whether or not the answer to the problems was correct. The results indicated that there was a similarity in the pattern of reaction times with multiplication to the pattern with addition (Parkman & Groen 1972). However, it remained inconclusive as to whether or not similar procedures were used in the solving of addition and multiplication problems. A differing line of thought proposed by Parkman (1972) was that simple 'addition and multiplication 'tables' were stored as hierarchical networks in long-term memory' (p. 443). The time taken to retrieve the answer to a problem would be dependent upon its location within the network.

The results of Moyer & Landauer (1967) suggested the use of an analogue scale for the comparison of pairs of digits. The analogue theory treats the task of magnitude comparison as a manipulation of an internally represented number line by the participant. Restle (1970) considered the concept of a number line as the basis of the solving of addition problems which is in contrast to the digital operations assumed by the counting theory.

Restle (1970) investigated the addition of two and three digit problems with each problem including one of the base numbers. The base numbers used were 13, 37, 83, and 153 and thirty-eight adult participants took part in the study. The addition problems appeared on a screen and reaction time and error scores were recorded. The problems to be solved varied in three ways. Firstly, Restle varied the magnitude or size of the digits to be added. It was considered that the larger the base number the more difficult the problems would be to solve. The reasons suggested were that larger numbers were less familiar and if an analogue system was used, larger numbers might require the manipulation of larger imagined quantities, which would result in more errors and a slower time recorded for calculation. The results showed that, when the base number was larger a longer time was taken to solve the problems and more errors occured. Restle considered that as larger numbers contained more digits, they would take a longer time to read. However Restle found this was not the case when the base number was 100. Reaction time and error scores decreased. Secondly, he varied whether the digits to be added were equal to each other or whether one was greater and the other smaller as in 37 + 7. The results showed that problems with digits equal to each other provided quicker reaction times. Thirdly, he varied the difference between the digits. For example, 6 + 7 are closely related but 13 + 27 are quite distant from each other. Here the results agreed with Moyer and Landauer (1967) in that the greater the difference between the numbers the greater were the reaction times before a response.

Restle considered that the number line was divided into segments of different sizes, as seen when reaction times and error scores increased as the size of the digits increased. Yet when the number 100 was included in the problems, the reaction times and error scores decreased. This was interpreted in terms of the number line containing markers so that a rapid comparison could be made with other digits in the problem. The sizes of the segments may vary by 10s, by 5s, and by 1s, therefore making it easier to compare two small numbers that were distant from each other on the number line, for example 2 and 9 as compared to 16 and 17. It was suggested that this was because the comparison of 2 and 9 involved a smaller number of sub division as only units are considered as compared to 16 and 17 where tens plus units had to be considered. The distance to cover on the number line was much shorter when considering single digits.

It appears from the above theories that the early digital and analogue theories may account for the solving of addition problems and the magnitude comparison task. The following section provides an account of the research methods used and explanations of theories of general numerical processing that provide insight into the organisation of numerical information and an elaboration of the theories that may account for the solving of addition problems.

1.6 Organisation of numerical information in long-term memory

1.6.1 Modularity

The concept of modularity has had a considerable influence on recent work in cognition and with models of numerical cognition to be discussed in Chapter 2. Modularity conceptualises the brain as being composed of different modules each supporting a different mental function. Fodor (1985) considered modularity as a part of a processing system that was solely concerned with a single function not performed elsewhere within that system. Each module engaged in its own form of processing independently of the activity in modules other than those it is in direct contact with. Modules were also distinct within the brain, so that brain injury could affect the operation of some modules while, at the same time, leaving the operations of other modules intact. One of the main sources of evidence in support of modularity comes from patients who have suffered brain injury. The consideration here is that impaired performance as a result of brain damage is reflected in the normal cognitive system having one or more damaged components. A selective deficit provides an indication as to the patterns of impaired and intact cognitive performance. This in turn can be reflected in a theory of normal cognitive functioning. The concept of modularity allows for the development of theories and for the alteration of theories dependent upon the patterns of normal cognitive performance and those deficits found in patients who have suffered from brain injury.

Warrington (1982) studied patient DRC who sustained damage to the left parietal-occipital lobes when a blood vessel in his brain ruptured. Following the injury DRC could read and write numbers accurately and was able to perform magnitude comparison accurately by judging which of two numbers was larger. Warrington tested basic addition and subtraction problems, for example, (5 + 7), finding that in tasks which required a quick response DRC was considerably slower and less accurate than a control group. DRC was able to work out the answer to problems by counting on. For example, to solve the problem 8 + 4 DRC had to count up 4 from 8. DRC was questioned regarding his difficulties in solving simple addition and subtract problems. He stated that he often knew the approximate answer to a problem but no longer knew the exact answer. He did, however, understand the basic principles of arithmetic operations and was able to give definitions for the meaning of addition, subtraction, multiplication and division.

The results were interpreted by Warrington (1982) as suggesting a distinction between knowledge of arithmetic operations (for example, knowing the definitions of addition, subtracting etc., which was intact) and the retrieving of stored knowledge of arithmetic 'table' facts such as 8 + 4, which was impaired. As a consequence of this impairment the facts had to be worked out by counting on. This research was an important contribution to the concept of modularity in relation to the study of numerical cognition. It provided some evidence for a distinction between operations involved in the solving of arithmetic problems.

1.6.2 Network theories and the problem size effect

As noted above the problem size effect refers to the finding that reaction time and errors increase as the size of the problem increases (Ashcraft 1992; 1995; Butterworth, Zori, Girelli & Jonckheere 2001). For example it will be more difficult to solve 9 + 8 than 2 + 3. Explanations for the problem size effect have suggested that it is related to the way arithmetic facts are stored in long-term memory (Groen & Parkman 1972; Ashcraft & Battaglia 1978; Campbell & Graham 1985; Ashcraft 1992; 1995). These researchers have hypothesised that arithmetic facts are stored in an interrelated associative network in long-term memory.

Ashcraft (1992) suggested that the time taken to produce the answer to a problem is dependent upon the distance into the network required to locate the answer. Therefore, the greater the distance to travel through the network to retrieve the answer the slower the reaction time.

Based on results from developmental and adult studies Ashcraft (1992) suggested that knowledge of arithmetic involved two main components – declarative knowledge and procedural knowledge. In relation to arithmetic problems declarative knowledge refers to knowledge of arithmetic in a network of stored facts; procedural knowledge refers to rules and the various procedures involved in the solving of problems, for example counting on and carrying operations in more complex problems. Aschraft hypothesised that the length of time taken to produce the answer to a problem, which involved searching and computing the answer, was dependent upon the amount of procedural knowledge that was required. The more dependent upon procedural knowledge an individual was the longer it would take to produce an answer, as it would be necessary to calculate the answer. For example, the answer to the problem 7 x 3 = could more quickly solved if the individual utilised declarative knowledge, previously learned table facts, than if table facts had not been learnt and procedural knowledge was relied upon, such as counting up to the correct answer.

According to Ashcraft (1992), as development and schooling proceed, the production of number facts involves a shift from using procedural knowledge to the retrieval of declarative knowledge. Initially young children produce answers to addition and subtraction problems by using counting procedures. In only a small number of problems do young children rely on declarative knowledge, for example in the solving of addition tied problems, 1 + 1, 2 + 2 etc. As a result of development the amount of declarative knowledge increases, so that older children become less reliant upon procedural knowledge. Reaction time studies involving children and adults showed a decrease in reaction time as an individual develops. This finding was attributed to an increase in declarative knowledge.

However convincing this theory may be, Baroody (1983) suggested that this model may underestimate the role of procedural processes in adults. The assumption has been that procedural processes are slower than declarative processes. An alternative model, according to Baroody, may be that the development of number fact retrieval is dependent on declarative knowledge but also on the development of procedural knowledge. As a child learns rules, principles and procedures they become well learnt and interconnected, with the result that they are more automatic and produce efficient problem solving and number fact retrieval.

According to Baroody (1983), this model can account for differences in reaction time in relation to factual knowledge. Ashcraft (1992) suggested that the larger the digits are, the greater the time taken to solve the problem due to the time taken to retrieve the answer from the network. Baroody suggested that time taken is dependent upon how automatic the rules and principles are.

The above view on the organization of arithmetic facts in an associative network has inspired recent research (Butterworth et al., 2001). The COMP model proposed by Butterworth et al is for the retrieval of single-digit addition facts suggesting that memory for arithmetical facts is domain-specific and reflects the numerical magnitude of the addends. To solve an addition problem the maximum (max) and minimum (min) addends are searched for such as 6 (max) + 3 (min) and 3 (min) + 6 (max). Both these problems have a single representation 3 (min) and 6 (max). In this model the crucial stage in solving addition problems is deciding which addend is the larger and the processing of the problem entails a comparison between the magnitudes of the addends. The assumption is that addition facts are stored in a network consisting of representations of the magnitudes of max and min. For a problem to be solved the numbers activate separate sets of units for max and min.

The model comprises three stages: Firstly the numbers in the problem are identified followed by finding the max and min numbers of the problem to be solved; secondly for the retrieval of the information the max and min numbers are used to access stored addition facts; thirdly an abstract representation of the answer to the problems is retrieved and the answer produced in written or spoken format.

The experimental work by Butterworth et al. (2001) tested the model with three production tasks for the addition of single-digit numbers and the production of the answer using spoken output. Firstly, there was a number naming task in which the researchers recorded the time taken to pronounce the name of the sum that included combinations from the numbers between 0 and 18. However, reaction times varied from 470 to 520 milliseconds as a consequence of reading '0' as either "nought" or "zero". Reaction times for naming the numbers were influenced by the sound of the number, and the magnitude of the number appeared not to make a contribution. Secondly, there was a number magnitude test. Participants were required to speak the larger of two single digits (the two digits were presented as addition sums that included the plus sign) for all pairs from 0 to 9. The results were consistent with those found by Moyer & Landauer (1967) who used the verification method. Thirdly, there was an addition task that involved participants saying the answer to 100 single-digit addition sums. The results were consistent with those reported by Aschraft & Battaglia (1978) and Groen & Parkman (1972) who used the verification method and with those of LeFevre, Sadesky & Bisartz (1996) who used the production method. The results showed that reaction times were longer for the larger sums and also for the larger minimum addends. Reaction times were found to be shorter for sums with the two digits the same.

A regression equation was used to assess the contribution of the three stages in the COMP model. This was achieved by taking the addition reaction times from task three and the magnitude comparison reaction times from task two together with task one, the naming task, for the larger of the two addends. The results of the regression equation showed that the tasks were good predictors in assessing the contribution of the three stages in the COMP model. The conclusion arrived at was that adults seemed to use simple fact retrieval processes with arithmetical facts arranged in a domain-specific way.

According to Butterworth et al. (2001) the above model for single digit addition problems provides evidence that numbers form a specific cognitive domain and follow distinct organization principles. The network interference theory described next, provides a different perspective on the organisation of numerical information in long-term memory.

1.6.3 Network interference theory

Campbell & Graham (1985) suggested that a number of the multiplication errors made by adults could be explained within a network interference theory. The implication was that each problem could become linked to a set of the participants' answers and that each answer produced could be linked to several problems, for example learning that 3 x 8 = 24 requires associating 3, x, and 8 to 24. Having learnt that sum, particular multiplication problem errors may be produced in subsequent problems that contain related numbers. Possible errors may include 3 x 9 = 24, 4 x 8 = 24, as one or other of these numbers have been encountered in the initial problems. According to Cornet, Seron, Deloche & Lories (1988), a further instance of confusion would be in the case of 3 x 8 = 24 which might produce errors in subsequent problems such as 6 x 3 = 24 or 4 x 8 = 24. The suggested reason for this is that 3, 4, 6, and 8 all produce the correct answer if written in the correct context. In terms of the network model the figure 24 has already been accessed and therefore has a high level of activation in contrast to the correct product of the other sums of either 18 or 32 which will have a low activation level as they have not previously been accessed.

Campbell (1987) conducted a verification study using 36 multiplication problems. The range of the multiplication problems was between 2 x 2 and 9 x 9. The 36 problems were divided into two sets, 18 in the practice set and 18 in the interference set. Analyses of practice set errors showed that 92.3% involved correct answers to other multiplication problems. Of the 92.3%, 88.5% were errors relating to the product of the same times-table. This lent support to Campbell & Graham's (1985) theory that errors were a reflection of a network interference by false association. A further finding contradicted Baroody's (1983) finding that procedural knowledge facilitated adults' performance on simple arithmetic tasks. Accordingly, if participants had relied upon rule based procedures derived from one set of problems, this knowledge should have been transferred and facilitated the problems not practised. The conclusion drawn was that reduced interference might produce faster reaction times and a decrease in the error score.

1.7 Conclusion

This chapter has provided an overview of a number of key themes associated with the study of numerical cognition. The introduction took a historical perspective followed by 20th century research that covered aspects of numerical cognition to be developed later in the thesis. Early and contemporary literature on the organisation of numerical information in memory and research methods used was reviewed. These issues are examined in greater depth later in the thesis. Chapter 2 provides a review of the key theories of numerical cognition. Chapters 8-11 includes empirical data which has a bearing on certain assertions made in each of the theories.

Chapter 2 – Models of numerical cognition

2.1 Introduction

In the previous chapter early work in this field was reviewed. The material discussed in Chapter 1 provided a historical perspective on the development of the study of numerical cognition and a number of concepts were introduced. These key concepts, for example, magnitude judgement and subitizing, are the focus of research studies to be discussed in Chapters 8-11 of this thesis. As a continuation of the early research in this field models of numerical cognition have emerged. It is important for the development of this thesis to review the models proposed in the last 20 years. The models of numerical cognition have inspired experimental research and neuropsychological case studies, and evidence for and against each model has been presented. A full review of this evidence is beyond the scope of the present thesis. The aim of this chapter is to provide an account of each of the models of numerical cognition and a summary of subsequent research developments. This is followed by an overview of the similarities and differences between the models. This account of the models will be referred to in the factor analytic study and in the experimental work.

2.2 Theoretical models of numerical cognition

The way in which numbers are mentally represented and manipulated has received recent attention. The current theories of numerical cognition vary with respect to the underlying assumptions made about the architecture of the models. Proposals that have been made relating to the organisation of number processing have led to conjectures about the effects of numerical input stimuli on the performance of arithmetic calculations. The models of numerical cognition are intended to explain ways in which individuals comprehend, represent, analyse, calculate and estimate numerical information. The numerals to be processed may be written as '1' or 'one' or spoken, with the various models providing explanations to account for the various possible systems that exist in a calculation process before the final production of an answer. Proposals have also been made relating to whether or not the processing stages occur independently from one another or are interlinked. Both modular and associative network models have been proposed. For example, McCloskey, Caramazza & Basili (1985) proposed an abstract modular model with a central abstract representation of numbers where number facts are stored in abstract format. Clark & Campbell (1991) argued against the existence of a central abstract representation and suggested that 'visuo-spatial, verbal and other modality-specific number codes are associatively connected as an encoding complex' (p.204). During calculation and retrieval this associative system is activated. Dehaene (1992) suggested that the representation of number is in three formats, verbal, Arabic and magnitude with each format resulting in specific procedures, such as number comparison or the solving of mental arithmetic problems. A contrasting approach was taken by Noel & Seron (1992) who suggested a preferred entry code model. The authors of this model suggested that there was access to number knowledge and calculation procedures which might be conducted either verbally or in Arabic depending on individual differences.

In this section existing theoretical models of numerical cognition are introduced. Descriptions and evidence in support of these models are given and the similarities and differences between the models identified.

- Abstract modular theory, (McCloskey, Caramazza & Basili 1985)
- Encoding complex approach, (Clark & Campbell 1991)
- Triple-code theory, (Dehaene 1992)
- Noel & Seron (1992) Preferred entry code model

2.3 Abstract modular theory (McCloskey, Caramazza & Basili 1985)

This theory of numerical cognition assumes the existence of an abstract internal representation mechanism and three other independent systems. The independent systems are considered to be a number comprehension system, a calculation system and a response system. The systems are assumed to be modular as the processing involved in each module can be disrupted independently by brain damage. The function performed by each module does not rely upon and has no access to the functioning of the others.

The numeral comprehension system is for encoding number input, for example, digits and written or spoken number words. The model proposed by McCloskey et al. (1985) draws a distinction within the numeral comprehension and production systems between lexical and syntactic processing. Lexical processing refers to the comprehension or production of the individual numeral in either digits or words, for example the digit 4 or the word four. Syntactic processing is required to produce an internal representation of the number words. For example, five hundred and forty is analysed to determine the word order so as to comprehend or produce a numeral as a whole. A further distinction is made within the lexical processing system for verbal numeral comprehension and production. McCloskey et al. (1985) distinguished between phonological and graphemic processing mechanisms. Phonological processing is involved when spoken number words are presented and graphemic processing is required when analysing written number words.

Prior to the calculation system, the input stimuli are thought to be converted to an abstract semantic representation which may represent the characteristics of the numeral formats. It is suggested that these representations may take the form of semantic representations, phonological representations of number words, or graphemic representations of digits that highlight a digit's abstract identity. The main assumption is that the internal representations specify basic quantities and their associated powers of 10. The calculation system includes memory for basic number facts and rules and incorporates procedures for more complex arithmetic, for example, multi-digit addition or multiplication. McCloskey et al. (1985) suggested components within the calculation system for the comprehension of symbols, for example, +, x, etc. and the words for symbols, for example plus, multiply, etc. and retrieval of arithmetic table facts and of calculation procedures. The production system recodes the abstract output from the comprehension system and calculation systems into Arabic written or spoken verbal number responses. As it is considered that calculation and response systems are utilised after the stimuli are transcoded to abstract form, then calculation and response processes are assumed to operate independently of input notation.

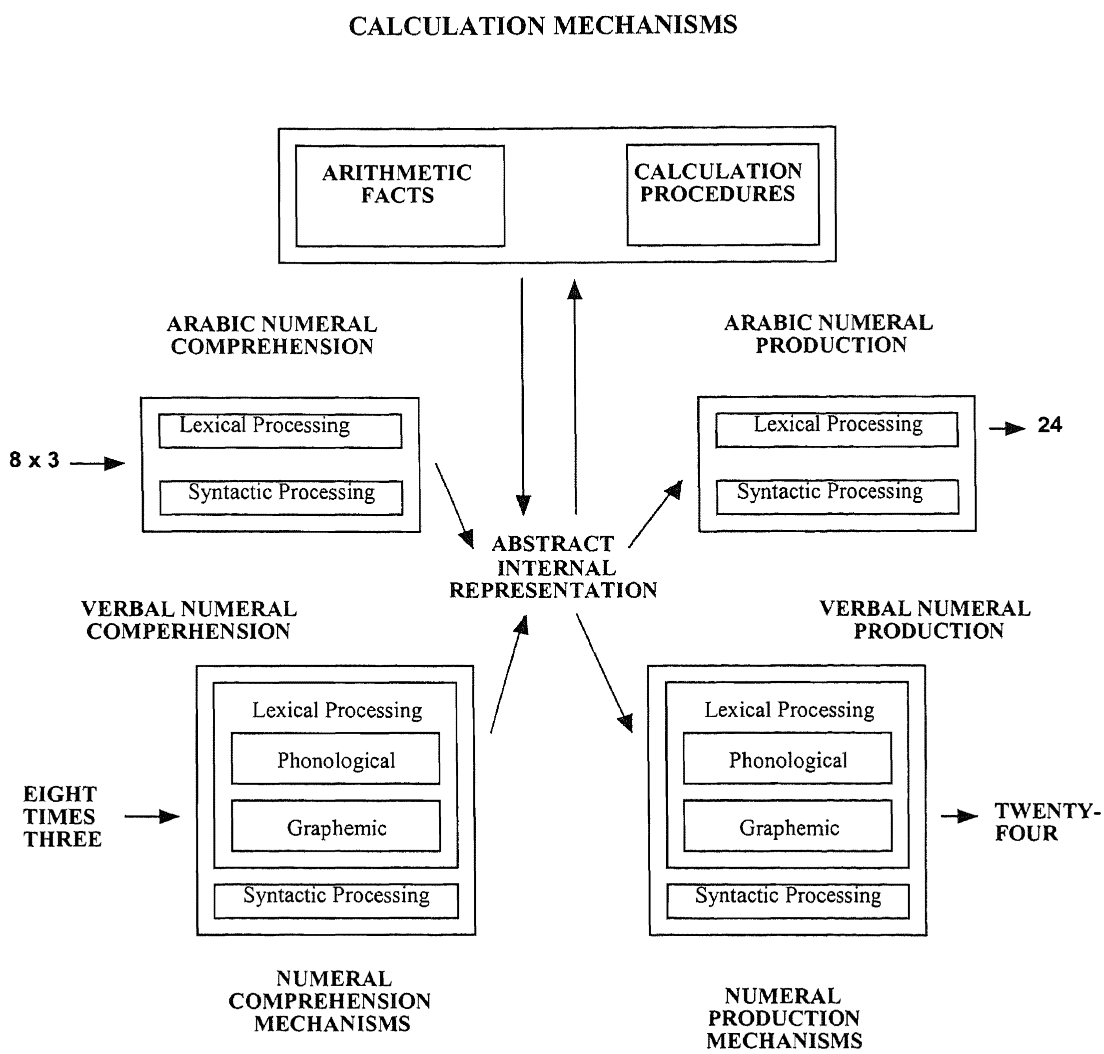


Figure 2.1 Diagrammatic representation of the processing components proposed in McCloskey, Caramazza & Basili's (1985) model of numeral processing and calculation. (After McCloskey 1992).

Support for this model comes from neuropsychological investigations. Sokol, McCloskey, Cohen & Aliminosa (1991) studied patient PS who demonstrated good performance on several numeral processing tasks with an understanding of how to calculate the answers. However there was a high error rate of 20% for single digit multiplication problems. This was considered to reflect a deficit in retrieving multiplication table facts. The fact retrieval deficit was also apparent when asked to calculate multi-digit multiplication problems. The conclusion drawn was that PS showed a dissociation between retrieval of arithmetic facts, which was impaired, and the execution of calculation procedures which was intact.

Dagenbach & McCloskey (1992) discussed patient RG who showed impaired arithmetic performance. Using verification and production tasks for addition, subtraction and multiplication problems a number of findings emerged. During the arithmetic production task patient RG performed considerably better in subtraction than in addition or multiplication. This suggested that stored arithmetic fact representations were possibly distinct from arithmetic operations. The results for addition, subtraction and multiplication in production tasks indicated that accuracy was much lower for problems requiring retrieval of individual stored facts, for example, 5 +7 = 12, than for problems that are solvable by reference to a general rule such as 3 + 0 = 3. This lent support for a distinction to be drawn between arithmetic facts and arithmetic rules. Finally there appeared to be a dissociation among arithmetic operations which was evident in the production tasks but was absent in the verification tasks. The most notable indication of this was that RG's verification performance was as good for addition as for subtraction. This finding is consistent with the view that production and verification tasks involve different retrieval processes (Campbell, 1987; Zbrodoff & Logan, 1990; 2000).

Temple (1989) described evidence of developmental dyscalculia in an 11 year-old boy named Paul. No head injury or serious illness was reported, but difficulties with arithmetic had been noted from an early age. A battery of tests showed that

Paul had no difficulties with literacy development and he was a fluent reader but his knowledge of arithmetic concepts and operations was poor. Through finger counting he was able to solve simple addition problems of less than ten. To try to understand Paul's difficulties a number of tests were employed which encompassed reading (both Arabic numbers and numbers written as words), writing and repetition. In all the tests 40 number stimuli were used.

The results showed that errors were of a particular format. When asked to read 34 he replied 'seventy-six', reading the number 1 the reply was 'nine' and for 8483 the reply was 'eight thousand four hundred and eighty four'. The interpretation of the observed pattern of errors was that the syntax of the number was correct in that the number selected was of the appropriate length. However, the error appeared to lie in the choice of lexical items which were embedded in these syntactical structures. This was consistent with the model proposed by McCloskey et al. (1985): the syntactic processing function was impaired. A further observation showed that there was impairment for the reading of number words and digits and a specific impairment for the reading of numeral words but not for the reading of other verbal items.

The conclusions drawn from this case study were that word reading and number reading involved quite separate components and that numeral words involved distinct components from other words. A further conclusion was that syntactic reading processes are distinct from lexical processes. These conclusions are consistent with the modular model proposed by McCloskey et al. (1985).

The neuropsychological case studies by Walen, McCloskey, Lindermann & Bouton (2002) reported below are relevant to the discussions of the models of numerical cognition. The authors related the results of the case studies specifically to two of the models discussed in this chapter. They presented two case studies of brain injured patients, KSR and JM, suffering from number processing impairments, in particular impairments associated with the inability to represent numerals phonologically. They sought to show that, contrary to superficial appearances, these case studies did not support the notion proposed by Dehaene & Cohen (1995) that arithmetic facts are stored in memory as a phonological representation of the problem. Walen et al. (2002) argued that instead, the pattern of results of these two patients was more successfully explained in terms of the abstract modular theory (McCloskey et al., 1985; McCloskey & Macaruco, 1995) which suggested that arithmetic facts are stored and retrieved in an abstract semantic representation.

The reason for the authors' suggestion, that the results from the case studies were in line with the predictions that arithmetic facts are stored and retrieved via an abstract representation of the problems, was that the patients' inability to represent numbers phonologically did not appear to affect arithmetic fact retrieval. As both patients showed relatively intact abilities to understand and produce Arabic numerals, it was considered that the patients were able to convert the problems from an Arabic form to an abstract representation which was then used to access the arithmetic fact from memory. The arithmetic fact was retrieved in an abstract semantic form, then converted into the Arabic numeral and the answer given.

2.4 Encoding complex approach (Clark & Campbell 1991)

The encoding complex approach is in contrast to the abstract modular theory. This associative network model assumes that arithmetic facts are stored in multiple representational formats and modality specific codes interconnected in a complex network. It is thought that number processing is based on visual, visuo-spatial and phonological codes which are primarily modality-specific processes. Different input numerical forms (for example, digits written as words, digits written as digits, and auditory input of numerical information) can influence the codes or strategies that are necessary for the completion of a task. A single numerical function may involve multiple associations within the network.

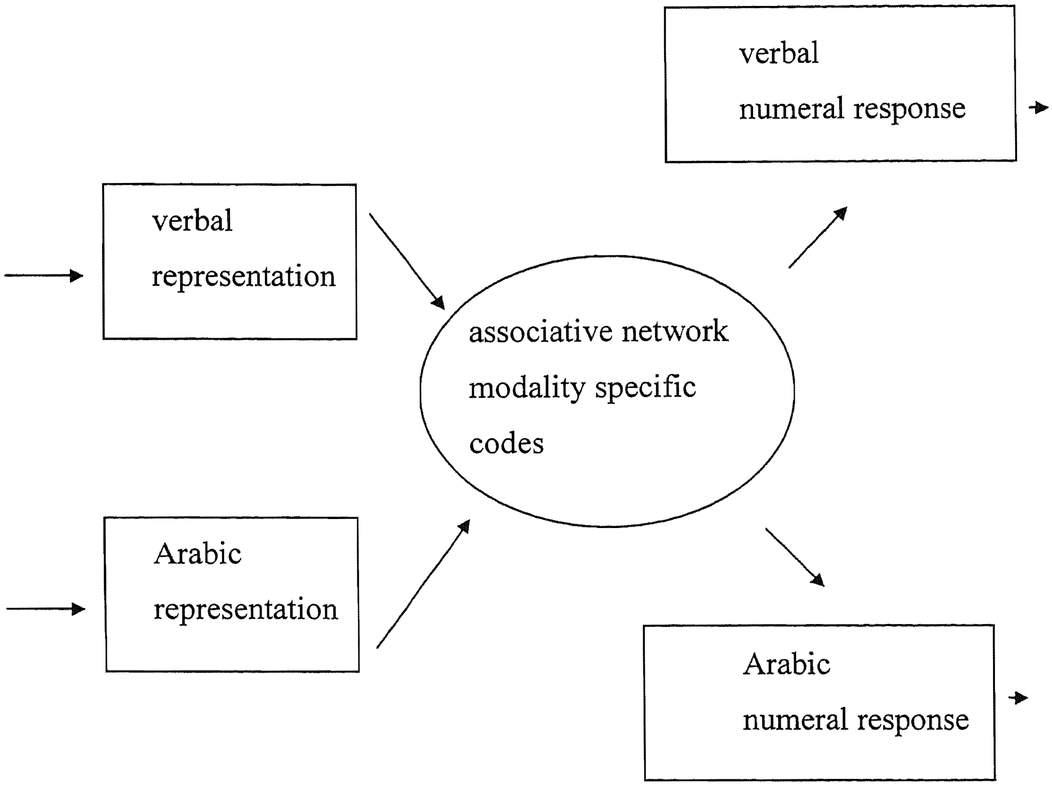


Figure 2.2 Simplified diagrammatic representation of Clark & Campbell's (1991) Encoding Complex Approach

This model is a theory of number fact retrieval and not a model of basic arithmetic skills. The assumption is that the model simulated number fact retrieval processes for single digit multiplication up to 9 x 9 and addition problems up to 9 + 9. When a problem is presented, memory codes that correspond to all the addition and multiplication facts in the network become activated. Memory for arithmetic facts was assumed to involve two codes. Firstly, there is a magnitude code that represents the approximate numerical size of the answer to a problem. This in turn primes the physical code that represents the exact answer. Secondly, there are the physical codes which are visual or verbal units associated with the operand pair, operation sign, for example, addition, multiplication etc. and the answer. The physical codes are referred to as nodes. Retrieval of an answer to a problem is thought to involve a number of cycles and on each cycle nodes receive excitatory and inhibitory input associated with both the physical and magnitude code. During the cycles the strengths of the excitatory and inhibitory inputs achieve a point where equilibrium is reached and a response to the arithmetic problem is given. This occurs when one of the nodes in the network reaches the critical threshold level of activation. It was suggested that these processes could account for a number of observations made from experimental work, for example, the problem size effect, errors and priming effects. This model reflects experimental work on number fact retrieval in normal adult participants when asked to produce correct answers to problems as quickly as possible.

Campbell (1994) conducted a detailed study to test this model in depth. The experiment consisted of eight blocks of trials, four blocks of simple addition problems alternating with four blocks of multiplication problems. In each block 36 possible problems from 2 + 1 to 9 + 9 were presented, once in Arabic digit format and once in number word format with the two formats alternating across trials. Therefore, each block consisted of seventy-two trials. Operand order was systematically varied across blocks and participants would be presented with 2 + 1 and 1 + 2 during the course of the experiment. Participants gave verbal responses with their reaction time and error scores recorded.

The results emphasised the patterns of errors across conditions, with number words producing slower reaction times and more errors than Arabic digits. Multiplication was slower with more errors than addition. Operation errors were more prevalent in the Arabic format where the participant added instead of multiplied or vice versa. There was evidence of negative priming. For example, the answer given on the previous trial was less likely to be produced on the current trial. There was also evidence of differential positive error priming where responses to number word trials were likely to intrude into future number word trials but this was not the case for Arabic digits. Campbell concluded that these results showed that specific types of error are tied to particular codes.

Campbell (1995) provided an extensive review of the literature surrounding the encoding complex approach and some applications of the model. The main assumptions of the model remained the same but a fuller discussion was given on the speed of correct retrieval across numerical information in relation to the frequenciy of specific retrieval errors. This was related to the relationship between the speed and accuracy of semantic memory retrieval. It was suggested that this revised model was able to accommodate a number of phenomena observed during problem solving for example, the problem size effect, errors and tied problems. In summary the author considered that the model provided a more detailed account of issues surrounding the processes involved in numerical facts.

Campbell, Kanz, & Xue (1999) investigated Chinese-English bilinguals across two types of numerical notations (Arabic and mandarin). According to the researchers, the way in which Arabic and mandarin notations are used is similar to the way in which Arabic digits and number words are used in many languages. This study is reviewed in greater detail in Chapter 9 where the implications of the results for the encoding complex approach are discussed.

A cross-cultural experiment by Campbell & Xue (2001) examined the role of retrieval versus procedural strategies in 72 adults. They examined the performance of Canadian university students of Chinese origin (CC), non-Asian origin (NAC) and Chinese university students educated in Asia (AC) across simple addition, multiplication, subtraction and division problems. Participants were also asked to report their strategies for solving the problems. The aim of the research was to address three issues. Firstly, the use of direct versus procedural strategies of memory retrieval. Secondly, they aimed to analyse the problem size effect and, thirdly, to examine any cross-cultural differences in simple arithmetic performance. Participants reported that they did not use calculation procedures for simple multiplication. However, the NAC group reported using calculation procedures for addition problems. The three groups reported using strategies for solving the larger, simple subtraction and division problems. Results for the problem size effect showed that the longer reaction times for the larger problems compared to the small arithmetic problems could be attributed to slower retrieval processes and slower procedural strategies for the larger problems. The authors concluded that answers to simple arithmetic problems did not always use direct retrieval from memory but involved procedural strategies.

The above research provides an example from experimental work on the performance of skilled adults in simple arithmetic. The studies have examined the representation of arithmetic information in memory and the use of direct versus procedural strategies for retrieving the answer.

2.5 Triple-code theory (Dehaene 1992)

This theory incorporates both representational and modular aspects of numerical processing. The visual Arabic number form represents numbers as strings of digits on an 'internal visuo-spatial scratchpad' (Dehaene & Cohen, 1995, p.85). The visual Arabic number form further allows for digital input and output, parity judgements and multi-digit operations. It is assumed that numbers are manipulated in a specific code in the Arabic format and that they are mediated by a specific set of numerical operations. The auditory verbal word frame represents numbers as number word sequences that can be mentally manipulated, for example four hundred and forty. The authors suggested that arithmetic facts are stored in phonological form and are accessed from memory using phonological representations of the problem (Dehaene & Cohen 1995, Cohen & Dehaene 2000). They also suggested that multiplication tables together with simple addition problems learned by rote at school are retrieved as automatic verbal associations. However subtraction problems are not learned by rote and are solved through the manipulation of numerical quantities represented as an analogue magnitude representation of the problem. Written and spoken input and output, together with counting processes, are processed by the auditory verbal code system. The triple-code model assumes that neither the Arabic number form nor the verbal word frame contain any semantic information. It is suggested that the meaning of numbers is represented in the third part of the triple-code model as an analogue magnitude representation.

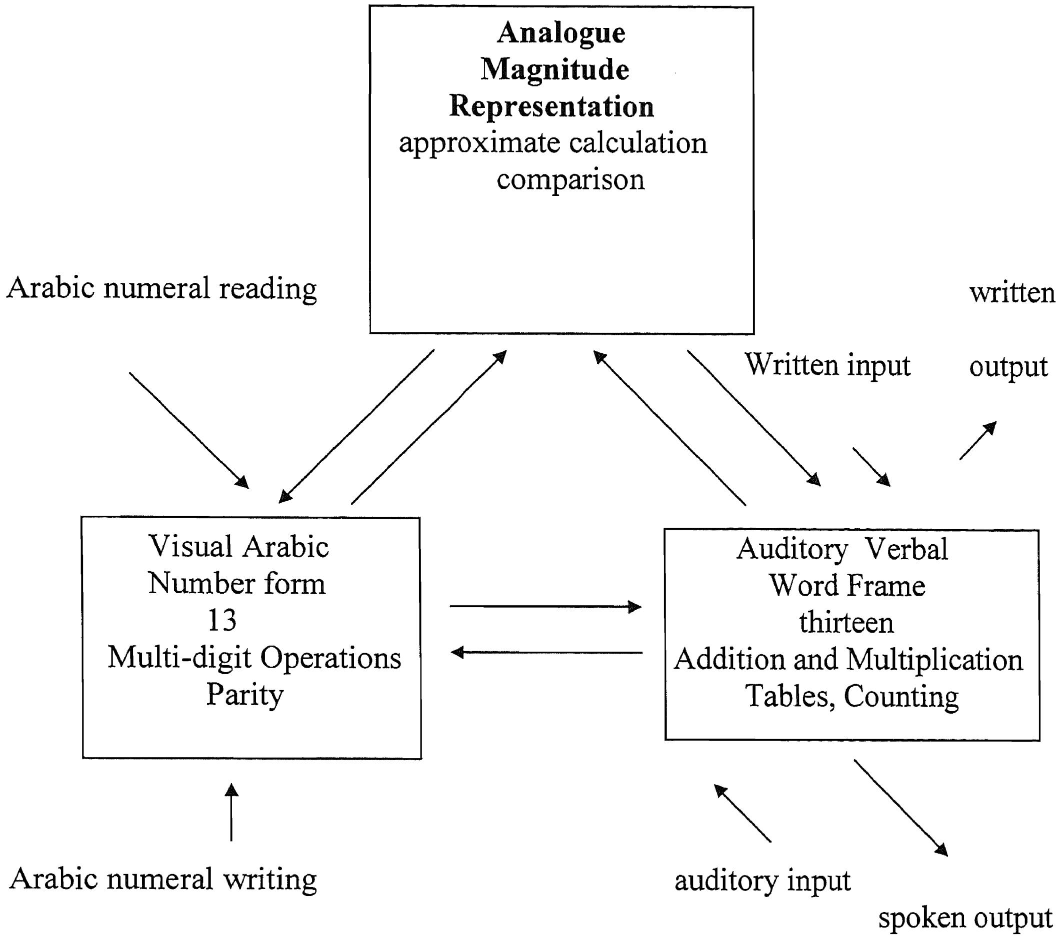


Figure 2.3 Simplified diagrammatic representation of Dehaene's (1992) Triple Code Theory

The analogue magnitude representation can be considered as equivalent to a number line. This representation lays the basis for approximate calculation and numerical size comparisons and may assist in subitizing tasks. Within the analogue representation the quantity or magnitude associated with a number is retrieved and can be put in relation to other numerical quantities. For example information that 43 is smaller than 64 and that 50 is half way between 0 and 100 is retrieved within this dimension of the model. Therefore, when numerical stimuli are presented through the visual or auditory modalities, their numerical quantities and size can be extracted and represented on the number line. Experiments with normal participants have suggested the existence of an analogue representation for numerical quantities as discussed in Chapter 1 (Moyer & Launder, 1967; Restle, 1970).

Within this model each code type is assumed to link to a specific set of numerical operations. This model assumes that the three different types of code activate one another directly and are not mediated by an abstract representational system as suggested by McCloskey et al. (1985).

As a result of the composition of the model a number of predictions can be made. For example patients may have a more severe impairment of multiplication compared to addition due to a fact retrieval deficit or greater impairment in subtraction compared to multiplication as a result of impairments to the representation of semantic information (Dehaene & Cohen, 1997). Evidence in support of the prediction that verbal associations assist in the retrieval of multiplication information comes from patients who have language deficits and impairments in multiplication. On the other hand individuals with preserved language tend to show intact multiplication skills (Dehaene & Cohen, 1991, 1997). Studies that elaborate on the predictions made by the triple-code theory are described below.

Dehaene & Cohen (1991) reported the case of patient NAU who lost all precise knowledge of numbers and arithmetic operations but was able to perform calculations after translating numerals into approximate numerical quantities. Patient NAU suffered from a lesion to the posterior left hemisphere resulting in severe alexia and speech comprehension and production deficits. In a number of tests the only knowledge NAU could access about a number was its approximate magnitude. Simple calculations produced incorrect answers, for example, 2 + 2 = 3, and during verification tasks he was unable to produce the correct reply to simple addition problems such as 2 + 2 = 5. However he was aware that 2 + 2 = 9 was incorrect knowing that 2 + 2 is smaller in magnitude than 9. Memory for a set of 3 digits was poor, for example 768, yet he was able to produce answers that were close in magnitude to the initial set of digits. His performance in simple addition problems, compared to that of a normal population, did not show the problem size effect. As the numbers in addition problems increased, his reaction times were not slower and there was no increase in the number of errors.

The conclusion arrived at was that NAU was able to produce only the approximate answer and that the degree of accuracy of the approximations fell with larger numbers. For example NAU would indicate that 43 + 21 = 69 was correct but 3 + 1 = 9 was incorrect. These findings align well with the concept of a number line (Restle, 1970; Dehaene, 1992).

Dehaene & Cohen (1995) proposed a model for arithmetic processes and neuroanatomical circuits. The components of the triple-code model were mapped onto hypothetical anatomical locations. The authors suggested that the left hemispheric visual system was able to recognise single and multi-digit numerals and printed words, the visual number component of the theory. These processes were anatomically represented in the left occipito-temporal region. The right hemisphere was also considered to identify visual symbols such as Arabic digits, some multi-digit numerals and some words. Both hemispheres were considered to process analogueical representations of numerical magnitudes and comparison processes for deciding which of two numerals was larger or smaller. It was suggested that these processes took place in the parieto-occipito-temporal area. The language areas of the left hemisphere process verbal numerals corresponding to the verbal word component. The hypothetical mapping of the components of the model to anatomical regions was supported from a number of neuropsychological case studies, for example split-brain patients (Seymour, Reuter-Lorenz & Gazzaniga, 1994), and patients with alexia (Cohen & Dehaene, 1995). The interpretation of patient NAU's deficits reported above (Dehaene &

Cohen 1995) was that NAU's preserved comprehension of numerical quantities was as a result of processing resources in the intact right hemisphere.

Seron, Pesenti & Noel (1992) investigated reports of individuals visualising numbers for calculation procedures on a visual number line. However these reports did not provide an insight into the functional or structural aspects of numerical representations. In the light of this uncertainty they studied a right-handed 17 year old male named JB. He described his visualisation of number as an infinite vertical scale made up of rectangular boxes. Each box contained one number written in digits with the positive numbers toward the top and negative numbers towards the bottom. The vertical scale was subject to the effect of perspective where numbers became smaller when located far from the viewing point. The numbers were also linked to colour with numbers written in black on a white background except for the numbers 11 to 20 and 111 to 120 which were written in white on a black background. JB also reported that he was able to ascend or descend within the scale by visualising and locating the area where the required digits fell.

Experimental work was conducted by presenting 64 pairs of numbers written in black. Of the 64 pairs 32 were pairs of small numbers from 2 to 30 and 32 of large numbers from 102 to 2500. The pairs of numbers were further subdivided with 16 pairs close to each other with a distance of ≤ 3 and 16 pairs were distant from each other with a distance of ≥ 15. When the first number of a pair was presented JB was required to press a key when he had visualised the number along his number line. The reaction time was recorded. A second number was immediately presented after his response and he again pressed a key when the number was visualised. A distraction task was used between each pair of digits to remove the previously presented digits from short-term memory.

Based on the description of how JB visualised numbers, predictions were made that small numbers would be visualised more quickly than large numbers. A further prediction was that reaction times would be less if the second number in the pair was close to the first visualised number than if the number was further away on the number line. The final prediction was that when asked to visualise two numbers in a row the ascending and descending procedures should not have any influence on the speed of processing.

The results, based on reaction time scores, indicated that JB took less time to visualise numbers between 0 and 100 than numbers above 100. This according to Seron et al. (1992) suggested that access to the number line requires two different mechanisms. Firstly, there must be a procedure to access directly the area of the number line holding numbers from 0 to 100 and, secondly, there must be a more complex and time consuming procedure required to locate numbers that are more distant.

Furthermore the results confirm that JB visualised more quickly the second numbers of close number pairs than those of distant number pairs. The result of the ascending and descending procedures within the number line indicated that ascending the scale took less time. Thus the evidence failed to confirm the prediction that the time taken would be equal for ascending and descending.

As a result of recent advances in neuroimaging methods the mapping of the components of the triple-code model has received attention. Pesenti, Thioux, Seron & De Volder (2000) conducted a PET study on normal participants to test the anatomical assumptions made by Dehaene & Cohen (1995). Cohen & Dehaene (2000) reported the case of a pure alexic patient VOL who had impaired naming and preserved comprehension abilities for numerical processing. The aim of this investigation was to try to explain the pattern of dissociations observed between the two processes, impaired naming and preserved comprehension, in terms of anatomical mapping proposed by Dehaene & Cohen (1995). The authors' investigations supported their anatomical hypothesis. This study was criticized by Pillon & Pesenti (2001). Firstly, they argued that VOL's pattern of performance could not be reliably interpreted as it was not analysed in sufficient detail and secondly, that Dehaene & Cohen did not provide an account of their data in relation to other models of numerical cognition. More recently Dehaene, Piazza, Pinel & Cohen (2003) reviewed the evidence for a subdivision of calculation processes in the parietal lobe. The data reviewed was from a number of sources, and included the bilateral parieto-occipital dysfunction seen in Turners syndrome, a PET study on five adult women having a glucose hypometabolism in bilateral parietal and occipital regions, and two MRI group studies with women who showed bilateral reduction in parieto-occipital brain volume. This review considered in detail numerical processing in the parietal lobe and the authors acknowledged that this research was inconclusive but provided a foundation for future research. A detailed account of neuropsychological evidence for neuroanatomical processes is beyond the scope of this thesis. The above studies have offered partial support for this position.

In summary the triple-code theory of number processing predicts that there are three separate systems of representation of numerical stimuli. Firstly a verbal system where numerals are represented in a similar way to words, for example, phonologically and lexically. Secondly a visual system where numbers are encoded as Arabic digits and thirdly a number line representing a nonverbal semantic representation of the size and distance between numbers.

2.6 Preferred entry code model (Noel & Seron 1992)

This model assumes that performance is based on modality-specific codes, for example, phonological and visual Arabic. Numerals are recoded from Arabic to verbal form by converting digit representations directly into word representations without the intervention of a semantic representational format. It is suggested that each individual has a preferred entry code. For example, input numerals may be converted to a verbal representation before the mental calculation is performed or they may remain in the Arabic notation format. This theory appears to highlight individual differences. Noel & Seron (1993) suggested that numerical processing may be achieved using a range of formats, but that individuals may have preferred codes for performing specific arithmetic operations, for example retrieving facts from long-term memory. Figure 2.4 below gives a diagrammatic representation of Noel & Seron's (1992) preferred entry code theory.

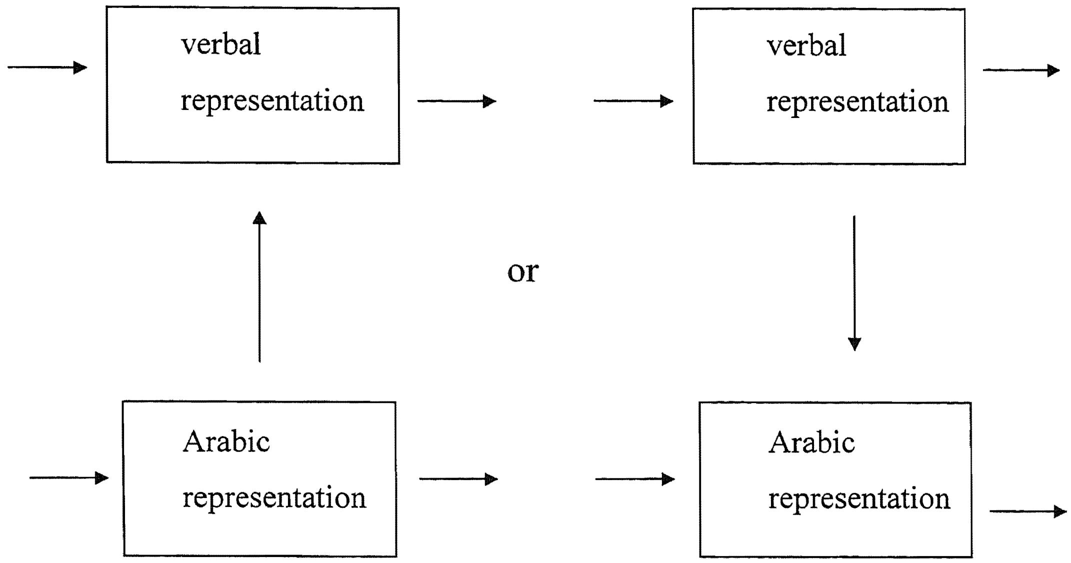


Figure 2.4 Simplified diagrammatic representation of Noel & Seron's (1992) Preferred Entry Code Theory.

Noel & Seron (1993) studied patient NR who seemed to convert all input numerals, including Arabic numerals, to a verbal representation before deciding upon the mental calculation procedures or forming a representation of quantity. In line with the preferred entry code theory it was suggested that NR accessed the semantic representation of quantities through verbal input. In the case of input in the form of Arabic numbers NR would transform the digits into a verbal input code before any required semantic representations. It was also thought that when NR was presented with three or more digits this stimulus would activate the syntactic frames of the digits which would be used to produce a verbal entry code, which would then be used to form a semantic representation.

Subsequent research by Noel & Seron (1993) focused on different aspects of the processing of numerical information. For example, they provided a review of the neuropsychological data for the representations and mechanisms involved in numerical processing (Seron & Noel, 1995). Another line of enquiry investigated was the interaction between language and arithmetic (Noel, Fias & Brysbaert, 1997; Noel, Robert & Brysbaert 1998). The results from the case studies reported by Whalen, et al. (2002), discussed in this chapter, were mapped onto the preferred entry code model. The authors suggested that arithmetic facts could be represented in a phonological format but in the cases of patients KSR and JM this seemed unlikely as both exhibited phonological production impairments and seemed to retrieve arithmetic facts using Arabic notation. It was suggested that the patients' preferred entry code for access to arithmetic facts was through an Arabic form of representation rather than phonological representations of the problem. According to Whalen, et al. (2002) there are omissions in the preferred entry code model which have not been investigated in subsequent work.

Since the development of the models of numerical cognition described above, research has continued to provide evidence for and against the theoretical framework associated with each of these models. The following section of this chapter examines in more detail the similarities and differences between the models.

2.7 Similarities and differences between the models of numerical cognition

2.7.1 The abstract-code and triple-code theories

The basic principle associated with the abstract-code model proposed by McCloskey, Caramazza & Basili (1985) and the triple-code theory of Dehaene (1992) is modularity. The abstract-code model and the triple-code theory assume three independent number processing systems. The abstract-code model assumes a comprehension system for encoding number input, a separate calculation system and an output production system. Dehaene (1992) introduced an alternative model suggesting that number processing involves an analogue magnitude representation supporting approximate calculations, numerical-size comparisons and subitizing tasks. The Arabic form mediates digital input and output, parity judgements and multi-digit operations. The auditory verbal system controls written and spoken input and output, counting processes, simple addition and multiplication facts. Both of these models suggest that number fact retrieval processes are the same irrespective of the different format of the input stimuli and they are based on the assumption that the initial representation of the problem is converted to an internal abstract code prior to calculation and response. For example, 2 x 3 = ? and two x three = ? are assumed to employ the same retrieval process. However, Campbell & Clark (1992) suggested that different tasks may use different representations of number which may vary from individual to individual. This appeared to be compatible with the preferred entry code Noel & Seron (1993). In both the abstract modular theory and the encoding complex approach calculation and arithmetic fact retrieval takes the format of an abstract representational system identified in McCloskey et al.'s (1985) abstract modular theory. However, the encoding complex approach is predominantly based on associated connections whereas the abstract representation system is one aspect of the abstract modular theory.

2.7.2 Modular versus associative network models

Different issues are continually being raised in the area of numerical cognition. One area open to discussion is whether the numerical processing system is modular or calculations are performed through an associative network. Campbell (1994) argued that digits and words elicit different degrees or strengths of activation in the same hypothetical associative network. However, McCloskey et al.'s (1985) and Dehaene's (1992) models predict independent memory processes for digits and words.

The encoding complex theory (Clark & Campbell, 1991) is in complete contrast to the modular theories and assumes an integrated network. Different forms of number input stimuli differ in their ability to activate the specific codes. For example 2 x 3 = ? would activate different specific codes as compared with two x three =. This model is without a separate central abstract representation system and a separate calculation system. It appears to rely entirely on a single network system. This model does not specify the different number representations and the way in which these representations may interact. It seems to be restricted to basic fact retrieval and automatic retrieval of previously learnt arithmetic knowledge without identifying how multi-digit problems are processed or how size comparison, subitizing and parity calculations are managed. In contrast the triple-code theory (Dehaene, 1992) attempts to explain multi-digit calculations, size comparison, subitizing and parity operations. It is suggested that the representation of the numerical input used in calculation is not abstract but is comparable to a visual image of the arrangements of the digits to be processed. This theory suggests that 'numerical quantities are represented as inherently variable distributions of activation over an oriented analogical number line' (Dehaene, 1992, p.30). An example of the concept of a number line is found when subjects are shown a pair of digits and asked to indicate which is the larger by pressing one of two buttons. Reaction times become shorter as the difference between the specified numerosities increases. For example, reaction times are shorter for numbers that are more distant in magnitude such as 9 and 3 than for numbers that are close in magnitude such as 9 and 8. In contrast to the other models that are discussed here the triple-code theory appears to account for more complex numerical processing.

2.7.3 Representation of numerical information

McCloskey & Macaruso (1995) argued that an important aspect of numerical cognition is the question of how numerical information is represented. This issue has attracted a good deal of attention, and different opinions have been expressed. Seron, Pesenti & Noel (1992) suggested that semantic representation plays a large part in numerical processing. The question then arises as to whether representations are used to perform the arithmetic operations or to aid memory in visualising the data. Representation may play a direct functional role in the processing of numbers that does not relate to the meaning of numerals. McCloskey et al. (1985) argued that numerical processing is mediated by number semantic representations. These semantic representations are thought to be reflected in the characteristics of the external numerical format, for example the phonological representations of number words and the graphemic representation of digits. These external formats are converted into internal semantic representations for further calculation, and the meaning of the number is accessed before the production of an answer. The Arabic numeral representation of, for example, 405 would be converted to the graphemic representation (4) (0) (5). It is this representation that specifies a digit's abstract identity. This is a similar concept to graphemic letter representation which specifies the abstract identity of a letter and not its visual appearance. It is from the graphemic digit representation that the digit's meaning is activated. The comprehension of Arabic digits involves firstly the identification of the digit and then the retrieval of the digit's meaning. This is in contrast to the comprehension of a written number word where there is the identification of the individual letters followed by the identification of the word as a whole and then the retrieval of the word's meaning.

Campbell & Clark (1992) suggested many different forms of internal representation were accessed via the associated network of numerical information. Accordingly the processing of Arabic or verbal numerals may activate 'visual and written codes for digits, imaginal analogue codes for magnitude (e.g. number lines), and combined visual-motor representations (e.g. counting on fingers; using an abacus)'. The representation may also include verbal codes that show 'visual and number word codes in literate individuals and unique codes in various specific groups (e.g. sign language codes for numbers)' (Campbell & Clark, 1992, p.459). They therefore considered that the processing of an Arabic or verbal numeral might activate visual or written codes for digits in various forms of representations which are interconnected in an associative network. This view is not without criticism. McCloskey & Macaruso (1995) argued that the encoding complex approach provided no specific account regarding the forms of representations that were involved in any particular task, or how that representation was used to complete a task. In contrast Dehaene (1992) proposed that 'numbers are manipulated in Arabic format on a spatially extended representational medium' (p.30). Deloche & Seron (1987) suggested that numerals were transcoded from Arabic to verbal form by converting digit representations directly into word representations without an intervening semantic representation.

2.7.4 The influence of input number notation

The above models show that there are differing opinions regarding the influence of input number notation. Dehaene, Bossini & Giraux (1993) conducted a series of experiments to investigate the numerical processing of input stimuli in the light of the different architecture of the models. The experiments were based on determining whether or not a number is odd or even.

Shepard, Kilpatric & Cunningham (1975) had suggested that there are differences in the way judgements are made about odd versus even digits regardless of whether the digits to be judged are presented as Arabic numerals, rows of dots or spoken words. Hines (1990) investigated this concept further in a series of experiments on the speed of response to odd and even digits. The results showed that judgements about even digits were processed more quickly and were more accurate than judgements about odd digits. This effect was also found when digit names were used. To judge whether a number is odd or even is known as a parity judgement. According to Clark & Campbell (1991) a mental division by 2 is performed during parity judgement: 'odd and even are in fact defined and presumably determined by numerical calculations (e.g. multiple or non-multiple of 2)' (p.210).

The experiments conducted by Dehaene, et al. (1993) were designed to investigate how parity and number magnitude are mentally represented and whether or not Arabic and verbal numerals are processed in similar or different ways. The results of a parity judgement task led them to reject Clark & Campbell's (1991) hypothesis that parity was determined by division by 2. Reaction times for single digits were fast for powers of 2 suggesting the direct retrieval of parity information from a semantic store of simple arithmetic knowledge. The result for two digit numbers suggested selective extraction of the units digit with some interference from the odd-even status of the tens digit. Further experiments examined whether similar representations were accessed when verbal numerals were used instead of Arabic numerals. Regardless of whether the stimuli were verbally presented numerals or written as Arabic input, parity judgements appeared to be executed in a similar way over the range of numbers 0 to 19. Reaction times for parity judgements were very similar with verbal stimuli as compared with Arabic stimuli. They also found that relatively small numbers were reacted to more quickly with the left hand than with the right hand. However, relatively large numbers provoked faster responses with the right hand than with the left hand. This effect is known as the Spatial-Numerical Association of Response Codes (SNARC). This SNARC effect is considered to originate from the fact that the mental number line is from left to right. So there was an association between small numbers and left hand responses and between large numbers and right hand responses. However, the SNARC effect did differ across notations with the spatial-numerical association stronger with Arabic numerals than with verbal numerals. It was considered that the representation of number magnitude was automatically activated only by Arabic numerals of relatively small magnitude.

These results according to Dehaene et al. (1993) were problematic for the encoding-complex model of Campbell & Clark (1992). It was suggested that different mental representations were activated depending upon the input notations. This therefore implied that number processing varies depending on the format of the input stimuli. This view was substantiated by results indicating that, regardless of input notation the processing of odd and even stimuli does not alter greatly. This result was in agreement with Macaruso, McCloskey & Aliminose (1993) who reported empirical evidence against the view that arithmetic facts and calculation processes vary as a function of the input number notation.

McCloskey (1992) suggested that input notation format information enters a central abstract representational system. This representational system is thought to process magnitude representations for Arabic and verbal numerals in the same way. However, the results obtained by Dehaene et al. (1993) suggested that there was a difference in reaction times depending upon notation input as shown by the SNARC effect. This implied that numerical stimuli Arabic and verbal numerals were represented differently.

In conclusion Dehaene et al. (1993) argued that the results did not support the hypothesis that input numerals are converted into a common format. If all numerals were processed in a similar way regardless of input format, then the SNARC effect should be identical in verbal notation and in Arabic notation. The results were in agreement with the triple code model proposed by Dehaene (1992).

From the above review it is clear that models of numerical cognition embody different ideas as to whether arithmetic facts are stored in an abstract format or in modality specific formats, for example, digits, or as a verbal sequence. McCloskey et al. (1985) proposed a central abstract representation system that was accessed in all calculation processes. Regardless of input format numerical information was converted into abstract representations that specified the magnitude of the number that was then processed by the calculation system. An alternative to the McCloskey et al. (1985) model was the encoding complex approach (Clark & Campbell 1991, Campbell & Clark 1992, Campbell 1994). This theory assumed that multiple codes, for example, phonological and visual, were involved in numerical processing. These codes become activated when numerical operations are performed. Dehaene 1992, Dehaene & Cohen (1995) suggested a triple-code model of numerical processing comprising a visual Arabic number code, an auditory verbal code and an analogue magnitude representation. The auditory verbal code was assumed to mediate verbal input, output, counting and previously learnt arithmetic facts. It was suggested that multiplication tables were stored as verbal associations. Noel & Seron (1992) proposed the preferred entry model. From their observations of neuropsychological case studies it was proposed that numerical operations might be achieved by using different formats. Participants may prefer a particular input format, such as verbal or auditory, in order to access a number's meaning and to then perform the numerical task.

2.8 Conclusion

This chapter has analysed the models of numerical cognition and provided a comparison between the models. These models have made significant contributions to the study of numerical cognition and have inspired a number of research studies that have examined various hypotheses in relation to the models. This analysis of the models suggests that there is no general consensus, for example, as to the format and representation of arithmetical facts. The results of the factor analytic study and the experimental work are analysed in relation to the models of numerical cognition and associated research material.

Chapter 3 reports the involvement of working memory in the study of numerical cognition. The association between working memory and numerical cognition is explored in the factor analytic study. The contribution of the visual spatial sketchpad component of working memory to numerical processing is studied in detail in Chapter 11.

Chapter 3 – Working memory and numerical cognition

3.1 Introduction

The notion of working memory refers to a limited-capacity system that is considered to be a centre for conscious activity and associated with memory, perception and attention. The system provides temporary maintenance and manipulation of information necessary for the execution of cognitive tasks. Miyake & Shah (1999) have provided an overview of the various models of working memory which have been proposed across a number of years. For the purpose of this thesis the working memory model (Baddeley & Hitch 1974; Baddeley 1986; 1996; 2000) will be used. This tripartite model is comprised of a central executive, and two slave systems – the phonological or articulatory loop and the visuo-spatial sketchpad. This chapter gives brief descriptions of the components of the working memory model followed by examples from research of the involvement in mental arithmetic of the components of the working memory model. The adaptation of the working memory model proposed by Ashcraft (1995) integrating mental arithmetic processes in relation to the components of the working memory model is discussed. The final part of this chapter investigates the association between the model of working memory and the models of numerical cognition discussed in Chapter 2.

3.2 Working Memory Model (Baddeley 1986)

3.2.1 Central Executive

The central executive is thought to be of limited capacity and responsible for reasoning, decision making, complex processing and problem solving, attention, and retrieval of information from long-term memory (Baddeley 1996). It is also concerned with the attentional control of information to be processed further in one or other of the systems. According to Baddeley (1992) most of the functions of the central executive are concerned with the attentional control of action. Accordingly, Baddeley considers the functioning of the central executive to be very similar to Norman & Shallice's (1986) Supervisory Activating System. Within this system an individual's responses can be controlled in two different ways. Firstly a large number of responses are under automatic control producing well-learned habitual patterns. Secondly there is an attentional controller which is capable of overriding habitual response patterns to initiate new behaviour. More recently Baddeley (2002) has introduced a third slave system termed the 'episodic buffer', that can be considered as an element of the original central executive. This buffer is thought to act as a temporary storage system capable of holding information from the phonological loop and visuo-spatial sketchpad and long-term memory. It is controlled by the central executive and can be accessed through conscious awareness. The cognitive processes associated with this component remain unclear and research continues (Baddeley, 1997; Miyake, Friedman, Emerson, Witzki, Howerter & Wager 2000; Baddeley, 2002).

3.2.2 Phonological or articulatory loop

The phonological loop is thought to be made up of two subcomponents. The first is a phonological memory store which is able to hold traces of speech-based material. However traces of the material are assumed to disappear within approximately two seconds unless they are rehearsed. The second subcomponent involves the process of articulatory sub-vocal rehearsal. This process can maintain the memory trace for speech-based material and material that is presented visually by subvocal repetition. The phonological store appears to be directly accessible either through auditory spoken information, which gains automatic access to the store, or indirectly by the articulatory coding of visually presented material.

Evidence for the role of the articulatory loop in short-term memory comes from studies investigating articulatory suppression. Baddeley, Lewis & Vallar (1984) found that when participants were required to suppress subvocal rehearsal by repeating an irrelevant sound such as the word 'the', immediate memory span was reduced. However, the effects of irrelevant speech and acoustic similarity were not present when the stimuli to be remembered were presented visually and articulation was suppressed.

Research has also investigated the word length effect. Words that take a longer time to repeat also take a longer time to rehearse so that less will be recalled. This phenomenon is seen with both visual and auditory presentation of stimuli and has been interpreted as reflecting the functioning of subvocal rehearsal. (Baddeley, Thomson & Buchanan 1975).

This component of the model has been extensively researched. The above studies provide examples of how the processes link to the functioning of the phonological loop. Baddeley, (1986) and Baddeley & Logie (1992) have published detailed reviews of the literature on the phonological loop.

3.2.3 Visuo-spatial sketchpad

The visuo-spatial sketchpad specialises in the visual and spatial coding necessary for recalling information relating to the spatial relationships between items. According to Logie, Gilhooly & Wynn (1994) this component of working memory may comprise two subsystems. The first subsystem may hold visual material, for example colour and shape, and a second subsystem may contain spatial information such as movement and the relationship to objects in space. According to Logie & Marchetti (1991) there is some evidence that temporary storage of visual information is disrupted by irrelevant visual input but there is no disruption if the concurrent task involves a motor process, such as hand tapping. In contrast spatial material seems to be disrupted by concurrent arm movement but not by concurrent irrelevant visual input. This component of the working memory model is researched in greater detail in Chapter 11.

3.3 The relationship between numerical cognition and working memory

The role of working memory in numerical cognition is a research topic that has aroused recent interest (e.g. Fürst & Hitch 2000; Seitz & Schumann-Hengsteler

2000; Noël, Désert, Aubrun & Seron 2001). However, the evidence for the involvement of the components of the working memory model in arithmetic operations remains scarce. The purpose of this section is to explore issues around long and short-term memory and to discuss the role of the three components of the working memory model in arithmetic. This discussion highlights the complex interlinking of the components and their involvement in solving arithmetic problems.

3.3.1 The role of long and short-term memory

Early research by Hitch (1978) reported participants' performance when given oral presentation of multi digit addition problems, for example 425 + 63. Speed of response to write their answers and error scores were recorded. Answers in one condition were to be written in the order of units, tens, hundreds and in another condition to be written in the reverse order, hundreds, tens and units. The aim of the reverse order of producing a response was to test the hypothesis that interim results are held in working memory. Within the condition there was the presence or absence of carrying operations and the position of the carries, whether it be on the units or the tens.

The results indicated that errors increased as the number of digits to be held in working memory increased. There also appeared to be an increased length of time when participants were required to write their answer down in reverse order, moving from the hundreds to the units with this delay increasing the number of errors. It was also found that as the carrying operations increased so did the error scores.

Research conducted by Nairne (1983) investigated participants' performance when asked to count backwards aloud beginning from 100 and ending with 0. Systematic errors were observed for numbers when the first and second digits were the same, for example, 99, 88, 77, etc. and for numbers that were decade numbers 90, 80, 70, etc. The possible assumption for these types of errors was that participants tended to control their position in the sequence by monitoring the second digit, for example, twenty-*five*, twenty *four*, twenty *three*, and so on. It may be, therefore that the second digit was the most important digit which provided the participant with the relevant information for the duration of that decade with the prefix ninety, eighty, seventy being of no relevance during the time that it was required. Nairne (1983) suggested that during counting participants rely on long-term memory for the appropriate counting sequence and on short-term memory to check their position in the sequence to determine which digit had just been said, and then to follow on with the next digit in the sequence.

These two studies examined the memory processes involved in the solving of arithmetic problems. The findings provided evidence to suggest that giving the answer to arithmetic problems involves a number of processes that utilise long and short-term memory. The research outlined below provides a more specific indication of the possible involvement of the various components of the working memory model in arithmetic.

3.3.2 The role of the central executive

Logie, Gilhooly & Wynn (1994) considered that the dual task methodology had much to contribute to the development of our understanding of working memory. Their view was that it is a successful method for identifying which components of working memory are involved in performing cognitive tasks. The rationale of this approach is to identify secondary tasks that have been shown through empirical studies to place large demands on the individual components of working memory. Participants are then asked to perform a concurrent cognitive task. From the results it is possible to assess the pattern of impairment or intact performance on the cognitive tasks. The resulting pattern allows the researcher to identify which, if any, of the components of working memory are involved in performing the cognitive task.

Using the dual task method Logie, et al. (1994) conducted experimental work to examine the functioning of the central executive in relation to the solving of complex arithmetic problems. The experiments also investigated mental addition performance with a range of concurrent secondary tasks to determine the nature of the various components of working memory.

The addition task involved participants adding together a series of two-digit numbers, for example, 13 + 18 (31) + 13 (44), to a maximum of six, two-digit numbers. The numbers were verbally presented and only the final answer to the addition of all the digits was reported aloud. The arithmetic task was accompanied by concurrent secondary tasks. Firstly, articulatory suppression was used to examine the role of subvocal rehearsal in mental arithmetic. Participants were asked to repeat the word 'the' once every second whilst calculating the answer to the number sequence until the final answer was produced. Secondly, random generation of letters was used to determine the role of the central executive. Participants were asked to say aloud a letter of the alphabet at a rate of one per second. They were discouraged from spelling out a word or producing the letters in alphabetic order. The random generation of the letters was recorded to determine exactly how random the production of them was.

An irrelevant pictures task was used to determine the involvement of visual imagery on mental arithmetic. In this task the participants were required to look at a screen where line drawings of objects and animals were projected. The instruction was for the participants to look at the pictures whilst adding the digits.

A hand movement task was designed to test whether mental arithmetic involved the spatial manipulation of images. Participants were required to press buttons in a specific sequence. However the participant's hand and the buttons were obscured from the participant's view to ensure the use of spatial rather than visual imagery. The rate of pressing the buttons was recorded.

The results indicated a serious disruption of performance in the mental arithmetic task when accompanied by the random generation of letters. This was interpreted in terms of the involvement of the central executive in the addition of arithmetic problems, as it was assumed that the random generation of letters also utilised central executive functioning. Articulatory suppression also appeared to disrupt the mental addition process. However, there appeared to be no disruptive effect from the concurrent hand movement or irrelevant pictures tasks. Logie et al. concluded that the phonological loop seemed to be responsible for holding the initial information of a sum and maintaining a running total. They considered that the central executive was involved in calculation and estimation procedures.

Kaufmann (2002) reports a case study of a 14-year-old boy (patient MO) suffering from dyscalculia. Patient MO showed great difficulty in retrieving arithmetic facts thought to be stored in long-term memory as a result of rote learning of arithmetic facts at school. Performance on non-numerical tasks was in the average range. The aim of this study was to examine MO's fact retrieval deficit during addition, subtraction, multiplication and division problems. The results showed relatively good retrieval for addition and subtraction facts with poor performance in retrieving multiplication and division facts. Other case studies have found similar results (Hittmair-Delazer, Semenza & Denes, 1994; McCloskey, Caramazza & Basili, 1985) and the opposite pattern which also indicated a dissociation (Dehaene & Cohen 1997). The authors suggested that MO's pattern of performance showed a preserved function of the phonological loop but a central executive deficit. It was suggested that to solve addition and subtraction problems fewer working memory resources were required, for example, there was less emphasis on central executive processes during addition and subtraction than was required for solving multiplication and division problems. The role of the central executive continues to be investigated through experimental studies and case studies. In the programme of research described in this thesis the specific role of working memory in mathematical operations is studied further.

3.3.3 The role of the articulatory or phonological loop

Logie & Baddeley (1987) used the dual task method to investigate the role of articulatory suppression in counting. In this research participants were asked to count the number of dots in a random set or to count the number of times a square appeared at irregular intervals on a screen. Participants were required to respond with the total answer. This counting task was conducted using concurrent secondary tasks, articulatory suppression, unattended speech and a hand movement task. As the target stimuli were interspersed with the concurrent secondary task, participants had to hold a running total before producing the final answer. In terms of counting errors, articulatory suppression produced the most disruption, irrelevant speech when it was phonologically similar to the target stimuli showed some disruption. However, the concurrent hand movement task caused no disruption to the counting.

The finding in that study, that more errors were reported under articulatory suppression, lent support to the hypothesis that the articulatory loop has a role during counting. According to Baddeley (1986) articulatory suppression may prevent a strong trace of the stimuli from being registered within the phonological store but it is still able to allow a weak trace to be registered, with the strength of the trace dependent upon the phonological characteristics of the digits or words. This interpretation would explain why articulatory suppression does not totally disrupt performance but merely impairs it.

Butterworth, Cipolotti & Warrington (1996) studied patient MRF who suffered from expressive dysphasia. Despite having a reduced digit span of 3 with apparently very little ability to rehearse the number to be remembered, he was able to calculate complex, auditorily presented multi-digit addition and subtraction problems such as 173 + 68. Patient MRF should not have been able to calculate the problems if it was considered that there was damage to the phonological loop. However, he did have a digit span despite its reduction from the usual 7 plus or minus 2 (Miller, 1956) and as a pharmacist it could be assumed that he was very familiar with calculating quantities and remembering formulas. Therefore he may have been more dependent on information stored in long-term memory and less dependant on short-term memory processes. This study raises issues relating to the cognitive processes involved in arithmetic calculation and in particular working memory resources.

Lemaire, Abdi, & Fayol (1996) conducted a verification experiment using simple addition and multiplication problems. In the control condition only the arithmetic problems were presented and in the experimental conditions concurrent secondary tasks were used. The participants were presented with, for example, 8 + 4 = 12 and had to respond 'true' or 'false' by pressing the appropriate key. The concurrent articulatory suppression task was designed to interfere with the phonological loop and a random letter generation task was considered to interfere with the central executive. Lemaire et al. (1996) found that random letter generation hampered the performance of both true and false answers but the most interesting result was that articulatory suppression only seemed to have an effect on the verification of true problems and not the false problems. From the results it was suggested that the phonological loop was involved only in verifying true problems and not false problems. The authors concluded that the apparent disruption in verifying true problems was because correct basic arithmetic facts were initially learned as a result of oral repetition. When participants did not recognise a problem from memory (because the answer was, in fact false), they had to employ other processes to check it and went beyond the phonological loop.

De Rammelaere, Stuyven & Vandierendonck (1999) replicated the above study using only addition problems to include 24 out of 64 possible single digit sums in the range of 2 to 9. Modifications were made to the experiment, in particular to how the researchers devised the answers to the false problems, and a new secondary task was introduced that required participants to tap randomly at different time intervals so as to produce a rhythm. Analysis of the results showed that random letter generation and tapping at different time intervals interfered with the verification of both true and false problems. The researchers also found that articulatory suppression had no effect on false problems. These results are consistent with those of Lemaire et al. (1996).

De Rammelaere, Stuyven & Vandierendonck (2001) further explored the effects of articulatory suppression on the verification of complete sets of 64 addition and multiplication problems. The concurrent secondary tasks were the same as used in the above experiment. The findings from the results suggested that the phonological loop was not involved in the verification of false arithmetic problems or in the verification of true problems – a result that was inconsistent with earlier findings.

The above accounts provide examples of investigations of the role of the articulatory loop in arithmetic. From the results of these studies there appear to be conflicting outcomes. These inconsistencies may be dependent upon the design of the experiments used, for example, there were variations in the type of secondary tasks the participants were required to undertake. Another possible factor is the number of arithmetic problems presented to the participants which varied between experiments, for example, all possible combinations from 2 to 9 or a selection from this range were used. The calculation of 'false' answers to problems varied between the experiments. A further possibility is whether the arithmetic task required a counting procedure or was a verification task. It seems likely that variable tasks, primary and secondary, have contributed to inconsistencies in experimental results. The following section provides examples from research of the involvement of the visuo-spatial sketchpad in arithmetic.

3.3.4 The role of the visuo-spatial sketchpad

Smyth, Morris, Levy & Ellis (1987) suggested that the addition of numbers might involve the use of the visuo-spatial sketchpad as the manipulation of digits might produce visual images that are organised into spatial arrangements.

Heathcote (1994) conducted dual task experimental work to investigate the role of visuo-spatial working memory in the mental addition of multi-digit problems. The problems were presented in both the visual and auditory modalities. Participants were required in one condition to produce only the final answer and in another condition to produce partial answers following carrying operations and then the final answer. The first experiment required participants to add mentally two three-digit numbers, for example, 346 + 457. The problems either involved no carrying operations or carrying in both the units and tens columns. The concurrent secondary tasks tested for the effects of visual interference, spatial interference and articulatory suppression. The results produced evidence that during the condition where participants were required to produce partial answers following the carrying operations both spatial and visual interference disrupted performance.

The second experiment used both lateral and columnar presentation of the addition problems, with concurrent secondary tasks to determine the nature of the spatial organisation of multi digit addition problems within working memory. Heathcote (1994) argued that in mental arithmetic visual imagery acted as a substitute for external visual cues that would normally be generated using paper and pencil. They considered that the digits were represented on the visuo-spatial sketchpad in a columnar spatial arrangement.

The final experiment considered the role of visual processing to determine whether information relating to the addition problems retained the visual features of physical digit stimuli in a visual Arabic numeral code. The problems were again the addition of two three-digit numbers but in this case the numbers were either similar or dissimilar. The permutations for visually similar numbers were taken from 5, 6, 8, 3, 9, and the dissimilar numbers from 1, 2, 4, 7, 9, 6. The only concurrent secondary task used here was articulatory suppression and a control condition of no suppression.

The results showed that there were significantly more errors when problems consisted of visually similar digits than when problems included only visually dissimilar digits. The similarity effect was more pronounced in the visual presentation of problems than in problems that were presented auditorily. This suggested that there was a greater dependence on visual working memory in the mental addition of problems presented in the visual modality. The results lent support to the notion that information to be manipulated was retained in working memory in a form which utilised the visual characteristics of digit stimuli. According to Heathcote the articulatory loop and the visuo-spatial sketchpad were used for the solving of multi-digit addition problems. The articulatory loop assisted in the retaining of the initial problem information and the partial results by rehearsal of the information and was also necessary in refreshing images stored in the visual spatial sketchpad. This according to Heathcote did not rule out the role of the central executive which appeared to be involved in the retrieval of numerical facts from long-term memory and also played a part in the choice of problem solving strategy to be employed. As the short outline of the role of the visuo-spatial sketchpad in arithmetic performance indicates there is the possibility of methodologically interesting ways of studying this component of the working memory model. The role of this component in arithmetic is investigated further in Chapter 11.

3.4 Implications for the models of numerical cognition

The above research examples provide support for the association of simple and complex mental arithmetic to working memory. Ashcraft (1995) details particular aspects of the working memory model in relation to arithmetic problem solving processes. The predictions and speculations proposed by Ashcraft (1995) are outlined below followed by recent research studies in this area.

3.4.1 Ashcraft's (1995) adaptation of the working memory model

The diagram shown in Figure 3.1 proposed by Ashcraft (1995), integrates the working memory model and predicted numerical calculation processes in relation to the three components of working memory. The italicised entries and asterisk entries in the diagram have been included by Ashcraft (1995) on a speculative and predictive basis.

From Ashcraft's (1995) application of the working memory model to numerical processing a number of possible predictions can be made. Ashcraft suggested that for solving problems requiring the carrying or borrowing of figures both the central executive and the visuo-spatial sketchpad are involved. A further suggestion was that the central executive requires the visuo-spatial component to hold a visual image of the problems to be calculated.

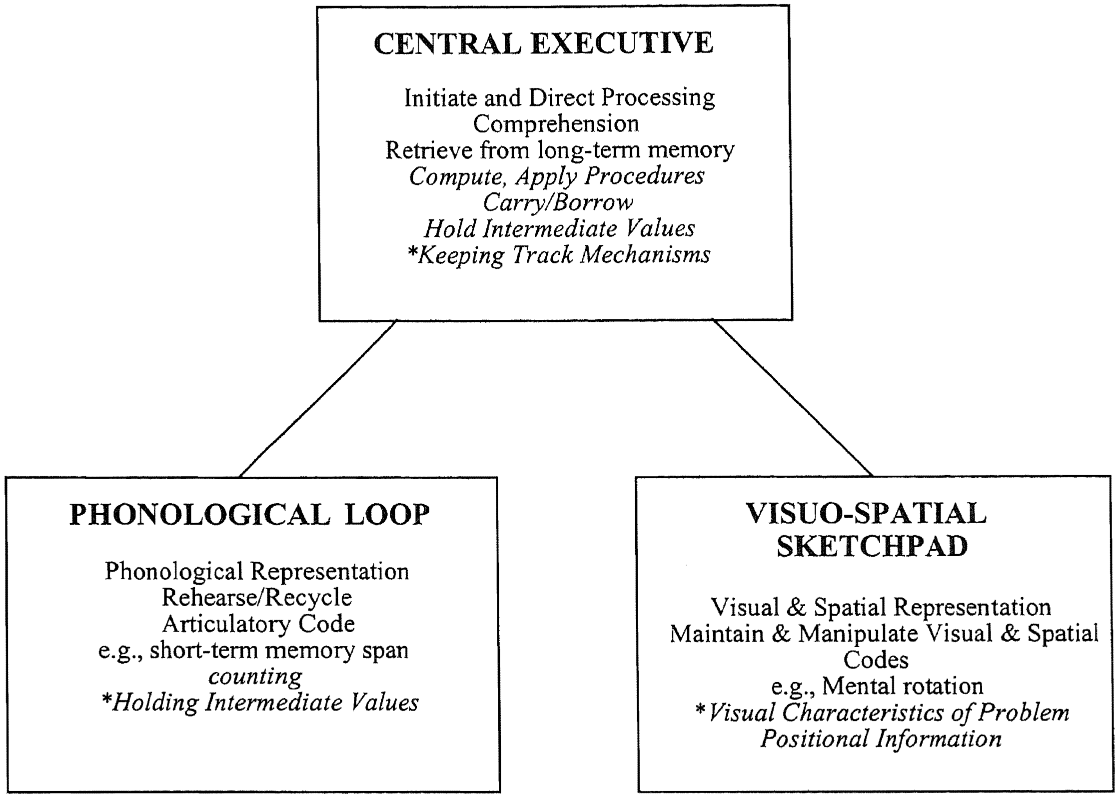


Figure 3.1 Ashcraft's (1995), adaptation of Baddeley's (1996) model of Working Memory (normal typeface) to effects in mathematical cognition (italicised); entries preceded by an asterisk represent predictions.

Ashcraft (1995) suggested that for the addition of numbers the central executive component retrieves necessary information from long-term memory and then manipulates this number and factual information. If the information to be retrieved is less well known so that the strength of activation in long-term memory is lower, this will utilise more central executive resources. Furthermore if retrieval of information is accompanied by a secondary task also requiring central executive processing, there may be an interference with normal retrieval operations.

The phonological loop, according to Ashcraft (1995), is involved in the holding of numerals to be counted. However, it is suggested that the central executive plays a role in maintaining the correct position within the counting sequence. The role of the visuo-spatial sketchpad in arithmetic is at present speculative. Ashcraft considered that the visuo-spatial component might be utilized for multi-digit problems that involve, for example, the addition of columns of numbers. It is unclear as to the role of the various components as it is thought that the central executive plays a role where carry operations are necessary. However, it is thought that there may also be a visual or spatial requirement for the execution of carry operations as multi-digit problems may be represented visually in columns in working memory. Empirical reports have supported Aschraft's formulation, for example, Baddeley (1996).

The capacity of co-ordination of visuo-spatial and verbal information was studied using a dual task method that combines a tracking task and an auditory digit-span task (Baddeley 1996). It was found that performance in the simultaneous execution of the tasks was impaired in patients who presented the dysexecutive syndrome. According to Baddeley (1996) the results suggested that impairment in simultaneous performance in the verbal and the visual tasks reflected a common contribution of the central executive plus utilization of the phonological loop and the visuo-spatial sketchpad. The requirement to perform two tasks simultaneously might involve resources other than those required for the same tasks executed separately. Furthermore, participants' performance under dual task conditions might be impaired because of heavy demands on cognitive processing in the single task condition (Logie, Gilhooly & Wynn 1994). Studies of performance in simultaneous verbal tracking tasks have shown that errors in tracking were a direct function of the difficulty of the verbal task. This was particularly shown when recall rather than encoding was involved (Johnston, Greenberg, Fisher & Martin 1970). These findings support the integration of the processing roles performed by the components of the working memory model to Aschraft's predictions.

Seitz & Schumann-Hengsteler (2000) investigated the role of working memory in mental multiplication. Two experiments were conducted using the dual task methodology. In the first experiment they studied the effects of spatial tapping (visuo-spatial sketchpad) and articulatory suppression (phonological loop) on simple multiplication (4 x 5 =) and complex multiplication (7 x 32 =) problems. Participants were required to solve the multiplication problems mentally and produce their answer aloud. The results for simple multiplication showed that there was no interference effect from the secondary tasks for the easy problems. Seitz & Schumann-Hengsteler (2000) suggested that visuo-spatial and phonological processing was not used in solving simple multiplication problems. The results on complex multiplication problems showed no interference effect for the visuo-spatial sketchpad but an effect was found during articulatory suppressions. The authors concluded that the phonological loop was involved in solving complex multiplication problems.

The second experiment focused on the role of the central executive in mental multiplication. Articulatory suppression with a concurrent central executive task (random letter generation) was used together with the same experimental conditions included in the first experiment. The results of the second experiment showed that both articulatory suppression (phonological loop) and random letter generation (central executive) impaired performance on the complex multiplication problems. The conclusion suggested by Seitz & Schumann-Hengsteler (2000) was that phonological and central executive processes are involved in solving complex multiplication problems as Aschraft had hypothesised.

Fürst & Hitch (2000) investigated the role of the phonological loop. They studied the contribution of the phonological loop in relation to solving complex addition problems. In the first experiment, articulatory suppression, participants were required to add two three-digit visually presented numbers (for example, 432 + 261). The problems were either presented for 4 seconds or until the answers were given. Participants were required to write down the answers from units to tens to hundreds. The results showed that articulatory suppression interfered with the solving of the problems when they were presented for 4 seconds but not when the problems remained until the answer was produced. This resulted suggested that the phonological loop is involved in the retention of digits during complex addition.

In the second experiment Fürst & Hitch (2000) used the Trails task to investigate central executive processes and the solving of complex addition problems. The nature of the arithmetic problems varied so that problems contained no carries (e.g. 235 + 133), one carry (e.g. 438 + 253) or two carries (e.g. 279 + 167). The secondary task consisted of a random letter and a random day of the week and participants had to continue the series by alternating between the letter and the days (e.g. B –Thursday, C – Friday). The findings suggested an interaction between executive load and carrying operations. The authors concluded that the central executive was involved in carrying operations, which relates to the predictions that Aschraft made.

Noël, Désert, Auburn & Seron (2001) investigated the involvement of the phonological loop and visuo-spatial sketchpad in the addition of two three-digit numbers. In this experiment the phonological and the visual similarity of the addends was varied. The findings suggested that it was harder for the participants to calculate the problem when the sum of the two addends was phonologically similar. However, visual similarity had no effect on the calculation performance.

Noël et al. (2001) proposed a possible association of the components of working memory to the triple-code theory (Dehaene, 1992). They considered that simple arithmetic facts that had been learnt by repetition for example, simple additions of 1 + 1 to 9 + 9 and multiplications of 1 x 1 to 9 x 9 were stored in a verbal format and that the transforming of visual Arabic stimuli into a phonological representation could be made through the artiulatory loop. They also considered that multi-digit calculations involved the mental manipulation of a spatial image of the digits and the retrieval of intermediate results. On this basis it was suggested that both the phonological loop and visuo-spatial sketchpad would be utilised. The phonological loop would be used for retrieving the simple arithmetic facts, as described above and the visuo-spatial sketchpad for keeping track of the spatial layout of the problems.

It appears from the above mentioned literature that there is evidence to suggest that working memory is modality and format specific. The disruptive effect of concurrent articulation on mental addition found by Hitch (1978) and the result on counting tasks reported by Logie & Baddeley (1987) suggest the necessity of phonological working memory codes in number processing. The results from research presented in this chapter seem to lend support for Ashcraft's adapted model. For example, research by Heathcote (1994) provided support for visuo-spatial working memory in the calculation of multi-digit problems. Seitz & Schumann-Hengsteler (2000) found from their research that the phonological loop assisted complex multiplication problem solving by keeping track of intermediate results of calculations. Research by Fürst & Hitch (2002) provided support for Ashcraft's view that the central executive is involved in carrying operations. However the role of the visuo-spatial sketchpad in complex arithmetic seems less clear and the results of the research by Seitz & Schumann-Hengsteler (2000) do not seem to support the prediction made by Ashcraft, that the visuo-spatial sketchpad is involved in identifying the 'visual characteristics of problems' and 'positional information' (Ashcraft 1995, p.17). The role of the visuo-spatial sketchpad in complex arithmetic is the focus of experimental work reported in Chapter 11.

3.4.2 Encoding complex theory (Clark & Campbell 1991),

According to Clark & Campbell (1991) the evidence for modality specific number codes and the involvement of the various components of working memory in calculation procedures presents a theoretical inconsistency with the abstract modular view. As Clark & Campbell point out if modality specific codes are utilised in the calculation of problems then the implication is that abstract codes cannot be stored briefly but require to be translated into a specific code. This makes calculationing a complex process fluctuating between modality specific codes and abstract representational formats.

It would appear that the encoding complex approach Clark & Campbell (1991) might explain the accessing of simple fact retrieval from the associative network. However if a more complex problem is presented, for example, 36 x 6 = ?, how does this model account for the calculation procedure in the absence of rehearsal of the problem and the visuo-spatial sketchpad component of working memory? It may also be appropriate to consider that the abstract modular theory of McCloskey, Caramazza & Basili (1985) is without a facility for visually representing the spatial arrangement of multi-digit problems. Both models appear to accommodate simple fact retrieval procedures and the basic principles of the models are based upon semantic representations but may require additional components (for example the phonological loop and the visuo-spatial sketchpad) to account for more complex numbers. The triple-code theory of Dehaene (1992) is less abstract in its arrangement than the other models mentioned and includes the concept of a number line. The operations associated with a number line may relate to the visuo-spatial sketchpad component of working memory. This model appears to take into consideration more complex processing than the proposals made from the other models.

3.5 Conclusion

This chapter has provided an overview of the literature on working memory and the recent research that has examined how working memory resources are employed during arithmetic calculation procedures. This review is of particular importance as specific themes are investigated in more detail later in the thesis. For example, the factor analytic study, includes detailed work on the components of working memory and Chapter 11 investigates the involvement of the visuo-spatial sketchpad in complex arithmetic.

3.6 Review of Chapters 1 - 3

In Chapter 1, the topic of numerical cognition was introduced and a historical overview of the development of numerical cognition was given. The models of numerical cognition were introduced in Chapter 2 and the way in which each model explains particular calculation procedures was discussed. Chapter 3 gave an account of the working memory model (Baddeley 1986) and how working memory and numerical cognition form a very close association. This association was developed by providing examples from research of how particular aspects of numerical cognition can be associated with the components of the working memory model.

Chapter 4 - Introduction to Study 1: A factor analytic study

4.1 Introduction

Chapters 4 to 7, provide a detailed account of a factor analytic study. A review of the literature to introduce the study is the theme of this chapter. The chapter highlights the diversity of opinions relating to numerical ability and numerical cognition amongst researchers and the methods of investigation used. Similarities and differences between the models of numerical cognition are discussed together with the predicted associations relating to working memory. Chapter 5 gives a detailed account of the method and of the tests used in the study. This study uses the factor analytical method, not with the intent of intelligence testing but to investigate cognitive processes used in the solving of arithmetic problems of differing kinds. The extensive battery of tests is specifically designed to cover a spectrum of arithmetic problem solving abilities in conjunction with specific memory processes, particularly relating to the working memory model. Earlier studies have investigated arithmetic problem solving and memory but have not integrated them in the way proposed here. The factor analytic study was designed to:

- Elucidate the factor structure of the processes that underlie numerical cognition.
- Investigate the various components of the working memory model in relation to arithmetic and include long-term memory processes.
- Examine the extent to which findings from the factor analysis are compatible with key features arising from the models of numerical cognition developed on the basis of experimental studies.

The factor analytical method, therefore, forms the basis for Study 1. Chapter 6 provides a detailed account of the analysis of the results together with a full discussion of the factors obtained from the analysis. Chapter 7 gives a summary of the factor analytic study and on the basis of the results of this study the development of specific areas of experimental work is introduced. The experimental work is reported in Chapters 8 - 11.

4.2 Rationale for Study 1

From the models of numerical cognition presented earlier there appears to be uncertainty and a diversity of opinions as to the structure and processing of numerical information. Attempts to conceptualise numerical cognition have included models for the suggested structure of numerical abilities as well as models identifying the various cognitive processes underlying numerical cognition. The use of the information processing paradigm is well established and has provided a method for identifying the processes underlying specific mental abilities. The factor analytic method, although extensively used in the study of human intelligence and personality, has been little used in the study of numerical cognition. Factor analysis has an extensive history with taxonomies of human mental abilities developed by Thurstone (1938, 1941) and Cattell, (1963) among others. According to the factor analytic studies carried out by Thurstone the same factors have repeatedly occurred in different test batteries. One factor that has consistently arisen is the numerical facility characteristically defined by tests of addition and multiplication. This indicates an important ability dimension according to Coombs (1941) who asked the question: 'Is the ability defined by addition and multiplication specific to numerical manipulations or does it represent some more fundamental unity that transcends numerical operations?' (p.161).

However, the factor analytic method has been criticized. The criticisms arise from the statistical criteria employed by researchers and the psychological basis of their inquiries. For example, Sternberg (1980) suggested that factorial theories of intelligence are in general compatible and not necessarily conflicting although the psychological interpretations of the meaning of factors may differ from one researcher to another. The various factor theories of intelligence slightly vary from each other primarily because of the way in which the factorial solutions are rotated. The decision to use a particular rotation is dependent upon the researcher's theoretical perspective. There may be a large number of alternative orientations of axes each of which could provide an equally acceptable mathematical representation of the data. In general terms, according to Sternberg, factors represent latent abilities that can produce measured individual differences in performance on intelligence tests.

Not only are there a number of factorial theories of intelligence but according to Sternberg there are a number of component theories of intelligence, for example, Carroll (1976), Pellegrino & Glaser (1979). A component is an information process that operates on internal representations of images or objects translating the sensory input into a conceptual representation resulting in a motor output. Sternberg (1980) argued that components can be distinguished on the basis of function into five different kinds, meta-components, performance components, acquisition components, retention and transfer components. According to Geary & Widaman (1987) the factor analytic method is appropriate for uncovering and differentiating between hidden or latent variables that represent psychologically distinct component processes. The factor analytic method is also useful for identifying associations between observed variables that may indicate psychologically similar processes in apparently quite different cognitive tasks.

As noted above factor analytic studies of ability measures have consistently identified a numerical facility factor. Experimental work focusing on speed of processing using reaction time data has been extensively used for the study of numerical abilities and has enabled the modeling of the processes involved in solving arithmetic problems (Ashcraft, 1982, Groen & Parkman 1972, Moyer & Landauer 1967). Process models for the solving of arithmetic problems discussed in Chapter 2 have included analogue, counting, and memory retrieval models. The development of these models was based on experimental research. This research has become more refined over the years and has focused on specific areas of numerical cognition. The programme of research that is reported here began with a factor analytic study which was a preliminary investigation into the relationship between aspects of numerical cognition and more general cognitive processes involved in working memory and long-term memory.

4.3 Previous factor analytic research

A basis for this research arises from the factor analytic studies of numerical ability by Coombs (1941), Geary, Widaman & Little (1986) and Geary & Widaman (1987, 1992). These studies used a broad range of measures. However the studies were not designed to assess directly the structure of numerical cognition or how the components of working memory reflect numerical processing.

4.4 Coombs (1941) - Description of tests

At an earlier stage Coombs (1941) suggested that tests such as addition and multiplication measure more fundamental processes than number manipulations. Based on this suggestion Coombs considered it possible to devise tests of a non-numerical nature which would measure a range of cognitive processes underlying numerical cognition. Coombs designed tests to investigate the nature of number ability and to investigate the effect that number ability has on calculation speed. The research included a battery of thirty-four tests given to 223 Chicago high school pupils. Sixteen tests were taken from the American Council of Education Tests for Primary Mental Abilities, (1938). This battery of tests was not described by Coombs (1941). The remaining eighteen tests that were designed to explore specific questions are described in the following section and listed in Table 4.1.

4.4.1 Tests 1, 2 and 3 - Addition

The aim of the addition tests, according to Coombs, was to investigate the possibility that the number process was essentially a serial response. If this were to be so then the results of these tests would progressively increase on the number factor. Participants were required to respond to whether or not the provided answers to the addition of 2, 3, and 4 single digit problems were correct or incorrect. This was a verification task.

4.4.2 Tests 4 and 5 – A B C tests

The aim of these tests was to investigate the manipulation of symbolic information. Coombs hoped to show that using familiar characters would provide a good measure of number ability. Both these tests are rule based using 3 familiar letters of the alphabet-A, B and C. For example one rule states 'that a combination of any two different letters is equal to the third letter' (Coombs p.167). Participants were required to solve a series of problems relating to the rule, for example, A B = C.

Table 4.1 18 tests designed to answer specific problems (Coombs 1941). This table does not include the sixteen tests taken from the American Council of Education Tests for Primary Mental Abilities, (1938).

Test number	Test	Category
1	Two digit addition	Verification
2	Three digit addition	Verification
3	Four digit addition	Verification
4	A B	Rule based problem
5	A B C	Rule based problem
6	Geometric forms	Manipulation of symbolic information
7	Alphabet I	Practice with a set of rules
8	Alphabet II	Practice with a set of rules
9	Alphabet III	Practice with a set of rules
10	Digit cancellation	Perceptual speed/relationship to a number factor
11	Cancellation of x's (rows)	Perceptual speed/relationship to a number factor
12	Cancellation of x's (pied)	Perceptual speed/relationship to a number factor
13	Identical numbers	Relationship to a number factor
14	Highest number	Relationship to number factor
15	Size comparison	Quantitative thinking
16	Substitution I	Familiarity of symbols to number ability
17	Substitution II	Familiarity of symbols to number ability
18	Substitution III	Familiarity of symbols to number ability

4.4.3 Test 6 – Geometric forms

This rule-based test contrasted with tests 4 and 5 and non-meaningful geometric designs were substituted for letters The rationale for this was to test the hypothesis that the more familiar the symbolic information is the better the measure of number ability.

4.4.4 Test 7, 8 and 9 – Alphabet I, II and III

These three tests were constructed to test the concept that practice with a set of rules would improve a test as a measure of number ability. If this were correct then the three tests would show progressively increasing validity as measures of number ability. These three tests were again based on letters of the alphabet, with each test containing problems based on three rules. The first rule stated that 'a combination of two letters in alphabetical order which has letters between them is equal to the letter in the alphabet which follows the second letter in the combination' (p.169). Two letters in the combination may be M P having the letters N and O between them. Therefore the combination is equal to Q as Q follows P in the alphabet, M P = Q. The second rule uses letters of the alphabet in reverse order. For example, 'a combination of two letters in reverse order which has letters between them is equal to the letter in the alphabet which precedes the second letter of the combination' (p.170). The two letters P M have letters between them, and therefore, the combination is equal to L as L precedes M in the alphabet, P M = L. The third rule states that 'a combination of two letters which have no letters between them is equal to the second letter of the pair' (p.170), for example, O P = P. Each test contained 192 similarly designed problems based on the three rules and participants were required to provide the missing letter to the problem.

4.4.5 Tests 10, 11 and 12 – Digit and letter cancellation

Participants in Test 10 were timed on the length of time taken to identify and place a parenthesis around a number 5 in a series of numbers. Participants in Test 11, letter cancellation, were required to place a parenthesis around the letter x. Test 12 also required participants to place a parenthesis around the letter x but this test used coloured letters.

4.4.6 Test 13 and 14 – Identical numbers and Highest number

Two tests investigated whether tasks involving number and numerical concepts necessarily had a relation to number ability if they did not involve manipulation of the numbers in the solving of arithmetic problems. Identical numbers, Test 13, consisted of 45 columns with 29 three-digit numbers in each column. The three-digit number at the top of each column was repeated one or more times in that column. Participants were required to identify as quickly as possible the repeated numbers. Test 14 consisted of 80 columns with 40 three-digit numbers in each. In this test participants were required to find the highest number in each column. Both tests measured the speed of identifying the target numbers, and manipulation of numerals was not required to complete these tests.

4.4.7 Test 15 – Size comparison

The size comparison test required participants to select the larger of two items, for example, answering the question, is a sardine larger than a shark? The test consisted of 69 similar questions. This non-numerical test was included to investigate the concept that the ability to differentiate between various sizes of items may show a relationship to number ability.

4.4.8 Tests 16, 17 and 18 – Substitution I, II and III

The final three tests consisted of 90 words in each test written in code. The code was built up using arbitrary symbols. Participants were required to translate the code into a word. The same code was used throughout the tests. The aim of these tests was to investigate whether or not practice in translating the code had any significance in relation to number ability.

4.5 Results

To produce the results the oblique rotation method was used utilizing the equipment available at this period of time. The tests for Primary Mental Abilities and the specifically designed tests produced ten factors. Coombs reported on the first seven of the ten factors produced by the analysis of the results from the 34 tasks. He selected factor loadings above 0.2. It is necessary to mention here that the statistical analysis of factor analytic studies is now greatly advanced in comparison to the methods of analysis available to Coombs over 60 years ago. According to Stevens (1996) factor loadings greater than .4 can be considered to exist independently. However, Coombs reports factor loadings above 0.2 producing great variation between the scores within each factor as can be seen in Table 4.2.

Table 4.2 The results of the rotated factor matrix showing 10 factors with factor loadings of above 0.2.

Test no.	Test name	F 1	F 2	F 3	F 4	F 5	F 6	F 7	F 8	F 9	F10
1	Two-digit	74									
2	Three-digit	72									
3	Four-digit	66									
22	Multiplication	64									
21	Addition	45						32			
23	Identical numbers		66								
24	Completion		64						21		31
15	Same-opposite		49								30
20	Size comparison		41			22					
26	Verbal			71			23				
25	Figures			65			36				
19	Cards			50		28					
28	Identical forms				55						
27	Number patterns				53						
11	Word-number			(19)		65					
12	Initials					55					
10	Scattered x's					46					24
32	Scattered x's						57				
33	Digit cancellation						56				
34	Identical numbers					(19)	30				
6	Identical forms							37	26		25
30	Letter grouping						29	30	24		
31	Verbal			20		20	25	26			
8	Number patterns								66		
9	Arithmetic								66		
7	Number series								64		
29	Cards					28		22	30		25
5	Mechanical							23	27		
17	Marks									78	
18	Number patterns									76	
16	Figures									69	
13	Forms	22				31					35
4	Marks								24		30
14	Addition										29

Table 4.3 below shows the seven factors described by Coombs. To facilitate the interpretation of the rotated factor matrix as presented in the 1941 paper only the factors and factor loadings discussed by Coombs are included in the table. It will

be noted that some overlapping correlations were ignored in his analyses and that there is no information on the proportion of the total variance accounted for each factor.

Table 4 .3 Results of Coombs (1941) Factor analytic study showing the seven factors to be described below. Factors 8, 9 and 10 are not described in detail by Coombs.

Test number	**Test name**	**Factor 1 number**	**Factor 2 verbal**	**Factor 3 space**	**Factor 4 memory**	**Factor 5 Perceptual speed**	**Factor 6 deductive**	**Factor 7 inductive**
1	Two-digit addition	.74						
2	Three-digit addition	.72						
3	Four-digit addition	.66						
22	Multiplication	.64						
21	Addition	.45						
13	Identical numbers	.22						
23	Completion		.66					
24	Same-opposite		.64					
15	Size comparison		.49					
20	Verbal enumeration		.41					
26	Figures			.71				
25	Cards			.65				
19	Identical forms			.50				
31	Number patterns			.20				
28	Word-number				.55			
27	Initials				.53			
11	Scattered x's					.65		
12	Scattered x's					.55		
10	Digit cancellation					.46		
13	Identical numbers					.31		
29	Identical forms					.28		
19	Letter grouping					.28		
20	Verbal enumeration					.22		
31	Number patterns					.20		
32	Arithmetic						.57	
33	Number series						.56	
25	Cards						.36	
34	Mechanical movements						.30	
30	Marks						.29	
31	Number patterns						.25	
26	Figures						.23	
6	Forms							.37
30	Marks							.30
21	Addition							.32
31	Number patterns							.26
5	A B C							.23
29	Letter grouping							.22

4.6 Coombs' (1941) Interpretation of the Factors

The following brief summaries are based on Coombs own interpretation and in some cases the anlysis would not meet current standards for analysing this data.

4.6.1 Factor 1 - Number

Addition, multiplication and identical numbers tests make up Factor 1 with the simpler tests of addition having a greater reflection on this factor. This is shown by two and three-digit addition problems having the highest loadings of .74 and .72 respectively. The more complex multiplication problems which included the solving of two digits by one digit had a loading of .64. The addition problems of six two-digit numbers had a loading of .45. The conclusion relating to number ability in this factor was that number ability was not necessarily associated with a serial response process. The correlation matrix of the A, B and A, B C tests and the non-meaningful geometric shapes test suggested that the more familiar or well established the representation of a symbol the better the test is as a measure of number ability.

4.6.2 Factor 2 - Verbal

Factor 2 was interpreted as the verbal factor with the two tests Completion and Same-Opposite designed to isolate the verbal factor. Included in this factor was the size comparison task. No numerical tasks were included in this factor.

4.6.3 Factor 3 - Space

Coombs suggested that there was not as yet a clear understanding of this factor which included tests based on Figures, Cards, Identical Forms and Number Patterns. It is interesting to note here that this factor may include aspects of visuo-spatial memory but that would not have been appreciated at that time as research on memory functioning had not yet fully developed.

4.6.4 Factor 4 - Memory

Factor 4, the memory factor, included only two tests from the Primary Mental Abilities battery, the Word-Number and Initials tests. Here Coombs suggested that this factor was a product of rote learning as both tests were tests of rote learning with immediate recall.

4.6.5 Factor 5 – Perceptual speed

Factor 5 was termed perceptual speed, which included the Letter and Digit cancellation tests. These tests were scored on the number of correct answers within a given period of time. Participants were required to scan as quickly as possible the content of the tests to produce their answers. All tests included in this factor required rapid scanning of the test material.

4.6.6 Factor 6 – The Deductive Factor

Factor 6 was identified as the deductive factor, reasoning from the general to the specific, by three tests included in the battery Arithmetic, Number Series, and Mechanical movements.

4.6.7 Factor 7 – The Inductive Factor

Factor 7 was interpreted as the inductive factor. However, the factor loadings range from Forms at .37 to Letter Grouping at .22. As there was only a small amount of variance accounted for by this factor Coombs considers that to make a clear interpretation of it as a factor would be difficult.

4.7 Conclusion to Coombs (1941) study

The conclusions reached by Coombs were that the number factor included very simple number tests. The perceptual speed factor was associated with cancellation tests. After examining the factor loading of each tests, Coombs concluded that tests were more efficient as measures of perceptual speed if they involved more scanning of material and less cancellation of target stimuli. These tests that included the manipulation of a familiar symbolic system (for example, alphabet letters) as opposed to non-meaningful geometric symbols would be better as a measure of number ability. It was further concluded that tests that involved following a set of rules, for example the A B C tests, reflected number ability.

This extensive study produced interesting findings with the results answering a number of the theoretical questions proposed by Coombs. It does have limitations perhaps as a result of the period of time in which it was conducted. With the restricted scope of statistical technology and theoretical frameworks at that time the researcher was unable to expand in detail on issues relating to, for example, memory, numerical processes, speed of response and problem solving. It is interesting to note that the results seemed to produce quite distinct factors, in particular, separate numerical and verbal factors. The early identification of these factors may provide some support for the approach taken by Dehaene (1992) in his triple code theory. The size comparison task loaded onto the verbal factor which may reflect the analogue scale proposed by Dehaene. One of the key issues associated with research into numerical processing is whether there is a functionally distinct number domain, for example, the analogue scale, (Dehaene 1992) or the abstract internal representation component of McCloskey et al.'s (1985) model, and how the components of these models relate to other cognitive functions and representations. While Coombs' study may be criticised on technical grounds, it indicated the potential value of a factor analytic approach in this field.

The study does, therefore, provide a justification for investigating numerical processing using the more sophisticated data analysis techniques and theoretical models that are now available in contemporary cognitive psychology. But for many years factor analysis was not used as a research tool to investigate theories of numerical cognition. In the interim period a range of theoretical models has been constructed through extensive experimental research that identified a wide range of numerical processing techniques.

4.8 Widaman, Geary & Cormier, (1986) - Computational model of mental addition.

Geary & Widaman (1987) provided the next major factor analytic and experimental study to assess the validity of the cognitive components model for addition initially proposed by Widaman, Geary & Cormier (1986). This model is based upon the network retrieval model described by Ashcraft (1982). It includes the same basic processing stages of encoding, search and compute, decision and response stages but provides a more elaborate description of arithmetic problem solving techniques.

Widaman et al. (1986) interpreted findings from research on adult performance to suggest that basic addition and multiplication facts are represented in memory in an organized network of information. This information is accessed and retrieved from a network through a process of spreading activation. The two most important aspects of the network theory involve the concept of strength of associations between nodes and the strength of frequency of past associations. The strength and amount of interconnection between the nodes was seen as a function of the frequency, and practice of the information with particular relationship to early education. Therefore, arithmetic education during the early school years was seen to have an effect on the strength of problem representation in long-term memory.

The computational model of arithmetic is an elaboration of the network retrieval model proposed by Ashcraft (1982). The first stage of this model is the encoding of the addition or multiplication problem and the first two digits to be summed, for example, the digits in the units column in complex problems. The researchers presented the problems in a verification format where the problem and a stated answer are presented simultaneously and the task for the participant is to assess whether or not the stated answer is correct. Once the two digits are held in working memory a total for the digits is obtained through either a counting or a memory search process. For simple addition involving two single digits participants compare their obtained answer with the given answer before producing their response.

The model is able to accommodate more complex problems by using recycling loops, which correspond to the summing of more than one column or more than two rows of digits. If multi-column problems are to be solved, the encoding and search/compute process for simple problems is recycled until sums are obtained for each column. The recycling loop can be modified in two ways. Firstly a carry operation will be necessary if the preceding column sum is greater than nine and secondly, complex multi-column problems may be ended if an error in the stated sum is encountered before the entire problem is processed. If a participant is asked to process a multi-column problem, it can be terminated when the first column-wise error is encountered. According to Geary & Widaman (1987) this self-termination of complex problems suggests the existence of a meta-cognitive process, which is involved in the mental solving of addition problems. This meta-cognitive process monitors the procedures used in problem solving, resulting in the selection and execution of the most efficient component process at each step.

According to the model for the addition of two digits (for example, 2 + 5 = 7), each of the numbers is encoded and the answer retrieved from a long-term memory network of arithmetic facts. For problems greater than two digits (for example, 4 + 7 + 6 = 17), the digits are scanned and the two digits with the largest value are encoded, in this case 7 and 6. The associated provisional answer is retrieved from long-term memory, 13, and held in working memory. The remaining digit is incremented onto the provisional answer until the final answer is obtained. The next stage involves a decision process as to the correctness of the answer. If the answer is not the same as the stated answer then the problem solving procedure is self-terminated and the response given by the participant would be 'incorrect'. If the answer obtained by the participant is the same as the stated answer and there are no further digits to be processed then a 'correct' answer can be given. However, if there are further digits to be processed then they are recycled through the described stages until all the columns of digits are processed. If an answer to a column exceeds 9 then the recycling loop is modified and a carrying operation is performed.

Geary & Widaman (1987) designed a research study to assess the validity of the computational model of mental addition across addition and multiplication problems and to assess the relationship between the component processes within the model and traditional measures of numerical facility. This research combined information processing tasks with psychometric measures. The aim was to assess the relationship between the time taken to answer simple addition and multiplication problems which included retrieval of information from a stored network of arithmetic facts and performance on psychometric tests that included tests relating to spatial relations, numerical facility and perceptual speed.

Undergraduate students participated in the study and were administered a battery of ability measures defining the Numerical Facility, Perceptual Speed, and Spatial Relations factors. The information processing section of the research was based on measuring reaction time to simple and complex addition and multiplication verification tasks.

The results of the verification tests demonstrated a relationship between elementary operations as defined by the computational model of mental arithmetic and performance on ability tests that define the Numerical Facility factor. The results demonstrated that arithmetic operations, addition and multiplication, were related to theoretically similar ability measures in the battery of tests and were unrelated to theoretically dissimilar ability measures, for example the spatial relations tests.

This study used the factor analytic and information processing methods to investigate the computational theoretical model. The results lent support to the model. However, a number of different and contrasting models of numerical cognition have more recently been proposed. This suggests the need for further investigation using a variety of research techniques to explore the recent theoretical models in greater depth.

A subsequent study by Geary & Widaman (1992) extended the findings from the study by Geary & Widaman (1987) and scrutinized the relationships in greater depth. The aim of this study was to examine the relationships between the tests included in the battery that have been traditionally used to define the Numerical Facility, Perceptual Speed, General Reasoning and Memory Span factors and specifically designed arithmetic tests. The involvement of working memory in solving arithmetic problems was also considered. All numerical facility, general reasoning and memory-span tests required the processing of numbers. The numerical facility and general reasoning measures included the arithmetic operations addition, division, multiplication and subtraction. The experimental arithmetic tests were designed to investigate the components of the computational model. The involvement of working memory in solving arithmetic problems was also considered.

The participants were 102 U.S. Air Force recruits, 54 were male and 48 were female, with a mean age of 20 years. The information processing arithmetic problems and the working memory task were presented on the computer. The arithmetic problems were all verification tasks with 80 problems in each set, 40 displayed with the correct answer and 40 with the incorrect answer. Participants were required to respond as to whether or not the answer was correct or incorrect. Reaction time and error scores were recorded.

4.9 Geary & Widaman (1992) tests used in the study

4.9.1 Arithmetic problems – simple addition

The 40 correct problems were selected from the 56 possible combinations of 2 through to 9, for example 6 + 3 = 9. No tie problems were included. The presentation of the digits was counterbalanced as each digit used in an addition problem appeared ten times, five times as the first digit in the sum and five times as the addend. The 40 incorrect sums were identical except that the sum was incorrect by plus or minus 1 or 2 digits, for example, $3 + 4 = 8$.

4.9.2 Multi-column complex addition

The 40 correct complex addition problems consisted of four different digits from 2 through to 9, for example $23 + 58 = 81$. The digits were counterbalanced for position within the sum. The 40 incorrect sums were identical but were incorrect by plus or minus 1 or 2 digits.

4.9.3 Multi-digit complex addition

Each of the 40 correct problems consisted of three different digits for example, $2 + 4 + 5 = 11$, from digits 2 to 9. The position of each digit in the sum was counterbalanced. The remaining 40 problems were identical but the sum was incorrect by plus or minus 1 or 2 digits.

4.9.4 Simple multiplication

The sums consisted of two single digits, for example $7 \times 3 = 21$. The problems were chosen from a possible 56 from 2 through to 9. No tie problems were included. The frequency of the digits was counterbalanced for position. The 40 incorrect problems were identical but incorrect by plus or minus 1, 2, or 10. Sixteen problems were incorrect by plus or minus 10 and 6 were incorrect by plus or minus 1 digit (12 problems) and plus or minus 2 digits (12 problems).

4.9.5 Complex multiplication

This test consisted of two digits to be multiplied by one digit, for example, $35 \times 7 = 245$. The 40 correct problems consisted of three different digits with the values 2 through to 9. The frequency of each digit was counterbalanced for position in the sum. The 40 incorrect sums were identical and the problems were incorrect by either plus or minus 1, 2, 10, 20 or 100.

4.9.6 Working memory task

The working memory task was used as a measure of working memory capacity that consisted of assigning numerical values to the letters A, B and C. For example, A = 72, B = 6 x 8, and C = B/2. Each section was presented separately on the screen for the participants to memorize and disappeared before the following section was displayed. This ensured that the participants were unable to re-examine previously presented material, therefore requiring the information to be held in working memory. Following the presentation of the various sections of the material individual questions were displayed on the screen, for example C = ?, and the participants were required to press the appropriate number keys to provide their answer.

4.10 Ability test battery

Ten ability measures were taken from the Educational Testing Service test battery (Ekstrom, French & Harman, 1976) considered to cover the Numerical Facility, Perceptual Speed, General Reasoning and Memory Span factors. Two to three measures from each of these factors were used.

4.10.1 Numerical facility

The three measures were the addition and division tests and the subtraction and multiplication test.

4.10.2 Perceptual speed

The three measures of Perceptual Speed were the Finding As test, and Number Comparison test, and the Identical Pictures test.

4.10.3 General Reasoning

Two measures were taken to assess general reasoning, the Arithmetic Aptitude test and the Necessary Arithmetic Operations test.

4.10.4 Memory Span

Two measures of memory span were used, the Auditory Number Span test, and the Visual Number Span test.

The 10 paper and pencil ability tests were given to groups of no more than 31 participants at any one time with the tests timed according to the instruction manual. The test battery was completed before the information processing tasks which were given individually. Table 4.4 gives a summary of the tests used by Geary & Widaman (1992).

Table 4.4 Summary of tests used in Geary & Widaman's (1992) study

Group	Test name
Arithmetic problems	Simple addition
	Multi-column complex addition
	Multi-digit complex addition
	Simple multiplication
	Complex multiplication
Working memory	ABC-assignment task
Numerical facility	Addition test
	Division test
	Subtraction and multiplication test
Perceptual speed	Finding As test
	Number comparison test
	Identical pictures test
General reasoning	Arithmetic aptitude test
	Necessary arithmetic operations test
Memory span	Auditory number span test
	Visual number span test

4.11 Results

The results are considered in four sections. The information processing tasks and the working memory test are included in the first section. The second section analyses the ability measures, the third section combines the information processing tasks and the battery of tests. The final section looks at a cross-sample comparison between the results of the 1992 research and that of Geary & Widaman (1987).

4.11.1 Information processing arithmetic tests and working memory test

The regression equation results of the computer based arithmetic problems indicated that the processing strategies adopted by the participants to solve the five problem types were compatible with the computational model. Using reaction time data and applying regression analysis to the various stages of the computational model it was found that complex addition and multiplication problems were processed using columns, for example, the addition or multiplication of the units column was completed before continuing to the tens columns. The answers to each column were retrieved from a long-term memory network of arithmetic facts. Complex problems were self-terminated when an error in the units column was encountered. Additional processes were used by participants, for example, encoding single integers, and for complex problems carrying to the next column and adding digits unit by unit onto a provisional sum.

The results of the error scores for the arithmetic tests used in this section were found to be consistent with previous research (Geary, Widaman & Little 1986). The results of the working memory task were found to be reliable and to be consistent with the results found by Woltz (1988).

4.11.2 The Ability Test Battery

The LISREL VI program (Joreskog & Sorbom (1984), was used to analyze the data. The confirmatory factor analytic method was adopted for the analysis. The ability test battery produced four factors, Numerical Facility, Perceptual Speed, General Reasoning and Memory Span. Table 4.5 shows the results of the confirmatory factor analysis of measures in the Ability Test Battery.

Table 4.5 The results of the confirmatory factor analysis of measures in the Ability Test Battery including factor intercorrelations (Geary & Widaman 1992)

Variable	Numerical facility	Perceptual speed	General reasoning	Memory span
Addition	.872 (.083)			
Division	.72 5 (.089)			
Subtraction/multiplication	.92 (.080)			
Finding As		.35 (.116)		
Number comparison		.737 (.127)		
Identical pictures		.433 (.116)		
Arithmetic aptitude			.807 (.067)	
Necessary arithmetic operations			.921 (.076)	
Auditory number span				.725(.076)
Visual number span				.747 (.078)
Factor Intercorrelations				
Factor				
Numericsal Facility				
Perceptual Speed	.650 (.109)			
General Reasoning	.310 (.102)	.210 (.133)		
Memory Span	.083 (.124)	.298 (.146)	.376 (.114)	

The interpretation of Table 4.5 by Geary & Widaman (1992) suggested that as each of the factor loadings were at least twice as large as their respective standard error this showed that all the factor loadings were significant. Analysis of the factor intercorrelations were significant at $p < .05$ except for the correlation between the Memory Span and Numerical Facility factors, and between the General Reasoning and Perceptual Speed factors, $p > .05$.

4.11.3 Analysis of combined data

The third analysis concerned the relationship between performance on the experimental tasks and the ability measures. This analysis suggested that there was a relationship between the information processing arithmetic tasks and the Memory Span ability factor with performance on the measures of general reasoning and memory span related to working memory capacity. All of these ability tests required the processing of numbers. However, the memory span tests did not involve arithmetic operations. The findings suggested that the rate of executing basic arithmetic operations was related to performance on traditional ability measures that required arithmetic and were not directly related to similar measures that required the processing of numbers but did not require arithmetic.

4.11.4 Cross sample comparison

The final analysis considered cross-sample comparisons of performance on the ability measures and experimental tasks that were administered to the U. S. Air Force recruits used in this study and the undergraduate students used in the Geary & Widaman (1987) study. A direct comparison was not possible as some of the tests differed between the studies. However, the data from the two studies were combined into a single data set and five composite scores were then analyzed representing numerical facility, perceptual speed, retrieval efficiency, and the speed of the carry operation and the time taken to encode, decide and then respond with the answer.

Geary & Widaman (1992) concluded that the processes that underlie the psychometrically derived numerical facility factor appeared to involve the operations of information retrieval from a stored network of arithmetic facts and carrying to the next column for complex problems. Information retrieval and carrying to the next column was not directly related to individual differences on theoretically different ability measures, for example the spatial tests in the previous study and the memory-span tests in the present study. The results indicated the presence of a close relationship between measures that require the manipulation of numerical information. There appeared to be a link between the experimental variables and the ability factors which suggested that individual differences in the speed of executing fact retrieval and carrying operations were related to individual differences on the measures defining both the Numerical Facility and General Reasoning factors. They concluded that Numerical Facility and General Reasoning factors use similar information processing processes. The correlation found between the General Reasoning and Memory Span factors could be considered in a similar way. However, it may be that these factors have similar working memory demands.

The above research has used the traditional individual differences strategy of including in the study generally unclear "general ability" measures, that may measure several aspects of cognition. A more specific approach could be adopted by aiming to use more narrowly focused measures. This was the strategy adopted in the current study reported in Chapters 5 and 6 below.

4.12 Differences between Geary & Widaman (1987) research and the present study

The aim of the research by Geary & Widaman (1987) was to assess the validity of the cognitive components model proposed by Widaman, Geary & Cormier (1986) across simple and complex addition and multiplication problems. It was also designed to assess the relationship between component processes identified within the model and the traditional factor analytic measures of numerical facility. The tests they selected were specifically chosen to verify their model and to investigate the retrieval of information from a stored network of arithmetic facts. The present study takes a different perspective with the foundations for the research derived from a number of different theoretical models of numerical cognition and memory, including McCloskey et al. (1985), Dehaene (1992), Clark & Campbell (1991) and with particular emphasis on Baddeley's (1986) Working Memory model.

Geary & Widaman (1987) did consider the concept of working memory. However, their choice of tests appears to have been based on the assumption that working memory is unitary in its functioning. Extensive neuropsychological research, for example, Baddeley (1986), and Logie, et al. (1994), suggests that this is not the case. The present study takes into account not only the models of numerical processing but also other cognitive theories, in particular the working memory model.

The selection of tests used by Geary & Widaman (1987) was specific to the requirements of their methodology, and the elaboration of their model. The test selection also reflected the literature available at that period of time. The tests included in the present study have been chosen from the wider range of tasks that is now available. The aim is not only to reflect the components of working memory, the central executive, the phonological loop and the visuo-spatial sketchpad, but also to investigate other aspects of the processes involved in a range of current theories of numerical cognition. This requires a battery of tests that covers a much broader sample of mental processes than Geary & Widaman (1987) measured.

Although the studies reported above have used a factor analytic approach to explore the issues surrounding numerical cognition, the focus of these studies has been very different from the study undertaken here. The previous factor analytic studies appear to have been primarily concerned with the relationship between numerical facility and intelligence while the study that is reported here aims to investigate current numerical processing theories with specific tests of arithmetic, memory and semantic processing.

4.13 Objectives of Study 1

As noted in Chapters 1-3, work in relation to numerical cognition and the involvement of working memory in the calculation process is in the early stages of research. The models of numerical cognition emphasize the representation of numerical information. McCloskey et al. (1985) suggested the involvement of an abstract semantic internal representation mechanism. McCloskey (1993) argued that research has not adequately considered the relationship between processes for numerical and non-numerical processing. This issue concerns whether numerical processing is separate from or integrated within the cognitive language processing system. McCloskey further suggested that research is required in the area of general cognitive processing and specifically on working memory and spatial processing. The objectives of the present study include investigating the issues raised by McCloskey. Tests were selected to assess the semantic elements of numerical processing, central executive functioning, verbal rehearsal and the phonological loop, visuo-spatial encoding and manipulation of information together with long-term encoding and memory retrieval processes.

Dehaene's (1992) triple code model highlighted format-specific representations and attempted to accommodate the abilities to compare and to approximate numerical quantities by way of the analogue magnitude representation component. Dehaene considered analogue magnitude representation to be important in understanding the quantity that a numeral represents and in checking the accuracy of calculations. This component was considered to provide a semantic representation of numbers. Yet the properties of McCloskey's abstract semantic representation system and Dehaene's analogue representational system appear to imply very different calculation procedures. To investigate the processing of arithmetic problems, tests of different aspects of processing are included in the new study, for example, mental arithmetic problems, addition and subtraction, subitizing and magnitude comparison.

The way in which numerical information is processed within these two models may be linked to the way in which information is represented within the components of the working memory model. In addition more complex problems may require specific working memory procedures. These issues are addressed in the factor analytic study. If the analysis produces a factor largely based on the representation of information, this would lend support to McCloskey's theory. However, if separate factors emerge for number tasks, for example subitizing and magnitude judgement tasks, then this would lend support to Dehaene's model. Clark & Campbell (1991) and Geary & Widaman (1992) suggest that information is accessed through an integrated associative network that would result in a number of different tasks appearing in the same factor. However, associative network theories appear to accommodate the retrieval of information stored in long-term memory with greater ease than immediate short-term memory functioning. This suggests that associative network theories have difficulty in accommodating the manipulation of numerals in the calculation of multi-digit problems. Within Dehaene's model multi digit calculations may utilize the visual spatial component of working memory together with central executive control. Simple arithmetic, however, may not require working memory operations but retrieval of previously learnt information from long-term memory, therefore complying with Campbell's associated network theory. In line with previous work Heathcote (1994) working memory tasks may form a factor alongside more complex problems for example, multi-digit addition as previously discussed in Chapter 3. The prediction is that simple and complex arithmetic problem solving will load onto a separate factor. Furthermore the results of the factor analysis will indicate that working memory plays a more important role in complex problem solving than in simple arithmetic and to a greater extent than the models discussed in Chapter 2 suggested.

4.14 Study 1 – A Factor Analytic Study

The first study in the research programme reported in this thesis is a factor analytic study that aimed to consider the possible structure of numerical cognition and to investigate the underlying relationships among the responses given to each of the tests used. It was further intended to explore the field of numerical cognition to discover the main dimensions using the framework provided by cognitive psychology. This framework provided theories, concepts and interpretations relating to the processing of differing and possibly interrelated information. For example, a size or magnitude judgment test, similar to the test used by Coombs (1941) was included in the battery of tests to assess if number ability had a 'characteristic of quantitative thinking' (p.173). This test used differing sizes of animal pictures to assess a possible relationship between this test and tests involving numerals. A similar test was included in the present study together with the magnitude judgement of numbers test to consider the active or passive processing of numerals in relation to, similar non-numerical tasks.

The intention was to map the field of numerical cognition by sampling a large range of variables. Where suitable tests were available in published material they were selected to cover the domains that were targeted. In some cases it was necessary to construct new tests in order to achieve the very broad sampling that was required. The aim, using the battery of tests, was to probe knowledge within the domain of arithmetic in conjunction with other cognitive processes that were known (or hypothesized) to be relevant. For example, it was decided to include cognitive processes that require discriminating between a word and a non-word. The processes used in such tests may require retrieving previously learnt information, in a similar way to the retrieval of learned number bonds used in the solving of simple addition problems. Two lexical decision tests were included in the battery of tests. These two tests were constructed using English and French words and non-words. They were included on the basis that to determine whether or not the stimulus is a word or non-word requires retrieval of previously learnt information stored in long-term memory. The question arises as to whether or not these processes are associated with solving of simple arithmetic problems. Subitizing of numbers was also included in the study. This test has a numerical content but it is not clear if the cognitive processes required for the execution of this test sample some of the cognitive processes required for the solving of simple arithmetic problems.

The arithmetic operations included in the test battery are addition and multiplication with both simple and complex problems to solve. To include subtraction and division problems would have completed the range of arithmetic operations included in test battery. However, it was not practically possible to do so due to the length of time that each participant would take to complete the battery of tests. According to Butterworth (1999) addition and multiplication form the basis of subtraction and division.

The tests for Study 1 have been selected against the following criteria:

1. The tests should explore a wide range of cognitive processes including all the key processes that have been hypothesized to contribute to numerical cognition in the theoretical work reviewed in Chapters 1 – 2.
2. The test battery should have balanced responses. For example, computer tests required equal numbers of 'yes' and 'no' keyed responses.
3. Methods of responding to the tests should be varied. Both computer and pencil and paper tests employed different methods, for example, the production of an answer, verification tasks and multiple-choice tasks.

4. Arithmetic tests should cover both short and long-term memory processes.
5. The battery should include tests designed to investigate the hypothesized components of working memory that were highlighted in the review in Chapter 3.

The tests included in this study are identified in Table 4.6. The tests are divided into two groups: Group 1: tests created for the purposes of this study and Group 2: tests selected from the literature of previously published tests.

Table 4.6 The tests used in Group 1 and Group 2

Group	Test name	Presentation	Response
1	English lexical decision task	computer	key
1	French lexical decision task	computer	key
1	Simple addition	computer	verbal
1	Simple multiplication	computer	verbal
1	Subitizing numbers	computer	key
1	Subitizing circles	computer	key
1	Magnitude judgement of numbers	computer	key
1	Magnitude judgement of animals	computer	key
1	Rotation of letters	computer	key
1	Abstract visual pictures	computer	key
1	Complex addition	verbal	verbal
1	Complex multiplication	verbal	verbal
1	Basic mathematical facts	verbal	verbal
2	Story (RBMT)	verbal	verbal
2	Tower of Hanoi	visual	practical response
2	Stroop Effect	visual	verbal
2	Trails Making Test (Part B)	visual	paper and pencil
2	Block design (WAIS-R)	visual	practical response
2	Forward digit span (WAIS-R)	verbal	verbal
2	Backward digit span (WAIS-R)	visual	verbal
2	Doors Test (Doors and People Test)	visual	verbal

4.15 Purpose for which the tests were designed

A statement of the purpose for which each test was designed is given below.

4.15.1 English Lexical Decision Test (SCOLP, Baddeley, Emslie & Nimmo-Smith, 1992) and French Lexical Decision Test.

The English and French lexical decision tests are based upon the Spot-the-Word Vocabulary Test a sub-test from the Speed and Capacity of Language-Processing

Test (Baddeley, Emslie & Nimmo-Smith, 1992). The words selected for both the English and French lexical decision tests are common words in everyday use. For example, in the English lexical decision test the words 'guard' and 'bowl' are included. The words used in the French lexical decision test were chosen in English based on their word frequency and then translated into French. For example the words 'chat' and lait' are included.

Both the English and French tests may utilize direct retrieval processes from memory and may show a correspondence to simple addition and multiplication problems. The relationship between numerical tests and language-based tests will be considered. The aim of the French lexical decision task is to investigate whether facts retrieved from long-term memory, show similar performance profiles across the verbal and numerical domains. If it is assumed that knowledge for the completion of both these tests requires access to previously learnt semantic knowledge a correlation may be found between the tests.

4.15.2 Simple Addition and Multiplication Problems

Mental calculation is tested using Arabic digit materials. These tests are designed to investigate the speed of response, to simple addition and multiplication problems. It is considered that long-term memory processes are involved in the execution of these problems.

4.15.3 Subitizing Numbers and Subitizing Circles (Mandler & Shebo, 1982)

Balakrishnan & Ashby (1992) suggest that the reaction time observed in subitizing studies is governed by encoding processes in pattern recognition rather than numerical processes. Subitizing numbers and subitizing circles tests have been included in this study of numerical cognition as they have not been included in other factor analytic studies, for example, Coombs (1941). The predicted association is between the subitizing tests and simple arithmetic.

4.15.4 Magnitude Judgement of Numbers and Magnitude Judgement of Animals (Paivio, 1975)

Two tests based on the judgement of magnitude have been included in the battery of tests. These are the magnitude judgement of numbers and the magnitude judgement of animals. The magnitude judgment of animals is used as an analogous test to balance against the number comparison test, with the rationale that relative size need not be restricted to numbers. Dehaene (1992) discussed the analogue magnitude representation component of the triple code model and considered within this component the distance effect. According to Dehaene the distance effect is consistent across different types of stimuli, for example, the length of lines, pitch or numerosity. Logie (1995) pointed out that the literature on magnitude judgements has focused on the extent to which participants form an image of the object or animal or make comparative judgements based on semantic knowledge of the physical characteristics of the stimuli.

4.15.5 Rotation of Letters (Shepard & Metzler, 1971)

This task was included to investigate the manipulation of mental images of visual stimuli and to assess any relationship that there may be between this process and complex addition and multiplication where a complex sum has to be held in memory for further processing. The rationale is that to perform complex arithmetic problems visual spatial imagery is required. According to Ashcraft (1995) the role of the visuo-spatial component of working memory in arithmetic problem solving is not confirmed. However, he suggested that when arithmetic problems are solved that require information to be held in columns or carrying operations from one column to another, the visuo-spatial sketchpad is involved.

4.15.6 Abstract Pictures Test

This visual spatial task, together with the rotation of letters task may possibly show a correlation with complex arithmetic problems. The rationale is that to produce the answer to a complex multi-digit problem it is necessary to hold the positioning and manipulation of the digits in memory. It is suggested that manipulation of visual imagery is required for the completion of these tasks. This may reflect a correspondence with carrying operations in complex arithmetic problems. This test requires the encoding and retrieval of abstract material and may, therefore, correlate with complex problem solving.

4.15.7 Complex Addition and Multiplication

These tests are designed to assess the manipulation of information and to assess whether or not there is a relationship between these tests and mental rotation of letters and the abstract pictures test. The rationale for including these tests is that the addition and multiplication of multi-digit problems requires not only the manipulation of numerals but may involve other cognitive processes. For example, the results may correlate with the key variables selected to test the various components of working memory. According to neuropsychological research (Hecaen, Angelergues & Houillier 1961 and Luria, 1966) spatial processing deficits may underlie calculation deficits.

4.15.8 Basic arithmetic facts

The questions in this test are designed to assess semantic arithmetic knowledge (e.g. how many sides has a hexagon). This previously learnt information is based on rules and general knowledge stored in long-term memory. It is possible that this test will show some relationship to other tests included in the study that may also rely on past experience and acquired knowledge.

4.15.9 Doors Test, Set B – (Doors and People sub-test, Baddeley, Emslie, and Nimmo-Smith, 1994)

This is a visual long-term memory test that reflects depth of processing. The stimuli are coloured pictures of doors with a variety of doors photographed from different types of buildings, for example, churches, barns, garages, houses, and sheds. This test is included to provide a balance with the abstract visual design task. The Doors test requires the encoding and retrieval of concrete objects, whereas the abstract visual design task requires the encoding and retrieval of abstract pictures.

4.15.10 Tower of Hanoi

The Tower of Hanoi test, according to Zhang & Norman (1994), is a distributed cognitive task that requires the processing of information about a visual internal representation of the problem. This task provides scope for the investigation of planning strategies, strategy choice, strategy execution, and the representation of problems (Scholnick & Friedman 1993). As a result of investigations on patients with frontal lobe damage, Shallice (1982) considered that this test is valid in the assessment of central executive processing and executive reasoning. As the central executive component of working memory is considered to play a role in the planning and selection of particular strategies (Baddeley 1993), it is speculated that this task may correlate with complex addition and multiplication problems. Both the Tower of Hanoi and complex arithmetic problems require strategy choice and planning abilities together with the manipulation of information.

4.15.11 Stroop Effect – (Stroop, 1935)

The Stroop effect is associated with speed of processing and automaticity. This test reflects executive control and inhibitory processes. For the purpose of this study the original test is used where cognitive interference is created using coloured ink to spell incongruous colour names.

4.15.12 Trail Making Test, Part B – (Reitan, 1958)

The Trail Making Test comprises Part A and Part B. Part A reflects a measure of speed and involves the linking of numbers in ascending order. Part B integrates two independent series, letters and numbers, and reflects mental flexibility and is considered to be a reliable measure of central executive processing. Part B was selected for this study as it requires control of attention for the ability to move between numbers and letters yet maintaining them in the ascending sequence. It is considered that this test requires visual-spatial processing together with central executive input. So it is predicted that this task will correlate with complex arithmetic problems.

The Trail Making Test has been used to differentiate cognitive abilities between control participants and patients suffering from brain damage. Reitan's (1958) study indicated a significant difference in performance between the two groups suggesting that some patients appeared to have difficulty with the spatial distribution task in the test.

4.15.13 Block Design, Design number 13 – (WAIS-R III 1997, Sub-test)

The block design task originally designed by Kohs (1923) appears to be a measure of the ability to analyze abstract figures visually and produce a reconstruction of the original stimulus pattern. According to Royer (1977) there are five functionally independent component processes – pattern perception, pattern re-coding, block selection and manipulation and pattern comparison. Strategies used in this task are thought to consist of a segmentation of pattern areas and a matching of blocks, one at a time, with the segmented parts. In using the block design task from the WAIS-RIII it is possible that some of the patterns may not be segmented into small component parts but 'chunked' by colour or shape. For the purposes of this study it is suggested that the block design task employs encoding strategies and visual spatial processing which may relate to the visuo-spatial sketchpad component of working memory. Furthermore, this task reflects central executive processing and the findings from the study may indicate a correlation with complex arithmetic problem solving.

4.15.14 Forward and backward digit span tasks (WAIS-R III 1997, Sub-test)

The forward and backward digit span tasks are included in the study to assess working memory capacity. These tests require the maintaining of information in short-term memory. The phonological loop component of the working memory model is considered to play a role in the rehearsal of the information to be remembered and retrieved. The backward digit span test in particular requires central executive control and the results may show a correlation to other tests that require executive functioning, for example, Trails Making Test and the Stroop Effect.

4.15.15 Short Story - (Rivermead Behavioural Memory Test sub-test, Wilson, Cockburn & Baddeley, 1991)

Collectively the tests from the Rivermead Behavioural Memory Test (RBMT) by Wilson, Cockburn & Baddeley (1991) are designed to assess impairment of everyday memory functioning. For the purpose of this study a short story was selected from the RBMT and read to the participants. Delayed recall of the story is intended to assess retrieval from long-term verbal memory.

Table 4.7 Summary of the tests included in the factor analytic study.

Test name
English lexical decision test
French lexical decision test
Simple addition
Simple multiplication
Subitizing numbers
Subitizing circles
Magnitude judgement of numbers
Magnitude judgement of animals
Rotation of letters
Abstract pictures test
Complex addition
Complex multiplication
Basic arithmetic facts
Doors test (Set B)
Tower of Hanoi
Stroop Effect
Trails making (Test Part B)
Block design (WAIS-R III Design number 13)
Forward digit span (WAIS-R III sub-test)
Backward digit span (WAIS-R III sub-test)
Short story (delayed recall) (RBMT)

4.15 Conclusion

This chapter has reviewed early factor analytic studies that have given a strong foundation on which to build in order to expand our knowledge of numerical cognition. On the basis of this review tests to be included in the present study have been selected in relation to arithmetic competence and the cognitive domains that were highlighted in Chapters 1, 2 and 3. The rationale of the first study is to incorporate into a single factor analysis precise measures of all the main hypothesised cognitive processes that have been postulated in the various theories of numerical cognition. Furthermore, to use alongside these precise measures cognitive tests relating to short and long-term memory processes in order to elucidate the extent to which the various cognitive processes may underpin the various numerical abilities. A detailed account of the method used for Study 1 is given in Chapter 5.

Chapter 5 – Method used in Study 1: A factor analytic study

5.1 Outline

Twenty-one tests covering a spectrum of cognitive processes were given to one hundred participants. The tests reflected a wide range of variables, to include not only arithmetic tests but tests involving the components of working memory and other cognitive processes. The statistical technique of factor analysis was used to analyze the data.

5.2 Research Methododology

5.2.1 Verification and production

Researchers have typically used two methods in the study of numerical cognition - verification and production. In the following paragraphs a brief review is given of these two methods both of which are used in the present study.

Using the verification method participants are shown arithmetic problems such as 3 + 7 = 10, or 5 x 3 = 15 and must indicate as quickly as possible by pressing an appropriate key on the computer keyboard whether or not the answer is correct or incorrect. It is suggested that the production of the answer utilises four stages (Parkman & Groen 1971, Ashcraft 1982, Campbell 1987).

encoding ⟶ computational or retrieval ⟶ comparison stage ⟶ response

Figure 5.1 Verification method - Four stage processing model

The first stage involves the encoding of the problem. The second stage is the computational or retrieval process associated with the presented problems. The third stage compares the retrieved answer with the presented answer and a decision process as to whether the retrieved answer matches the presented answer. The fourth stage is the response.

In contrast the production method requires participants to state the answer to a given problem. This, therefore produces, a slightly different picture as shown in Figure 5.2.

encoding $\longrightarrow$ computational or retrieval stage $\longrightarrow$ verbal response

Figure 5.2 Production method - Three stage model

This model proposes only three stages as the comparison stage is excluded. According to Campbell (1987) this poses the question as to whether or not verification and production tasks provide an equal insight into the skills and processes utilised in solving arithmetic problems. To assist in answering this question Campbell (1987) conducted two reaction time experiments to compare the relative difficulty of multiplication problems using verification and production task methods. The design was repeated measures using 36 simple multiplication problems. The 36 problems were divided into two sets - 16 high error problems and 20 low error problems. The relative difficulty of each problem was ascertained from a pilot study. The aim was to investigate whether there was an interaction between the different task methods and the difficulty of the problem. If, as seen in Figure 1, the verification method includes a comparison stage that influences each trial equally, then production and verification methods should show similar patterns across reaction time and error scores.

The results showed that in the production task there were significant differences in error scores between the low error and high error problem sets. However, in the verification task there were no differences in error scores between the two sets. A comparison of reaction time scores indicated no significant interaction between the verification and production method. Analyses of reaction times for every problem across participants showed a greater difference between the reaction times of problems with fast times using the production method as compared to their corresponding verification times than between problems with slow reaction times in production and the verification reaction times. The conclusion reached, based on the difficulty of the problems, was that there are differences in accuracy and speed between the two methods.

The results suggested that the presented answer in a verification task can act as a priming stimulus which in turn directly affects the process of generating an answer. However, what is the stage or stages within the two models (see Figures 5.1 and 5.2) that affects the differences between the models? According to Campbell (1987) the verification method affects the computational or retrieval stage. This finding does not lend support to Ashcraft, Fierman & Bartolotta's (1984) research using verification and production methods to study mental addition. According to Ashcraft et al. (1984) the presented answer using the verification method affects the comparison stage but has no effect on the computational or retrieval stage utilised in both models. The general conclusion drawn by Campbell (1987) was that patterns of reaction time and error scores may differ depending upon which method of stimuli presentation is used. Zbrodoff & Logan (1990; 2000) suggested that verification stimuli affect retrieval processing differently from production stimuli. This seems to be connected to the priming mechanism. The reason for this was considered to be connected to the presentation of the answers, as it is necessary in a verification task to present incorrect answers on one section of the trials. According to Ashcraft (1995) verification tasks are a recognition process and production tasks rely more on recall. It has also been suggested (Campbell & Tarling 1996; Butterworth et al. 2001) that arithmetic verification tasks can be achieved by the use of strategies and being familiar with the numbers in the problem and therefore do not require retrieval of specific facts. The present study uses both the verification and production methods.

5.3 Participants

Twenty-eight students volunteered to participate in a pilot study and one hundred students volunteered to participate in the completed study, 67 females and 33 males, from university, further education and schools ('A' level students). The ages ranged from 16 to 50 with a mean age of 23 and standard deviation 7. Within the sample population there were diverse cultural and employment backgrounds. Due to the inclusion of the French lexical decision task participants were asked if they had learned French. All the participants in the study had learnt French during their secondary education.

5.4 Ethical considerations

- Participation was voluntary.
- Participants were clearly informed as to the content of the tests and length of time it would take to complete them. (Up to 2 hours).
- Participants were informed that at any time they could terminate the testing. (No participant asked to terminate the battery of tests).
- Participants were asked to sign a consent form indicating their agreement to proceed with the tests.
- Participants were informed that their results would be confidential and numbers and not names were assigned to all taking part in the study.
- Participants were asked at regular intervals if they wished to rest. (No one took up this option).

5.5 Materials/Apparatus

Twenty-one tests were administered to all the participants. Ten computer based tests were specifically written for the study with the stimuli presented using the Superlab software programme. Three verbally presented tests were specifically written, with the remaining eight tests carefully selected from published material.

Superlab software is designed for the generation of stimuli. The programme not only allows for participants to produce their responses using the computer keypad but it also records verbal responses. When participants respond using the keypad, reaction time data and correct and error scores are recorded and automatically transferred to Excel for further analysis. However, when verbal responses are given, the programme records reaction time data but correct and error scores are recorded by the experimenter. Factor analysis of the data was conducted using SPSS (The Statistical Package for the Social Scientist). Table 5.1 shows the order in which the tests were presented to the participants.

Table 5.1 Order of presentation of the tests

Test number	Test name
1	Simple addition
2	Simple multiplication
3	English lexical decision
4	Subitizing numbers
5	French lexical decision
6	Subitizing circles
7	Magnitude judgement of numbers
8	Rotation of letters
9	Magnitude judgement of animals
10	Abstract visual pictures
11	Short story (RBMT)
12	Complex addition
13	Basic arithmetic facts
14	Complex multiplication
	Memory recall of test number 11
15	Tower of Hanoi
16	Stroop Effect
17	Forward digit span (WAIS sub-test)
18	Trails Making Test (Part B)
19	Backward digit span (WAIS sub-test)
20	Block design (WAIS sub-test, design number 13)
21	Doors Test (Set B)

The presentation of the tests was counterbalanced with the first fifty participants completing the computer based tests (1 – 10) first followed by the verbally presented tests (11 – 21) while the remaining fifty completed the verbally presented tests followed by the computer based tests. The aim of counterbalancing was to even out order effects. Due to the large battery of tests used and in particular the size of the computerised battery the order of presentation of the tests within the verbally presented tests and the computer based tests was not randomized. For analysis the data set was combined and analyzed as a whole.

As the battery of tests took up to two hours to complete, participants were given a rest period of fifteen minutes between the computer presentation of tests and the verbally presented tests. The reverse procedure was adoped for the remaining participants who began with the verbally presented tests. During the rest period participants did not carry out any arithmetic related activities.

5.6 Pilot Study

A pilot study using the battery of tests was conducted on 28 participants who were to be representative of those taking part in the main study. From this pilot study it was found that the three specifically written verbally presented tests (complex addition and multiplication and basic arithmetic facts) and the tests from previously published material did not produce floor or ceiling effects so no alteration was required and a satisfactory spread of scores was achieved.

The pilot study showed that the computer based tests, with the exception of tests of simple addition and simple multiplication required alteration. The findings suggested that the remaining computer tests, with the exception of the abstract picture test, were not generating sufficient data from each of the participants as the length of time the stimuli were presented on the screen was too short. Therefore, participants were not able to make their responses within the given time. The length of time that the stimuli were presented on the screen was increased from 2 seconds to 5 seconds. This was consistent across all the computer tests that required a keyed reaction time response.

Subitizing numbers and subitizing circles tests were re-written. These tests were originally designed so that the participants produced their answer with a keyed response. The response was to press keys 1, 2, 3, or 4 to confirm how many objects were present on the screen during each of the trials. This method of response, because of the need to select of one of four keys, seemed to be confusing to the participants, making reaction time data and correct and error scores unreliable. The tests were altered to verification tests. The participants were presented with the stimuli on the screen and above the stimuli an answer was provided. Participants were required to respond with the 'M' key if the presented answer was correct or the 'Z' key if the presented answer was incorrect. The alterations were made so that the response system would be consistent and comparable to the other computer tests. A full description of these tests is given later in this chapter.

The results from the pilot study for the abstract visual pictures tests, which consisted of twenty different stimuli, indicated that five of the stimuli were consistently too easy and five stimuli were consistently too difficult. This result was producing ceiling and floor effects and these 10 stimuli were removed from the test. The remaining 10 stimuli were considered to be an adequate representation of the test and to discriminate across the sample.

5.7 Procedure

Participants were tested individually. Fifty participants were presented with the computer-based tests to commence with followed by the verbally presented tests. The reverse procedure was adopted for the remaining fifty participants. At the point of changing between either the computer based tests or the verbally presented test participants were given a fifteen-minute rest period to eliminate fatigue effects. During the rest period participants were not involved with any arithmetic related activities. Instructions for all the tests were consistent across the participants. Practice trials were given for all the computer-based tests, but these trials were not included in the analysis of the data. The order of presentation of the stimuli used in the computer-based tests was consistent for all participants.

5.8 Descriptions of the 21 tests

5.8.1 Simple addition (Test 1)

There are 64 possible single digit simple addition problems taking combinations from 2 through to 9. From the possible 64 simple addition problems, 28 were selected for use in this test. The number one, zero or same number combinations were not used.

Examples of simple addition stimuli
9 + 2 =
6 + 7 =
8 + 3 =
2 + 4 =

Twenty-eight s ingle d igit a ddition p roblems w ere presented individually on the computer screen. The stimuli remained on the screen for a maximum of 5 seconds or until the participants produced their answer verbally. The Superlab programme automatically recorded reaction time data for the solving of each addition problem, and the experimenter recorded correct scores produced by the participants' verbal responses. As the programme is sensitive to noise when voice response is required participants were asked to keep fairly quiet throughout this task. The microphone was kept to a standard level. Data used for analysis were reaction time scores for correct answers in milli-seconds.

5.8.2 Simple multiplication (Test 2)

There are 64 possible single digit simple multiplication problems taking combinations from 2 through to 9. From the possible 64 simple multiplication problems, 28 were selected for use in this test. The number one, zero or same number combinations were not used.

Examples of simple multiplication stimui
5 x 3 =
9 x 6 =
8 x 2 =
3 x 2 =

The same procedure was used as in Test 1, simple addition.

5.8.3 *English lexical decision task (Test 3)*

Participants were shown in total a series of twelve words and twelve non-words presented individually on the computer screen. If the word on the screen was a word which could be found in the English dictionary participants were required to press the 'M' key on the keyboard and if the word on the screen was a non word they were required to press the 'Z' key. The stimuli remained on the screen for 5 seconds or until the correct response was made. Reaction time data for correct responses was used in the analysis.

Examples of English lexical decision stimuli

Words	**Non-Words**
Vase	Haffle
Drip	Verd
Bowl	Ooly

5.8.4 *Subitizing numbers (Mandler & Shebo, 1982) (Test 4)*

Participants were shown a series of groups of numbers with twenty-four trials presented to each participant. The numbers used for each group varied from two to nine and within each group the same number was used. The total number of digits within each group did not exceed four. For example, in one item three 3's were presented on the screen. The spatial organization of the groups varied across trials. Participants were asked to calculate the total of the numbers presented in each group. In the example described above 3 + 3 + 3 = 9. On the screen above each group a number was shown in a box. If the number in the box was the same as the total of the numbers in the group added together the participant was required to press the 'M' key. If the number in the box did not equal the total of the numbers added together participants were required to press the 'Z' key. Twelve stimuli were correct and twelve stimuli were incorrect. The stimuli remained on the screen for 5 seconds or until the correct response was given. Reaction time data for correct responses were collected for analysis.

Examples of subitizing numbers

Example requiring a correct response

$\boxed{27}$

9

9

9

Example requiring an incorrect response

$\boxed{7}$

7

7

5.8.5 French lexical decision task (Test 5)

Examples of French lexical decision stimuli

Words	**Non-Words**
Chat	Flombie
Lait	Delphie
Cheval	Ebit

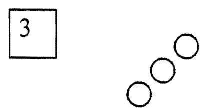
Participants were shown in total a series of twelve words and twelve non-words presented individually on the computer screen. If the word on the screen was a word which could be found in the French dictionary participants were required to press the 'M' key on the keyboard and if the word on the screen was a non word they were required to press the 'Z' key. The stimuli remained on the screen for 5 seconds or until the correct response was made. Reaction time data for correctly identifying if the stimuli were words or non-words was used in the analysis.

The English and French lexical decision tests were separated by the subitizing numbers task to reduce learning effects.

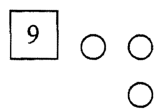
5.8.6 Subitizing circles (Test 6)

Examples of subitizing Cirlces

Example requiring a correct response



Example requiring an incorrect response



Participants were shown a series of groups of circles. Each circle was 0.25 inches in diameter. Twenty-four trials were presented. The number of circles used in each group ranged from two to four and the spatial organization of the groups varied across trials. Participants were asked to add up the number of circles presented in each group. On the screen above each group a number was shown in a box. This number did not exceed 27. Participants were required to identify if the number in the box above the circles represented the exact number of circles presented on the screen. If the number in the box was correct participants pressed the 'M' key. If the number in the box did not equal the total number of circles participants were required to press the 'Z' key. Twelve stimuli were correct and twelve stimuli were incorrect. The stimuli remained on the screen for 5 seconds or until the correct response was given. Reaction time data for correct responses was collected for analysis.

5.8.7 *Magnitude comparison of number (Test 7)*

Numbers	Difference between numbers	Examples of pairs of numbers
Small numbers Difference of 1	Low difference between the numbers	3 4
Small numbers Difference of 6	High difference between the numbers	3 9
Large numbers Difference of 1	Low difference between the numbers	53 54
Large numbers Difference of 6	High difference between the numbers	53 59

Participants received 80 trials in the magnitude comparison of numbers task and selected the larger number from a pair. Each stimulus consisted of two numbers (both Arabic numbers) in the range from 2 to 9 to represent low numbers and in the range of 51 to 98 to represent high numbers. Forty trials were constructed to represent low numbers and forty trials to represent high numbers. Numbers including zeros were excluded from the trials (for example, 10, 50, 60). Pairs of high numbers consisted of numbers from the same decade, such as 54 57, and 87 88. The numbers were presented horizontally on the computer screen and were separated by 4 centimetres. The larger operand appeared in the right position in forty trials and in the left position for the remaining forty trials.

The distance between the pairs of digits was identified as a small distance if the numerical value between the digits was one or two. A large difference was identified if the numerical value between the digits was four, five, six, or seven, (i.e. the absolute numerical difference between the two numbers). Table 5.2 below outlines the stimuli used in the eighty trials.

Table 5.2 Stimuli used in the magnitude comparison of numbers

Block of trials	Numerical distance between the pairs of numbers	Example of pairs of low numbers	Example of pairs of high numbers
10	1	2 - 3	
10	1		87 - 88
10	2	2 – 4	
10	2		77 – 79
2	4	5 - 1	
2	4		75 – 71
6	5	3 - 8	
6	5		57 – 52
6	6	2 - 8	
6	6		68 – 62
4	7	1 - 8	
4	7		89 – 82
4	8	1 - 9	69 - 61

Within each block of trials half required a right hand response and the remaining blocks a left hand response.

Twenty trials had a numerical distance of one (2 3, 87 88) with 10 of the trials made up of low numbers and 10 trials made up of high numbers. There were 20 pairs with a distance of two (2 4, 77 79), 10 trials made up of low numbers and the remaining 10 of high numbers. Four pairs had a distance of four (5 1, 75 71), with 2 trials of low numbers and 2 trials of high numbers. Twelve pairs had a distance of five (3 8, 57 52), with 6 trials of low numbers and 6 of high numbers. Twelve pairs had a distance of six (2 8, 68 62), with 6 trials of low numbers and 6 of high numbers. Eight pairs with a distance of seven (1 8, 89 82), 4 trials of low numbers and 4 trials of high numbers and four pairs had a distance of eight (1 9, 69 61). The 80 trials were randomly presented with the order of presentation consistent across participants.

The Superlab programme was coded with the above information as shown in Table 5.3

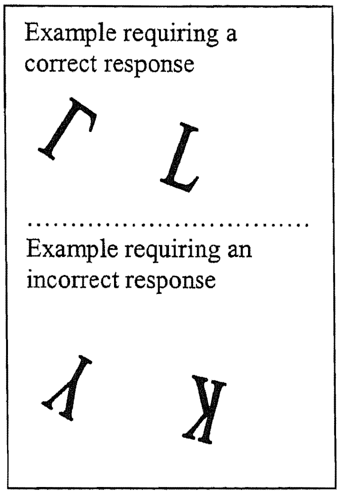
Table 5.3 Coding procedure for the magnitude comparison of numbers.

Code	Code title
1/response	Response with the right hand (Mm key)
2/response	Response with the left hand (Zz key)
1/large number	Large number – from 51 to98
2/small number	Small number – from 1 to 9
1/large distance	Numerical distance – 4, 5, 6, 7, or 8
2/small distance	Numerical distance – 1 or 2

For each pair of stimuli three codes were selected to provide the required data for analysis. There was an interval of 1500 milli-seconds before the presentation of the stimuli and an interval of 2500 milli-seconds following each pair. The stimuli remained on the screen for 5 seconds or until the participants made the correct response. During this time participants made their response. Participants were required to press the 'M' key with their right hand if the larger of the numbers was on the right of the screen or pressed the 'Z' key with their left hand if the larger of the numbers was on the left of the screen. Reaction time data for the number of corrct responses received from the right and left hand were used in the analysis.

5.8.8 Rotation of letters (Shepard & Matzler, 1971) (Test 8)

This task required participants to respond as to whether or not two letters presented on the screen were the same or not. In this test participants were shown a series of twenty pairs of letters with all the letters written in upper case. Ten pairs contained the same letter and ten pairs were of different letters. The pairs of letters were rotated to different angles with the angle ranging from a rotation of 40 degrees to 300 degrees. Participants were required to identify if the pairs of letters presented on the screen were the same or different letters. If the letters were the same they were required to press the 'M' key if they were different they were required to press the 'Z' key. Ten stimuli required a right hand response and the remaining ten a left-hand response. Analysis of the data was based upon reaction time data for correct responses.



5.8.9 Magnitude comparison of nouns (Paivio, 1975) (Test 9)

Participants received 40 trials in the magnitude comparison of animals task and selected the larger animal from a pair. The stimuli were written as words. This task assessed the comprehension of size with the size of the animal based upon its relative size and not the length of the animal word. Twenty pairs required a right hand response and the remaining twenty pairs a left-hand response.

Examples of animal stimuli		
Size of animals	**Difference between the animals**	**Examples of pairs of animals**
Small animals	Low difference between the animals	rabbit squirrel
Small animals	High difference between the animals	mouse cat
Large animals	Low difference between the animals	wolf sheep
Large animals	High difference between the animals	elephant goat

The stimuli for this test are taken from research by Paivio (1975). This research provides ranked mean scores for 174 objects including 60 animals. To obtain the mean scores participants were asked to rank the objects on a 9-point scale where 1 represented the smallest size and 9 the largest size. From the calculated mean scores Paivio (1975) constructed size ratios with a ratio of 1.0 to 3.99 for the smallest size range, from 2.0 to 6.99 for the intermediate range and from 3.0 to 8.99 for the largest size range. For the purposes of the present experiment the stimuli have been drawn from only two size ratios, the smallest and largest size ratio. The smallest size ratio is from 1.00 to 3.99 and the largest from 4.00 to 8.99.

The 40 trials were sub-divided into 20 trials to include small animals. 10 of the trials represented small animals with a small size distance between the animals, (e.g. lobster-turtle) and a further 10 trials represented small animals with a large size distance between the animals (e.g. mouse-squirrel). The remaining 20 trials were made up of 10 trials to include large animals with a small size distance between them (e.g. dog-fox) and 10 trials of large animals with a large size distance between them (e.g. elephant-goat). Table 5.4 below shows examples of the stimuli used in the blocks of trials for the magnitude judgement of animals.

Table 5.4 Stimuli and ranked mean scores (Paivio 1975) used in the magnitude comparison of animals.

Trials	**Range of ranked mean scores**	**Pairs of small animals -small distance between the animals**	**Pairs of small animals - large distance between the animals**	**Pairs of large animals - small distance between the animals**	**Pairs of large animals - large distance between the animals**
10	.02 - .10	Lobster – turtle			
10	.50 – 2.50		Mouse – squirrel		
10	.02 - .10			Dog – fox	
10	.50 – 2.50				Elephant - goat

The distance between the animals was arrived at by subtracting the ranked mean scores from a pair of animals where both animals belonged to either the small or the large group according to the research by Paivio (1975). Therefore 10 trials consisted of pairs of small animals with a ranked mean difference ranging from .02 to .10 to represent a small distance between them and 10 trials with a ranked mean difference of .50 to 2.50 to represent a large distance between small animals. A similar procedure was used for large pairs of animals where 10 trials consisted of a ranked mean difference ranging from .02 to .10 for a small distance between large animals and 10 trials with a ranked mean difference of .50 to 2.50 for a large animal with a large distance between the animals. Within each block of 10 trials five trials required a right hand response and five a left-hand response. The presentation of the trials was randomized with the randomization consistent across the participants. The Superlab programme was coded with the above information as shown in Table 5.5.

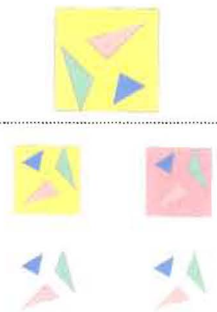
Table 5.5 The coding procedure for the magnitude comparison of animals.

Code	Code title
1/response	Response with the right hand (Mm key)
2/response	Response with the left hand (Zz key)
1/large animal	Large animal
2/small animal	Small animal
1/large distance	Distance between animals
2/small distance	Distance between animals

For each pair of stimuli three codes were selected to provide the required data for analysis. There was an interval of 1500 milli-seconds before the presentation of the stimuli and an interval of 2500 milli-seconds following each pair. The stimuli remained on the screen for 5 seconds or until the correct response was made. During this time participants made their response. The participants were required to press the 'M' key with their right hand if the larger of the animals was on the right of the screen or pressed the 'Z' key with their left hand if the larger of the animals was on the left of the screen. Reaction time data for the number of correct responses received from the right and left handwere used in the analysis.

5.8.10 Abstract pictures test (Test 10)

Examples of abstract pictures test



The abstract visual picture test required participants to look at and remember a single abstract picture presented on the computer for 5 seconds. The picture then left the screen and there followed a blank screen for 5 seconds. During the 5 seconds participants were required to memorize the picture. Following the blank screen four pictures appeared on the screen labeled, A, B, C and D. Participants were required to select the picture that exactly matched the first picture displayed and press the corresponding key to denote their answer. Number correct were used in the analysis. The test comprised 10 trials in total.

5.8.11 Short Story (Rivermead Behavioural Memory Test sub-test (Wilson, Cockburn & Baddeley, 1991) (Test 11)

Short Story

Mr. Brian / Kelly / a Security Express employee / was shot dead / on Monday / during a bank raid / in Brighton. The four raiders / all wore masks / and one carried / a sawn-off / shotgun. / Police detectives / were sifting through / eye witness accounts / last night. / A police spokesman said / 'He was a very brave ma. / He went for / the armed raider / and put up a hell of a fight.'

The participants were asked to listen to and remember a short passage of prose. At the end of the story participants were asked to remember as much of the story as possible and were told that they would be asked to recall it later on in the battery of tests. After the completion of tests 12, 13, and 14 participants were asked to recall the story. The story contained a total of 21 ideas. Score sheets were marked with the number of correct ideas recalled and the total of the correct ideas recalled out of a possible 21 for each participant was used in the analysis of the data.

5.8.12 Complex addition (Test 12)

Examples of complex addition

36 + 49 =

86 + 93 =

42 + 89 =

49 + 57 =

27 + 31 =

Participants were required to complete 20 two-digit verbally presented addition problems. Participants were asked to mentally calculate without the use of paper and pencil the answer to the problem and give their answer verbally. There was no time limit set for the participants to provide a response. Correct scores were recorded.

The twenty problems varied in difficulty depending on the involvement of carrying operations. Four of the addition problems required carrying operations on the units column. Four required carrying operations on the tens column and not the units column. Four required carrying both on the units and tens columns. Four required c arrying operations on t he u nits c olumn w hich c aused carrying o n t he tens column and the remaining four required no carrying operations.

5.8.13 Basic arithmetic facts (Test 13)

Ten questions in total were verbally presented to the participants. These questions were not based on arithmetic table facts such as multiplication tables but were designed to assess participants' arithmetic knowledge, After the participants were given each question they were required to produce their answer verbally. Participants did not use paper and pencil to work out their answers. Correct responses were used in the analysis.

> **Examples of basic arithmetic facts**
>
> How would you work out the area of a square?
>
> How many sides has a hexagon?

5.8.14 Complex multiplication (Test 14)

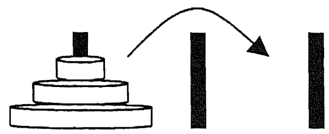
Participants were required to complete 20, two-digit verbally presented multiplication problems. Participants were asked to mentally calculate the answer to the problem and give their answer verbally. There was no time limit set for the participants to provide a response. Correct scores were recorded.

> **Examples of complex multiplication**
>
> 14 x 5 =
> 73 x 3 =
> 54 x 6 =
> 23 x 3 =

The twenty problems varied in difficulty depending on the involvement of carrying operations. Three of the multiplication problems required carrying operations o n the units column. Three required carrying operations on the tens column and not the units column. Eleven problems required carrying operations on both the units and tens column and three required no carrying operations.

5.8.15 Tower of Hanoi (Test 15)

Example of the Tower of Hanoi



Participants were presented with three vertical pegs in a row the first of which had three disks stacked on it in order of size. The goal of the problem was to have all the disks stacked in descending order on the last peg. There are three constraints:

- Only one disk may be moved at a time.
- Any disc not being currently moved must remain on a peg.
- Thirdly a larger disc may not be placed on a smaller disk.

The length of time taken and the number of moves to successfully complete the task were recorded in seconds.

5.8.16 Stroop Effect (Stroop, 1935) (Test 16)

Examples of the Stroop effect

RED	Blue
YELLOW	Green

Participants were shown a series of 24 colour words, for example, red, blue, green and yellow. The colour words were written in different coloured ink, for example, the word red was written in blue ink and the word yellow was written in green ink. Participants were required to say aloud the colour that the word was written in and not read the colour word. The time taken to read the list correctly was recorded.

5.8.17 Forward digit span - WAIS-R III sub-test (Test 17)

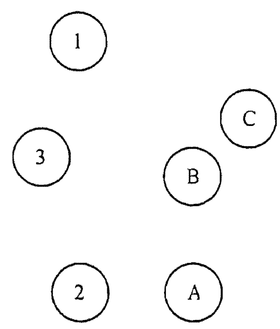
Examples of forward digit span

5 – 8 – 2
6 – 9 – 4
6 – 4 – 3 – 9
7 – 2 – 8 - 6

The forward digit span task required participants to listen to a series of number sequences read at a rate of one digit per second. Participants were then required to repeat each sequence in the same order to the experimenter. The series began with three digits, for example, 5-8-2 and increased to 9 digits in the sequence. Each item consisted of two trials with each trial consisting of the same number of digits. The scoring was recorded as 2 when participants correctly recalled the two series of numbers within section. One was recorded if only one of the series was correctly recalled and 0 if neither of the series were recalled in the correct order. Scores were totalled with a maximum of 14 correct for each participant. The test was discontinued after failure on both trial 1 and trial 2 of any item.

5.8.18 Trail Making Test, Part B (Reitan, 1958) (Test 18)

Trail Making Test



Numbers from 1 to 13 and letters A to L were randomly distributed on a sheet of A4 plain paper. Participants were required to commence at number 1 and join the numbers and letters together in an ascending sequence. For example, number 1 will be joined to letter A and letter A to number 2 and number 2 to letter B and so on. This task requires control of attention for the moving between numbers and letters yet maintaining them in the ascending sequence. There was no time limit placed on the participants and the time taken to complete the task was recorded.

5.8.19 Backward digit span task - WAIS-R III sub-test (Test 19)

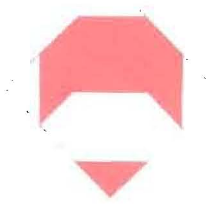
Examples of backward digit span

2 – 4
5 – 8
6 – 2 – 9
4 – 1 – 5

The backward digit span task required participants to listen to a series of number sequences spoken at a rate of one per second. Participants were then required to repeat the digits in the reverse order of presentation, for example, 2-4 would be recalled as 4-2. The number of digits in each series increased from 2 to 8. Each item consisted of two trials with each trial consisting of the same number of digits. The scoring was recorded as 2 when participants correctly recalled the two series of numbers within a section. One was recorded if only one of the series was correctly recalled and 0 if neither of the series were recalled. Scores were totalled with a maximum of 14 correct for each participant. The test was discontinued after failure on both trial 1 and trial 2 of any item.

5.8.20 Block Design, design number 13 WAIS-R III Sub-test (Test 20)

Examples of block design



The design chosen for this test used nine blocks. In this sub-test the participants were shown a two-colour design on paper and asked to replicate it using the nine blocks. The design remained in front of the participants during the time they were replicating it. The time taken to complete the design was recorded in seconds.

5.8.21 Doors Test, Set B – (Doors and People sub-test, Baddeley, Emslie, Nimmo-Smith 1994) (Test 21)

Examples of Doors test

For the purpose of this study Set B from the Doors and People Test was used. Participants were shown a series of 12 different kinds of doors. Each door was shown for 3 seconds before moving on to the next one. Following the presentation of the 12 doors participants were shown 12 pages with each page containing four different photographs of doors. One of the four doors exactly matched a previously seen door with the other three used as distracters. Participants were required to recall the target door. The presentation of the 12 pages containing the four different photographs was given in random order and did not follow the presentation of the individually presented target doors. Participants were scored on correct responses with a possible maximum score of 12.

Table 5 .6 T able indicating the method of data collection for each of the tests described above. Reaction time data was based on the correct responses received from each trial.

Test	Test name	Data received
1	Simple addition	Reaction time
2	Simple multiplication	Reaction time
3	English lexical decision	Reaction time
4	Subitizing numbers	Reaction time
5	French lexical decision	Reaction time
6	Subitizing circles	Reaction time
7	Magnitude judgement of numbers	Reaction time
8	Rotation of letters	Reaction time
9	Magnitude judgement of animals	Reaction time
10	Abstract visual pictures	Correct responses
11	Short story (RBMT)	Read to participants
12	Complex addition	Correct responses
13	Basic arithmetic facts	Correct responses
14	Complex multiplication	Correct responses
	Memory recall of short story (test number 11)	Correct number of ideas recalled
15	Tower of Hanoi	Time taken in seconds
16	Stroop Effect	Time taken in seconds
17	Forward digit span (WAIS-R III sub-test)	Correct sequences of numbers recalled
18	Trails Making Test (Part B)	Time taken in seconds
19	Backward digit span (WAIS-R III sub-test)	Correct sequences of numbers recalled
20	Block design (WAIS-R III design number 13)	Time taken in seconds
21	Doors Test (Set B)	Correct number recalled

5.9 Conclusion

This chapter has given a detailed account of the procedure employed in the factor analytic study and the tests included. Full descriptions of the tests have been given together with the examples of the stimuli used. Chapter 6 discusses the analysis of the data obtained for the twenty-one variables.

Chapter 6 – Results of Study 1: A factor analytic study

6.1 Results

The data produced from the battery of tests was entered into SPSS and subjected to factor analysis. This analysis was employed to identify the common elements that underlie the variables in the data set.

The decision to use exploratory factor analysis arises from the aims of the study. Firstly, it was intended to incorporate into a single analysis precise measures of all the main hypothesised cognitive processes that have been postulated in the various influential theories of numerical cognition. Secondly, alongside these precise measures, cognitive measures relating to short and long-term memory processes were used in order to elucidate the extent to which various basic cognitive processes may underpin numerical operations. Thirdly, since no previous study had focused on this specific combination of determinants of numerical cognition, it was planned to augment the limited prior understanding of how the variables relate to one another.

The exploratory factor analysis method is, according to Stevens (1996), a theory-generating method which is in contrast to confirmatory factor analysis which is considered to be a theory-testing procedure. Confirmatory factor analysis allows the specification of an exact factor model in advance of statistical analysis together with knowing which variables will load onto which factors.

Exploratory factor analysis is the preferred method as it is used here as a method of determining the number of underlying factors that summarise the twenty one variables. This is achieved by showing the intercorrelations among the variables without having prior specifications of what these factors might be. Therefore, the aim is to identify the main factors that reflect the latent, hypothetical, underlying concepts found behind the correlation matrix.

The method of factor analysis used to analyse the data set is principal component analysis with Varimax rotation, as this is the recommended technique for exploratory analysis (Stevens 1996). This method aims to discover which sets of variables form subsets that are relatively independent of one another. This method is exploratory and is used as a tool to reduce a large set of variables to smaller sets in order to identify groups of inter-related variables. The first principal component that is extracted accounts for the most variance, and the further components are extracted in order of the size of the variance that they account for.

The Varimax rotation keeps the axes orthogonal and 90 degrees apart. Varimax aims to maximise the sum of variances of squared loadings in the columns of the factor matrix. This produces in each factor, loadings that are either high or near zero. This is particularly helpful for an exploratory analysis as it maximises the probability of identifying potentially distinct underlying factors.

6.1.1 Tests of appropriateness

Prior to the analysis of the data set, tests were applied to the data to assess the suitability of the input variables.

6.1.2 Bartlett Test of Sphericity

The Bartlett Test of Sphericity looks for differences within the correlation matrix with the aim of establishing that not all the correlations are the same. This test investigates the hypothesis that the correlation matrix is an identity matrix with the components uncorrelated and all having equal variance. The null hypothesis is that the correlation matrix is a unity matrix. The null hypothesis can be rejected if the results are $p < .05$. The result in this case was that for the Bartlett Test of Sphericity $p < .0001$.

6.1.3 Kaiser-Meyer-Olkin Measure of Sampling Adequacy

The Kaiser-Meyer-Olkin Measure of sampling adequacy compares the magnitudes of the observed correlation coefficients to the magnitudes of the partial correlation coefficients. The data set is suitable for factor analysis if the correlation matrix contains some correlations between variables greater than .30. This provides an objective index of the adequacy of the correlations. For factor analysis to be appropriate for this data set the result of the Kaiser-MeyerOlkin test is required to be 0.5 or above. In this case the result of the Kaiser-Meyer-Olkin Measure of Sampling Adequacy was .72.

This result indicates that the magnitude of the observed correlation coefficients is higher than the partial correlation coefficients. The value of .72 suggests that the correlations of pairs of variables can be explained by other variables. Therefore the data is satisfactory for factor analysis.

6.2 Descriptive statistics for the 21 variables

The mean score and skewness for the 21 variables, (standard deviations in parenthesis) are shown in Table 6.1. The data collected from nine of the tests was reaction time data recorded in milli-seconds, for a further nine tests correct responses to the stimuli were recorded and the data from the three remaining tests was recorded in seconds.

A diverse range of tests was required for the factor analytic study to elucidate the factor structure of the processes that may underlie numerical cognition. The mean reaction time data indicates that participants took a longer time to complete the simple addition and multiplication, subitizing numbers, French lexical decision and the magnitude judgement of animals tasks compared to the English lexical decision task, subitizing circles, rotation of letters and the magnitude judgement of numbers. The remaining tests show a range of mean scores. This range of scores recorded for the 21 variables may reflect the diverse nature of the tests that the participants were asked to complete for the factor analytic study. The standard deviations and skewness show the distribution of the data for each of the variables.

Table 6.1 Mean scores, and skewness for the 100 participants (standard deviations in parenthesis)

Test	Test name	Data received	Mean scores (standard deviation in parenthesis)	Skewness
1	Simple addition	Reaction time (milli-seconds)	1430.3 (547.5)	1.6
2	Simple multiplication	Reaction time (milli-seconds)	1923.1 (631.8)	.940
3	English lexical decision	Reaction time (milli-seconds)	805.7 (172)	1
4	Subitizing numbers	Reaction time (milli-seconds)	2988 (556)	-.004
5	French lexical decision	Reaction time (milli-seconds)	1163.1 (374.7)	1
6	Subitizing circles	Reaction time (milli-seconds)	935 (178.9)	.4
7	Magnitude judgement of numbers	Reaction time (milli-seconds)	668.1 (124.5)	1.3
8	Rotation of letters	Reaction time (milli-seconds)	867.5 (178)	.6
9	Magnitude judgement of animals	Reaction time (milli-seconds)	1437.5 (289.5)	-.4
10	Abstract visual pictures	Correct responses	4.5 (1.5)	.22
11	Short story (RBMT)	Correct number of ideas recalled	11.3 (3.3)	.1
12	Complex addition	Correct responses	16.3 (2.7)	-.8
13	Basic arithmetic facts	Correct responses	9.3 (2.7)	-.3
14	Complex multiplication	Correct responses	13.8 (3.2)	-.06
15	Tower of Hanoi	Time taken in seconds	18.8 (15.7)	2.7
16	Stroop Effect	Time taken in seconds	20.7 (4.6)	.6
17	Forward digit span	Correct sequences of numbers recalled	8.9 (2)	.37
18	Trails Making Test B	Time taken in seconds	41.7 (13.2)	.1
19	Backward digit span	Correct sequences of numbers recalled	7.5 (2.3)	.2
20	Block design	Time taken in seconds	123.7 (134.5)	5
21	Doors Test	Correct number recalled	9.1 (1.9)	-.4

The following sections examine the correlation matrix for the 21 variables followed by the factor extraction.

6.3 The correlation matrix

Based on the results of the tests of appropriateness a factor analysis was conducted on the data set. A correlation matrix for the 21 variables was produced that represented the relationships among the set of variables used in this study. In this correlation the values on the diagonal are 1.0 as each of the variables correlates perfectly with itself. The off-diagonal or column elements are the correlations between all variable pairs.

The correlation matrix is computed for all variables (see Table 6.1 for the variable number and title).

Table 6.2 Variable number and title.

Variable number	Variable title
1	Simple addition
2	Simple multiplication
3	English lexical decision task
4	Subitizing numbers
5	French lexical decision task
6	Subitizing circles
7	Magnitude judgement of numbers
8	Rotation of letters
9	Magnitude judgement of animals
10	Abstract visual pictures
11	Story (RBMT)
12	Complex addition
13	Basic arithmetic facts
14	Complex multiplication
15	Tower of Hanoi
16	Stroop effect
17	Forward digit span (WAIS-R III sub-test)
18	Trail Making Test (Part B)
19	Backward digit span (WAIS-R III sub-test)
20	Block design (Design number 13)
21	Doors Test Set B

Variables that do not appear to be related to the other variables can be identified from this matrix. The correlation matrix (Table 6.2) displays the correlation coefficients between each variable and each of the other 20 variables included in the analysis. The correlation matrix was the basis for a preliminary study of the associations between the constellation of tests included in the study. It provided an insight into the processes underlying numerical cognition which will be elucidated by the revealed factor structure.

Table 6.3 Correlation matrix. Column labels refer to the same tasks, in the same order, as the row labels.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1																				
2	**.757**$^{\varnothing}$	1																			
3	.157	**.276**$^{\varnothing}$	1																		
4	.142	.074	-.044	1																	
5	.096	.126	**.615**$^{\varnothing}$	.131	1																
6	**.318**$^{\varnothing}$	**.168***	**.508**$^{\varnothing}$	.027	**.372**$^{\varnothing}$	1															
7	**.204***	.145	**.598**$^{\varnothing}$	.011	**.380**$^{\varnothing}$	**.555**$^{\varnothing}$	1														
8	.090	.023	**.534**$^{\varnothing}$	-.077	**.390**$^{\varnothing}$	**.549**$^{\varnothing}$	**.464**$^{\varnothing}$	1													
9	**.220***	**.205***	**.350**$^{\varnothing}$	0.78	**.281**$^{\varnothing}$	**.391**$^{\varnothing}$	**.388**$^{\varnothing}$	**.298**$^{\varnothing}$	1												
10	**-.184***	-.094	-.015	-.102	-.031	**-.204***	-.036	-.114	.001	1											
11	-.027	-.127	-.058	-.072	-.013	.038	-.149	-.021	-.099	-.042	1										
12	-.143	-.125	-.053	**-.202***	-.032	-.119	-.198	.015	-.153	.067	.137	1									
13,	-.087	-.113	-.157	-.161	**-.229***	**-.174***	-.137	-.020	-.001	.159	.186	**.402**$^{\varnothing}$	1								
14	**-.269**$^{\varnothing}$	**-.269**$^{\varnothing}$	-.099	-.127	-.106	**-.236**$^{\varnothing}$	**-.270**$^{\varnothing}$	-.004	-.135	**.187***	**.201***	**.516**$^{\varnothing}$	**.487**$^{\varnothing}$	1							
15	-.790	-.024	**.175***	-.018	.062	.112	**.236**$^{\varnothing}$	0.58	.066	-.010	**-.247**$^{\varnothing}$	-.139	-.035	-.121	1						
16	.024	.127	.048	-.003	-.320	.070	**.255**$^{\varnothing}$	-.001	.080	.039	**-.263**$^{\varnothing}$	**-.250**$^{\varnothing}$	-.137	**-.266**$^{\varnothing}$	.085	1					
17	**-.324**$^{\varnothing}$	**-.320**$^{\varnothing}$	-.091	-.097	.055	-.034	-.104	.138	.042	.055	.153	**.378**$^{\varnothing}$	**.314**$^{\varnothing}$	**.427**$^{\varnothing}$	.068	**-.303**$^{\varnothing}$	1				
18	-.044	.118	.123	.083	.043	.078	**.253**$^{\varnothing}$	**.173***	.008	-.149	**-.255**$^{\varnothing}$	**-.326**$^{\varnothing}$	**.269**$^{\varnothing}$	**-.374**$^{\varnothing}$	**.203***	**.405**$^{\varnothing}$	**-.288**$^{\varnothing}$	1			
19	**-.170***	-.225	-.152	-.026	-.027	-.052	**-.181***	-.062	.051	.003	**.395**$^{\varnothing}$	**.417**$^{\varnothing}$	**.222***	**.415**$^{\varnothing}$	-.016	**-.353**$^{\varnothing}$	**.555**$^{\varnothing}$	**-.399**$^{\varnothing}$	1		
20	.058	.085	.091	.079	.153	.131	**.206***	.016	.090	.025	-.020	-.143	-.144	-.120	.161	.097	.088	**.212***	-.006	1	
21	-.071	-.116	**.318**$^{\varnothing}$	.095	**-.210***	-.210	**-.275**$^{\varnothing}$	-.143	-.159	.157	**.186***	.136	**.203***	**.221***	.092	-.017	.072	-.149	.109	**-.258**$^{\varnothing}$	1

* significant at $p < .05$ $^{\varnothing}$significant at $p < .01$

A number of patterns can be discerned in the correlation matrix. The correlations between variable 1, simple addition, and variable 2, simple multiplication, have markedly high magnitudes and are significant at $p < .01$. It was expected that these two tasks would show a high relationship with each other as they both are related to the accessing of previously learnt numerical information from long-term memory. The magnitude judgement of numbers, variable 7, and the magnitude judgement of animals, variable 9, are significant at $p < .01$. The rationale previously discussed in Chapter 4 suggests that relative size will not be restricted to numbers if the distance effect is consistent across different types of stimuli. The correlations observed here suggest that this is the case with different stimuli producing the same effect. The analysis of the two magnitude judgement tests will be discussed in greater depth in Chapter 8.

Subitizing numbers, variable 4, and subitizing circles, variable 6, show no significant correlation. A possible explanation for this is that these tests, as described in Chapter 5, were verification tests. The reaction time recorded was not based only on the speed of visualising the number of visual objects presented on the screen but incorporated additional processes of decision making to produce a response. It is possible that the slower reaction times recorded for the subitizing numbers task compared to the subitizing circles task may reflect additional cognitive processes incorporated into the subitizing numbers task.

It was predicted that the subitizing tests would correlate significantly with the simple addition and multiplication tests. This association is not apparent in the correlation matrix with respect to subitizing numbers, variable 4. Subitizing circles, variable 6, is highly correlated with simple addition at $p < .01$ and simple multiplication at $p < .05$. There is a significant correlation between subitizing numbers and complex addition, test 12, at $p < .05$ and a nearly significant correlation with basic arithmetic facts variable 13, of .056. Subitizing circles and complex multiplication indicate a correlation at $p < .01$.

Subitizing circles shows a significant correlation with the English and French lexical decision tasks at $p < .01$. This suggests that the relationship between the two tests may be associated with the semantic element of the tests. An alternative explanation would be that both tests required a decision process. This decision process may involve whether a word was a word or a non-word in the English lexical decision task or to decide on the number of circles presented on the screen as required in the subitizing circles task. The concept of subitizing is investigated further using experimental methods in Chapter 10.

The six variables significantly correlated with the English lexical decision task, variable 3, at $p < .01$ are the French lexical decision task, subitizing circles, magnitude judgement of numbers and animals rotation of letters and the Doors test. The Tower of Hanoi is correlated with the English lexical decision task at $p < .05$. These tests could be considered as similar, with the exception of variable 21, the Doors and People test, and variable 15, the Tower of Hanoi, in that the tasks produced reaction time responses. However, the design of the tests and the stimuli used were very different and consisted of either numerical, word or letter stimuli. The English and French tests were included on the basis that they may utilise direct retrieval processes from memory and may show a correspondence to simple addition and multiplication problems. This association is not reflected in the correlation matrix.

The Doors test, a multiple choice test, is significantly correlated to the English lexical decision task at $p < .01$. This test is a visual long-term memory test that reflects depth of processing. It is interesting to note that six of the seven tasks correlated with the English lexical decision task were either language based or numerically based yet they show a significant relationship with one another. The tasks do appear to draw on a common resource.

Complex addition, variable 12, is highly correlated with six other variables, knowledge facts, variable 13, complex multiplication, variable 14, the Stroop effect, variable 16, forward and backward digit-span, variables 17 and 19, and

Trails Making test part B, variable 18, at $p < .01$. Four of the variables are consistent with the view that they are associated with working memory functioning. Forward and backward digit span tests, tests 17 and 19 respectfully, are associated with the phonological loop component of working memory that plays a role in the rehearsal of information to be remembered and retrieved. The backward digit span test also requires executive control. Similarly the Stroop test, variable 16, and the Trails Making Test Part B, variable 18, are generally accepted as requiring central executive processing. Basic arithmetic facts, variable 13, is also significantly correlated to complex addition and multiplication which had not been predicted as this test was designed to assess arithmetic knowledge based on rules and general knowledge stored in long-term memory. However, executive processing and visuo-spatial memory may have played a role in the execution of this test.

A prominent and interesting feature arising from the significant correlation between complex addition and complex multiplication is that according to the correlation matrix, both these tests are linked to the generally accepted tests that are associated with working memory. This is in line with the theoretical prediction that the solving of complex arithmetic problems will utilise working memory. The association between complex arithmetic and working memory is investigated further in Chapter 11.

The short story recall task taken from the Rivermead Behavioural Memory test, variable 11, is highly correlated at $p < .01$ to 4 variables traditionally associated with working memory. These variables are the Tower of Hanoi, Stroop test, Trail Making Test B and backward digit span. Complex multiplication, Doors test and basic arithmetic facts are significantly correlated to variable 11, short story at $p < .05$. This association of variables was not predicted as the delayed recall of the story was intended to assess retrieval from long-term verbal memory. However the correlation matrix indicates an association with short-term memory tests.

Examination of the correlations between variable 10, abstract visual pictures, variable 20, block design, and variable 21, the Doors and People test, indicates a low and insignificant correlation. This is contrary to what had been expected. A possible explanation for this may be that each of these tests is using a different processing procedure. It could be argued that block design requires, in part, visual spatial memory but in conjunction with aspects of problem solving involving central executive processing. On the basis that the block design test utilises working memory processing it would be expected to show a correlation with complex arithmetic. However, this is not reflected in the correlation matrix. The Doors and People test may require not only visual spatial memory but also the phonological loop for rehearsal of information to be remembered and then later recalled. The abstract visual pictures test is perhaps the only measure of visual spatial memory in this battery, which does not substantially involve other mental processes.

6.4 Discussion of the correlation matrix

The correlation matrix highlighted expected and unexpected associations between variables with interesting associations emerging. Significant correlations were found between the language-based tests, magnitude judgement tests, subitizing circles and rotation of letters. Although these tests are based on the use of different stimuli, for example numerical stimuli, words, circles and letters, it is reaction time data that is used in the analysis, therefore reflecting variations between participants in speed of processing. However, this association between the above variables may reflect not only the common element of speed of processing but also the speed of retrieval of stored information from long-term memory. However, to explain the association between the two tests of magnitude judgement, animals and numbers, a more in-depth study is required as merely reflecting speed of processing appears to be inadequate. The common element associated with these two tests may be related to the distance effect.

The correlation matrix further highlights the strong and significant correlation between simple addition and simple multiplication. The data analysed from these tests is reaction time data perhaps reflecting speed of processing and retrieval of stored knowledge. If the language based tests, magnitude judgement tests, subitizing circles and rotation of letters tests reflect speed of processing variations between participants it could be expected that tests of simple addition, simple multiplication and subitizing numbers, also producing reaction time data, would show significant correlations to the above tests. However, this is not reflected in the correlation matrix.

Complex addition and multiplication show strong associations with tests generally assumed to reflect short-term memory, for example, Stroop, forward and backward digit span and the Trails Making Test Part B. This apparent relationship suggests that short-term memory plays a part in the solving of complex arithmetic problems. Relating the above tests to the components of working memory, Stroop and the Trails Making Test are considered to reflect central executive processing with forward and backward digit span reflecting the phonological loop. However, at this stage it is unclear as to the role of working memory in the solving of complex arithmetic problems. This issue will be investigated in greater depth in Chapter 11.

The correlation matrix shows a number of interesting associations that may be linked to underlying phenomena. To gain a fuller insight into the relationships between the variables a factor analysis of the data is appropriate to investigate the factor structure.

6.5 Factor extraction

Based on the results of the tests of appropriateness the 21 variables meet the criteria for inclusion in the factor analysis. To determine how many factors to extract in Table 6.3 displays the proportion of variance accounted for by the 21 variables together with the corresponding eigenvalues.

Table 6.4 Eigenvalues of the 21 variables

Variables	Eigenvalue	% of variance	Cumulative %
1	4.505	21.453	21.453
2	2.774	13.208	34.661
3	1.788	8.515	43.176
4	1.375	6.546	49.722
5	1.178	5.608	55.330
6	1.142	5.439	60.769
7	1.052	5.010	65.779
8	.971	4.626	70.405
9	.867	4.129	74.534
10	.764	3.639	78.173
11	.670	3.191	81.364
12	617	2.937	84.301
13	.568	2.704	87.006
14	497	2.367	89.373
15	.450	2.145	91.518
16	.442	2.106	93.624
17	.363	1.727	95.351
18	.290	1.382	96.733
19	.280	1.333	98.066
20	.261	1.245	99.311
21	.145	.689	100.000

The eigenvalue indicates the total amount of variance explained by each variable. Factor 1 has an eigenvalue of 4.505 accounting for 21.453% of the total variance of the 21 factors. The eigenvalues and percentages of variance decrease gradually from factor 1 to factor 21. However the eigenvalues remain at above one for factors 1 to factor 7. It is necessary to give consideration to the number of factors to be included in the factor analysis. The issue of how many useful and meaningful factors to include in a factor analytic study is a question that is greatly debated.

Kaiser (1960) recommended that factors should be included for rotation if the eigenvalues are greater than one. On this basis 7 factors meet the eigenvalue criterion of one. However, Cattell (1978) has shown that the inclusion of all eigenvalues above one can overestimate the number of factors. A scree plot proposed by Cattell (1978) is intended to identify the number of factors for rotation. Figure 6.1 shows the scree plot for the factors included in this study. The conventional way for interpreting a scree plot is that the cutoff point for factor rotation is where the line of the slope changes. Cattell's scree plot is generally accepted to provide a relatively reliable criterion for factor selection (Stevens 1996). The scree plot provides a helpful visual representation of the reported eigenvalues for each of the variables, which with careful interpretation can aid the decision making process.

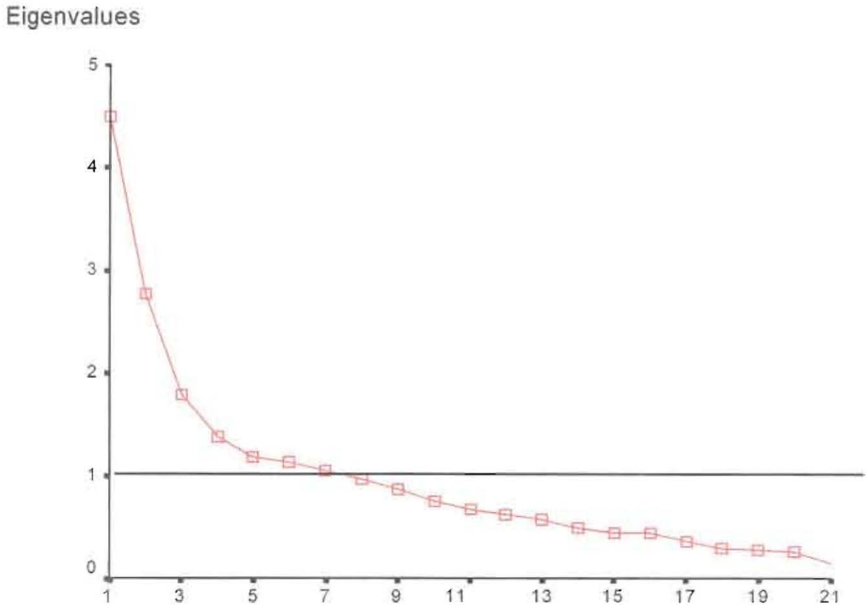


Figure 6.1 Scree plot showing the 21 variables

The Cattell scree plot above, Figure 6.1, shows the factors or components as the X axis and the corresponding eigenvalues as the Y axis. This plot includes the 21 unrotated factors or components showing that the first seven factors have eigenvalues above one. Moving to the right of the plot toward later components the eigenvalues decrease quite sharply. However, when the sharp drop ceases and the curve forms an elbow toward a less steep decline it is suggested by Cattell that all further components after the one commencing the elbow should be excluded from the analysis. In general, where the line in the graph changes slope this coincides with the eigenvalues becoming less than 1. However, in this case, it appears that the slope changes after the fourth component yet eigenvalues are remaining up to component seven.

The above discussion highlights the factors relevant to a judgement on the number of factors to be rotated. Based on these factors and the information provided by the scree plot both a four factor and a seven factor solution will be examined. A comparison analysis will be made of the two solutions and justification will be provided for a full examination of the seven factor solution.

6.6 Results of the factor analysis (principal components, Varimax rotation)

6.6.1 Four factor solution

Factor analysis of the 21 variables was conducted on the data from 100 participants. A four factor solution was selected based on the findings from the scree plot. Table 6.4 presents the four factors and their percentage of variance after rotation.

Table 6.5 Eigenvalues and % Variance accounted for after the 4 factor rotation

Factor	Eigenvalue	% of variance	Cumulative %
1	4.50	21.45	21.45
2	2.77	13.20	34.66
3	1.78	8.51	43.17
4	1.37	6.54	49.72

As can be seen from this table the four factors account for 49.72% explained variance. The first 2 factors account for the most explained variance of 34.66%. The remaining 2 factors account for 15.06% of the explained variance.

The results of the principal components, Varimax rotation is shown in Table 6.5. The figures have been given to two decimal places. All factor loadings exceeding a positive criterion level of .50 have been included as well as all negative loadings in excess of -.50. All variables loaded onto only one factor (> .50) with the exception of variable 14 complex multiplication that reached the .50 loading criterion on Factor 2 as well as a slightly higher loading on Factor 3.

Table 6.6 Rotated 4 Factor Matrix showing factor loadings, eigenvalues and % variance accounted for by each factor.

Test	Variable	Factor 1	Factor 2	Factor 3	Factor 4
3	English lexical decision task	**.82**	-.14	.02	.05
7	Magnitude judgement of numbers	**.75**	-.29	.06	.02
8	Rotation of letters	**.74**	.01	.08	.06
6	Subitizing circles	**.73**	.02	-.19	.16
5	French lexical decision task	**.67**	.08	-.20	.03
9	Magnitude judgement of animals	**.57**	.00	.00	.18
19	Backward digit span	.00	**.76**	.05	-.21
16	Stroop effect	.04	**-.68**	.06	.00
18	Trails making test B	.11	**-.62**	-.28	-.13
11	Story (RBMT)	.09	**.61**	.07	.08
17	Forward digit span	.13	**.59**	.20	-.469
13	Basic arithmetic facts	.07	.28	**.67**	.01
14	Complex multiplication	.09	**.50**	**.57**	-.19
12	Complex addition	.00	.49	**52**	.06
10	Abstract visual pictures	.07	.15	**50**	-.13
1	Simple addition	.23	.04	-.18	**.83**
2	Simple multiplication	.21	-.19	.09	**.77**
4	Subitizing numbers	.06	.06	-.48	.13
20	Block design (WAIS-R)	.21	.05	-.39	-.24
15	Tower of Hanoi	.23	-.27	.05	-.38
21	Doors Test (Doors & People Test)	-.33	.15	.32	.143
	Eigenvalue	4.505	2.774	1.788	1.375
	% of total variance accounted for	21.45	13.21	8.52	6.55
	Number of tests	6	6	4	2

The first factor accounting for 21.45% of the variance has loadings from six variables. The variables are the English and French lexical decision tasks, the magnitude judgement of numbers and animals, rotation of letters and subitizing circles. The variables within this factor appear to relate to the speed of processing information as reaction time data was collected from these tests. However, the two variables in Factor 4, simple addition and multiplication, are also based on reaction time data, although they form a separate factor accounting for 6.55% of the variance.

Six variables load onto the second factor accounting for 13.21% of the variance. Five of these variables are generally associated with working memory processing with the exception of complex multiplication. Complex multiplication only reaches the loading criteria of > .50. Complex addition fails to reach the loading criteria at .49. However, it seems likely that the solving of complex arithmetic problems does involve working memory processing.

Factor 3, accounting for 8.52% of the variance, contains four variables. This factor highlights the relationship between the complex addition and multiplication tests and the abstract visual pictures test. This relationship suggests that there may be a visual spatial processing element in the solving of complex problems. The basic arithmetic facts task, variable 13, was considered to require the accessing of previously learnt information from long-term memory.

Factor 4 highlights a strong association between simple addition .83 and simple multiplication .78. This suggests that both variables share common elements. A possible explanation is that they both relied on previously learnt knowledge in a single school subject during early education.

6.6.2 Seven factor solution

A principal component factor analysis was conducted on the correlations of the twenty one variables. Seven factors were initially extracted with eigenvalues equal to or greater than 1.00. The percentage variance accounted for by each factor is shown in Table 6.6.

Table 6.7 % Variance accounted for after rotation taking eigenvalues greater than 1

Factor	Eigenvalue	% of variance	Cumulative %
1	4.50	21.45	21.45
2	2.77	13.21	34.66
3	1.79	8.52	43.18
4	1.38	6.55	49.72
5	1.18	5.61	55.33
6	1.14	5.44	60.77
7	1.05	5.01	65.78

As can be seen from this table the seven factors account for 65.78% of the total variance. The first 3 factors account for the most variance of 43.18%. The remaining 4 factors account for 22.60% of the variance.

Orthogonal rotation of the factors yielded the factor structure given in Table 6.7. The figures have been given to two decimal places. All factor loadings exceeding a positive criterion level of .50 have been shown in bold as well as all negative loadings in excess of -.50. All variables loaded onto only one factor (> .50) indicating that the solution had a simple structure. It should be noted that variable 11, Rivermead Behavioural Memory Test (story), and variable 13, basic arithmetic facts only just reached the .50 loading criteria on their factors.

Table 6.8 Rotated 7 Factor Matrix showing factor loadings, eigenvalues and % variance accounted for by each factor.

Test	Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7
3	English lexical decision task	**.80**	.08	.12	.07	.00	-.32	.18
8	Rotation of letters	**.79**	.01	.10	-.18	.04	.01	.17
6	Subitizing circles	**.76**	.03	.16	.07	.05	.02	-.30
7	Magnitude judgement of numbers	**.73**	-.26	.08	.00	.26	.03	.00
5	French lexical decision task	**.66**	.12	.00	.27	.04	-.37	.18
9	Magnitude judgement of animals	**.57**	.03	.25	.21	.19	.26	.12
19	Backward digit span	.04	**.79**	-.12	.01	.17	-.13	.12
17	Forward digit span	.08	**.67**	-.32	-.13	.32	.11	.01
16	Stroop effect	.07	**-.66**	.00	.05	.20	.25	.07
18	Trails making test B	.09	**-.65**	.07	.02	.30	.02	-.22
14	Complex multiplication	.08	**.58**	-.19	-.35	.06	.21	.26
12	Complex addition	.04	**.56**	.02	-.49	-.10	.06	.11
11	Story (RBMT)	.03	**.50**	.04	.08	-.24	.15	-.36
1	Simple addition	.15	.08	**.90**	.09	.05	.01	-.16
2	Simple multiplication	.10	-.16	**.89**	.00	.00	.08	.03
4	Subitizing numbers	.01	.01	.09	**.77**	.04	.13	.02
13	Basic arithmetic facts	.08	.37	.06	**-.50**	.06	.48	.16
20	Block design (WAIS-R)	.02	.05	.11	.21	**.69**	-.28	.00
15	Tower of Hanoi	.12	-.17	.11	.06	**.66**	.03	.02
21	Doors Test (Doors and People Test)	-.16	.09	.09	.16	-.26	**.75**	.09
10	Abstract visual pictures	.05	.02	.09	.02	.00	.12	**.83**
	Eigenvalue	4.505	2.774	1.788	1.375	1.178	1.142	1.052
	% of total variance accounted for	21.453	13.208	8.515	6.546	5.608	5.439	5.010
	Number of tests	6	7	2	2	2	1	1

Factor 1 accounts for 21.45% of the variance. Six variables load onto this factor. This factor contains the same variables as in the four factor solution. The variable loadings indicate only a slight difference between the four and seven factor solution.

Seven variables load onto Factor 2 accounting for 13.21% of the variance. Four of these variables are generally associated with working memory processing with the exception of complex multiplication, complex addition and the story taken from the Rivermead Behavioural Memory test. The variables contained in both the four and seven factor solution are the forward and backward digit span tests, the Stroop effect, Trails Making test Part B and complex multiplication. Complex addition does load onto the seven factor rotation in Factor 2 at .56. The story (RBMT) only just meets the loading criteria at .50 in the seven factor solution yet shows a stronger association to the other variables that load on this factor in the four factor solution at .61.

Factor 3, accounting for 8.52% of the variance, indicates a strong association between the variables simple addition at .90 and multiplication at .89. This factor is similar to Factor 4 in the four factor rotation where the association between the two variables is simple addition at .83 and simple multiplication at .78.

Factor 4 contains two variables that appear to be very different in their composition. Variable 4, subitizing numbers, is based upon reaction time data and variable 13, basic arithmetic facts, is based on previously learnt knowledge scored using correct responses. There is no equivalent factor in the four factor solution.

Two variables loading onto Factor 5 account for 5.61% of the total variance. Block design and the Tower of Hanoi are tests that are generally associated with central executive functioning. Factor 6, the Doors test, accounts for 5.44% of the variance and Factor 7, abstract visual pictures, accounts for 5.01% of the variance.

6.7 Comparison of the two solutions

The rotated factor loadings of the four and seven factors share common elements, in particular Factors 1 and 2. Factor 1 in both solutions contains the same variables, the English and French lexical decision tasks, magnitude judgement of numbers and animals, rotation of letters and subitizing circles. The variable loadings between the two solutions are very similar all meeting the loading criteria of > 0.5.

Factor 2 produced similar results in the four and seven rotated factor matrix with forward and backward digit span, Trail Making Test B, Stroop, short story and complex multiplication loading onto this factor in both solutions. Complex addition only just fails to meet the loading criteria in the four factor rotation at .49.

The tests of simple addition and multiplication in both rotations result in forming a separate factor. These tests form Factor 4 in the four factor solution and Factor 3 in the seven factor solution.

A difference between the four and seven factor rotation is evident with respect to complex addition and multiplication. In the four factor rotation the complex arithmetic tests share common elements with the abstract visual pictures and basic arithmetic facts tests together forming Factor 3. Complex multiplication also falls into Factor 2 in both the four and seven factor solution. With respect to complex arithmetic the results from both rotations suggest that a close relationship exists between the variables which may perhaps be linked to working memory processing.

Factor 4 in the seven factor rotation indicates associated elements between subitizing numbers and basic arithmetic facts. It is clearly evident that subitizing numbers, although analysed using reaction time data, shows no association with the tests loading onto Factor 1 which appear to be associated with speed of processing. The basic arithmetic facts test was analysed using correct responses and it is, therefore, difficult to interpret the association between these two tests.

Factor 5 of the seven factor rotation appears to be associated with central executive processing. The two tests that load on Factor 5 did not reach the loading criteria in the four factor rotation. The Doors test, which is the only test loading on Factor 6, did not reach the loading criteria in the four factor rotation and accounts for a very small percentage of the variance in the seven factor rotation. This is similar to the visual abstract pictures test. However, it is interesting that although associated with the complex arithmetic tests in the four factor solution, in the seven factor solution it forms Factor 7 accounting for only 5.01% of the variance.

Based on the comparison between the four and seven factor rotations it would seem that three factors remain consistent in both the rotations. These factors are Factor 1 and Factor 2 in both solutions and Factor 3 in the seven factor rotation and Factor 4 in the four factor rotation comprising the tests of simple arithmetic. The remaining factors in the seven factor solution are less stable. However, it seems appropriate to discuss the seven factor rotation in depth. This rotation is more informative as it teases out three additional factors and draws on results from more of the tests. A further justification is that all variables have been included with eigenvalues equal to or greater than 1. Table 6.8 provides an interpretation of the seven factors.

The following section discusses the seven factors individually to identify the model produced from the factor analytic study. The concluding section provides a general discussion of the factors and proposes further areas for experimental investigation. An overview of the implications for different models is presented in Chapter 7.

Table 6.9 Interpretation of the seven factors.

Factor	Proposed factor name	Test	Test name	loading
1	Access to representations	3	English lexical decision task	.80
		8	Rotation of letters	.79
		6	Subitizing circles	.76
		7	Magnitude judgement of numbers	.73
		5	French lexical decision task	.66
		9	Magnitude judgement of animals	.57
2	Working memory	19	Backward digit span (WAIS-R III)	.79
		17	Forward digit span (WAIS-R III)	.67
		16	Stroop effect	-.66
		18	Trail making test (Part B)	-.65
		14	Complex multiplication	.58
		12	Complex addition	.56
		11	Story (RBMT)	.50
3	Basic number facts	1	Simple addition	.90
		2	Simple multiplication	.89
4	Retrieval of mathematical knowledge	4	Subitizing numbers	.77
		13	Basic arithmetic facts	-.50
5	Central executive (planning)	20	Block design (Design number 13)	.69
		15	Tower of Hanoi	.66
6	Visual long-term memory	21	Doors test (Set B)	.75
7	Visual spatial short-term memory	10	Abstract visual pictures	.83

6.8 The seven factor solution

6.8.1 Factor 1 – Access to representations

The tasks loading onto this factor are the English and French lexical decision tasks, the magnitude judgement of numbers and animals, rotation of letters and subitizing circles. The stimuli used in these tasks included words and non-words, letters, arrangements of circles presented on a computer screen up to a maximum of four at any one time, pairs of numbers and animals used in the magnitude judgement tasks. All the stimuli were visually presented on the computer. To explain the diversity of the tasks included in this factor a number of concepts will be examined. Firstly how the numerical and word based information may be represented in memory and secondly visual attentional processes. These concepts are discussed in more depth in Chapters 9 and 10 respectfully.

Factor 1 appears to be linked to the speed of access and retrieval of information stored in long-term memory. This information may be stored in the form of representations. However, the question arises as to nature and form of the stored representations. In general, information stored in long-term memory may relate to episodic or semantic memory with the information represented using analogue or analytical processes. Imagery is also an important consideration particularly with reference to mental rotation and the encoding of stimuli.

The magnitude judgement of numerals task required participants to decide which of two numerals was larger in magnitude, for example, 8 or 5. Responses are reported to be slower the closer in magnitude the numerals are (Moyer & Landauer 1967, Dehaene 1992). Therefore response times are slower for stimulus pairs such as 9 and 8 than for pairs like 3 and 9. Dehaene (1992) drew on their finding when he developed the triple code theory and suggested that number semantic representations are often characterised as analogue representations. The numerical distance effect produced by analogue representations discussed in Chapter 2 suggests that numerical comparisons are carried out on internal semantic representations that reflect magnitude or quantity relations among numbers. According to McCloskey & Macaruso (1995) the semantic representation of 8, 9 and 3 may reflect that 8 and 9 are similar with respect to the quantities they represent, while 3 represents a much smaller quantity. The result is that the relative magnitude judgement is more difficult for 9 and 8 than for 9 and 3.

A contrasting view is the abstract modular theory of McCloskey et al (1985) who suggested that representations reflect the c haracteristics of numeral formats, for example phonological representations of number words and graphemic representations of digits. The view is that the external formats, number words or digits, are converted to internal semantic representations. The required calculation process is then carried out with the answer converted to either a phonological or graphemic representation depending on the format of output.

The encoding complex view proposed by Clark & Campbell (1991) assumes interaction among many forms of numerical representation. For example, the processing of Arabic or verbal numerals may activate "visual and written codes for digits, imaginal, analogue codes for magnitude, (e.g. number lines) and combined visual-motor representations (e.g. counting on fingers; using an abacus)" (p.205). According to this view the various forms of representation are interconnected in an associative network so that activation of one representational format leads directly or indirectly to activation of other formats, producing a multi-component "encoding complex".

If it is assumed that the judgement of magnitude involves analogical representations, then the question arises as to how the judgement of other quantities, for example the sizes of animals, is represented. It may be that these two tasks share the common element of semantic analogue representation.

The speed of word recognition was measured using two lexical decision tasks. According to Ellis & Young (1996) the reading of a word involves a visual analysis system, a visual input lexicon and a grapheme-phoneme conversion component. The visual analysis system identifies individual letters that make up a whole word. As these letters and subsequently the words become familiar, visual representations of words are established in the visual input lexicon. Each word becomes activated by its own representation. Therefore, the visual input lexicon receives input from letter recognition in the visual analysis system that in turn activates stored representations of meanings in the semantic system. However, in the skilled reader word recognition becomes more efficient and may rely more heavily on attentional demands rather than on accessing the semantic system.

Herdman & Lefevre (1992) examined the attentional demands of word recognition using a dual-task paradigm. Their prediction was that a highly skilled reader with efficient word recognition, assessed on speed and accuracy, would show little or no decrement in performance on concurrent secondary tasks. However, a less skilled reader with lower lexical decision scores would show larger decrements under dual task conditions as a result of having to attend to more than one stimulus at any one time. Dual task paradigm differences in single and dual-task performance are considered to identify the availability of attentional resources. The results of the study did suggest that accuracy measured by lexical decision performance was related to the attentional demands of word recognition. This lent support for the assumption that the efficiency of reading processes is related to attentional resources.

Mandler & Shebo (1982) outlined a subitizing model based on the recognition of the spatial configurations of the visual items to be enumerated. For example, the visual system may recognise three items as a triangular configuration. Whether the items are animals, different types of shapes or circles, three items can be transferred into a triangular shape very quickly. Four items can also be visualised as forming a shape, for example, a square. The ability to enumerate decreases as the number of items increases as the likelihood of producing a visual configuration at a glance is reduced. If the recognition of the spatial layout of the items involves recognition of shapes, this in turn will utilise semantic representations of images.

An alternative subitizing model proposed by Trick & Pylyshyn (1994a) suggests that up to four items can be enumerated with very few errors. However, enumerating larger number of items is much more susceptible to errors. They further suggested that visual analysis plays a large part in enumeration of items based on two stages. Firstly a spatial parallel, pre-attentive stage where identification of features such as colour, line orientation and brightness are derived from the items simultaneously at the image location. In other words all items to be enumerated are identified by their features at every image location at the same time. The second stage is the spatial serial, attentive stage where analysis occurs only at one location at a time. Here the attentional focus can be moved from one location to another in the image, but the focus of attention is the place in the image where the majority of attentional resources are concentrated or items to be enumerated exist. It is interesting to note that Variable 4, subitizing numbers, does not load onto this factor suggesting that there may be an underlying unique element to subitizing numbers.

Rotation of letters, Variable 9, can be interpreted in terms of mental representations. Shepard & Metzler (1971) presented participants with images of 3D objects in various orientations. On each trial a pair of images appeared and participants were required to decide whether the two images depicted the same or different objects regardless of any difference in orientation. The two patterns were to be judged as the same if they were the same shape though one was rotated but different if they were mirror images. The time it took participants to make same or different judgements was measured. The reaction times to decide same or different were found to relate to the amount by which the second pattern was rotated from the first. The results provided support for the view that participants mentally rotate one or both objects until they are mentally aligned with one another before the same-different judgement is made. Shepard & Metzler (1971) suggested that the mental rotation process is an internal analogue of physical rotation. In other words participants use a continuous yet direct process of rotation that brings the objects into alignment.

Cooper & Shepard (1973) conducted an experiment using single English letters or digits. On each trial letters or digits were presented either correctly written, or as a mirror reflected image. Both were shown at various rotated angles. Participants were required to decide whether the rotated characters were written in the standard manner or as a mirror image. The results were found to be similar to those of Cooper & Shepard (1973) with mean response times for judging a standard or mirror image increased as the test characters were rotated further from their upright orientations. According to Cooper & Shepard (1973) this suggested that participants were mentally rotating the characters in the shortest direction to the upright position regardless of whether the rotation was clockwise or anticlockwise. Furthermore the results provided some evidence for the use of a continuous mental rotation processes that brings images of two objects into correspondence or the image of a single object into alignment with an internal representation.

In the present test, rotation of letters, participants were presented with pairs of letters and were required to decide whether the two letters presented were the same letters or different letters regardless of any difference in orientation. It appears that this test is associated with the other tests loading onto this factor that utilise representational processes.

The common cognitive processes that appear to relate to the tasks loading onto Factor 1 are representations of the stimuli, semantic long-term memory and short-term visual attentional resources. The theoretical interpretations proposed above for the association between the variables loading on to Factor 1 are considered in greater detail through experimental work reported in Chapters 9, and 10.

6.8.2 Factor 2 – Working memory

Tests loading onto this factor are associated with the working memory model and include elements of executive control. Stroop (Stroop 1935) and Trails B (Reitan 1958) are considered to require central executive processing with forward digit span and backward digit span utilising the rehearsal processes associated with the phonological loop. Complex addition and multiplication may require the phonological loop for the rehearsal of numbers to be included in the calculation of multi-digit problems and the visuo spatial component for visualising the spatial layout of the problems. The relationship between the working memory model and complex arithmetic is discussed in more depth in Chapter 11.

The phonological loop according to Baddeley (1992) is assumed to comprise two sub-components. Firstly a phonological memory store, which hold traces of speech-based material. This material is assumed to disappear within approximately two seconds unless it is refreshed by the second component, a process of articulatory sub-vocal rehearsal. The assumption is that the forward and backward digit span tasks utilise both the components of the phonological loop.

The Stroop task investigates the concept that the naming of words can interfere with the task of naming the colour of the ink in which a word is written. The Stroop effect highlights many of the characteristics of automatic processing, demonstrating that conscious intentions have only limited control of the way information is processed. The difficulty appears when trying to avoid reading the colour words indicating that participants are unable to completely concentrate their attention on the requirements of the ink-naming task. As previously discussed automatic processes, such as reading words, can occur without conscious intention, therefore making it difficult to override this process when it is inappropriate for the Stroop task.

The central executive component of working memory is considered to be responsible for the integration and attentional control of information. It is further considered to mediate the planning of response sequences and supports the capacity of selective attention together with the temporary holding and manipulation of information (Baddeley 1992). The Stroop task requires attentional and executive control processes for successful completion. It is, therefore, suggested that it loads onto this factor because of the involvement of working memory processing.

The Trail Making Test B requires motor and spatial tracking skills and sequencing abilities. It also requires additional planning strategies for the participant to move between well rehearsed sequences, (numbers and letters). This test is generally accepted to reflect executive planning, therefore having an association in Factor 2 with the Stroop test.

The story taken from the Rivermead Behavioural Memory Test only just reached the loading criteria of > 0.5. The story was included as a delayed memory recall task intended to assess retrieval from long-term verbal memory. However, it is possible that for participants to refresh the story before recall they were utilising the phonological component of working memory. The results of this test may include both retrieval from long-term memory and working memory processes.

The most interesting aspect of this Factor for this research programme is the inclusion of complex addition and multiplication. Aschraft (1995) speculated that the central executive might be involved in carrying and borrowing procedures with the phonological loop holding intermediate values and the visuo-spatial sketchpad responsible for the visual characteristics of the problems. The relationship of complex arithmetic to working memory will be investigated in greater depth in Chapter 11 with particular emphasis on the involvement of the visuo-spatial sketchpad component.

A summary of the tests loading onto Factor 2 is presented in Table 6.9. This summary illustrates the involvement of the various components of working memory in relation to the tests included in this factor. Entries preceded by an asterisk represent predictions.

Table 6.10 Tests associated with the components of the Working Memory Model (Baddeley 1986).

Central Executive	Phonological Loop	Visuo-spatial Sketch Pad	Long-term memory
Stroop effect	Forward digit span		
Trail Making Test B	Backward digit span		
	Story (RBMT)	Story (RBMT)	Story (RBMT)
* Complex addition	* Complex addition	* Complex addition	
* Complex multiplication	* Complex multiplication	* Complex multiplication	

The above predictions surrounding the association between complex arithmetic and the components of working memory are discussed in more depth in Chapter 11.

6.8.3 *Factor 3 - Basic number facts*

This factor seems to be clearly a simple arithmetic factor indicating processes that are independent from those tests identified in Factor 1 and Factor 2. The tests loading onto Factor 1 and Factor 3 all required reaction time responses. However, simple arithmetic shows no correspondence to the tests in Factor 1. Although this factor accounts for a small percentage of the variance (8.52%) it has provided an interesting element within the study. The remaining numerically based tests appear to have been filtered out leaving the two simple arithmetic tests in one factor. This would seem to suggest that the remembering of number bonds may involve only long-term memory processes where other mathematical tasks are more complex. However, the models of numerical cognition are diverse in how they account for the retrieval of numerical information.

The abstract modular theory proposed by McCloskey et al. (1985) suggested a general cognitive architecture for numerical processing that holds assumptions about the form in which arithmetic facts are retrieved and answers are produced. This theory proposes that numerical processing is composed of three main components, numerical comprehension, calculation and numerical production. According to this theory all stimuli, regardless of the manner in which they are presented, are converted into an abstract semantic code in order to perform calculations. Arithmetic facts are retrieved using an abstract representation of the problem, and the answer is retrieved in an abstract semantic form that is subsequently converted into an appropriate form for output, for example written or spoken numerals.

Dehaene's (1992) triple code theory proposes that numerical processes are performed using one of three types of numerical code, an analogue magnitude representation, a visual Arabic number code or an auditory verbal word code. Dehaene suggested that arithmetic facts are stored and retrieved in an auditory-verbal form. To retrieve an arithmetic fact from memory a problem presented in Arabic numerals, for example 7 x 3, is converted into an auditory-verbal code (seven times three). Once converted into an auditory verbal code the arithmetic problem can be used to retrieve the appropriate answer from memory (for example seven times three is twenty-one).

The encoding complex theory of Campbell & Clark (1992) proposed that numerical information is stored in many different ways including visual, semantic, auditory and written forms. It is suggested that arithmetic fact retrieval involves complex interactions of many different format specific codes. Campbell (1995) suggested that arithmetic facts are represented in every form in which arithmetic expressions may be described, for example, visually, auditorily, in written words, in imaginary number lines, and in colour. However, how the various codes might interact to produce the answers to problems has not yet been detailed.

Although the exact retrieval processes cannot be speculated within the context of this study it is interesting that both the simple arithmetic tasks form an independent factor with the exclusion of all other tests. While this is a point of interest there are no significant aspects of each of the models that discriminate between them in order to clearly identify the processes involved in simple arithmetic.

6.8.4 Factor 4 – Retrieval of mathematical knowledge

Factor 4 accounting for 6.55% of the variance includes subitizing numbers, Variable 4, with the data based on reaction time scores. Basic arithmetic facts, Variable 13 was scored using correct verbal responses. The questions asked in the basic arithmetic facts tests included 'what is the area of a square', 'what is the area of a triangle', and 'how many sides has a hexagon'. The nature of the questions may involve a number of cognitive processes, for example retrieval of previously learnt information from long-term memory, the visual representations of spatial configurations and the semantic representation of images. For the completion of subitizing tasks it was earlier suggested that both spatial configurations and semantic representations of images may be involved. The nature of the questions in the basic arithmetic task may also involve the same cognitive processes of spatial and semantic representations. However, the extensive attentional resources that are generally associated with subitizing are not necessarily reflected in the basic arithmetic facts task.

6.8.5 Factor 5 – Central executive planning

Both the Tower of Hanoi and block design tasks are considered to reflect the central executive component of working memory. The Tower of Hanoi task is widely used as a tool to gauge problem solving and planning processes. The rationale underlying the use of this task is that to complete the task participants are required to visualise a number of 'moves' ahead before physically moving the disks. By visualising the 'moves' to complete the task participants are utilising planning and problem solving strategies.

For the successful completion of the Tower of Hanoi and the block design task participants were required to visualise patterns and form planning and problem solving strategies. As these two tests are traditionally associated with central executive processing it is plausible that they should load onto the same factor.

6.8.6 Factors 6 and 7 – Visual long-term memory and visual spatial short-term memory

Factor 6 and 7 account for a small percentage of the variance (Factor 6, 5.44% and Factor 7, 5.01%). Although both tests are visual spatial memory tests they are different in their composition. Therefore it is not surprising that they form separate factors. The Doors test is a visual long-term memory test that reflects depth of processing. The stimuli are coloured pictures of doors with a variety of doors photographed from different types of buildings. However, the abstract pictures test, a visual spatial memory task, uses abstract and not concrete pictures as in the Doors test. Further experimental work into the processing systems utilised for concrete and abstract pictures would be required.

6.9 Conclusion

This chapter has detailed the results of the extensive factor analytic study. This study was designed specifically to investigate the structure of numerical cognition and to apply the findings to the models of numerical cognition and the working memory model. The factor analytic study included a range of variables described in Chapters 5 and analysed in Chapter 6. A factor analytic study on this scale that includes a carefully selected range of variables has not been undertaken before. Analysis of the results has shown important findings that form the basis for the empirical work described in Chapters 8 - 11. Chapter 7 provides a summary of the findings from the factor analytic study.

Chapter 7 – Summary of the Factor Analytic Study

7.1 Introduction

The aim of this chapter is to highlight the main findings from the factor analytic study and to introduce how the findings have led to further investigations. The results of the factor analytic study have provided a solid foundation from which to explore in more depth specific aspects of numerical cognition. The further investigations involved reanalysis of some data reported above (Chapter 8) and four new pieces of experimental work (Chapters 9-11). The experimental work, using the dual task methodology, explores the main themes that emerge from the factor rotated matrix.

The way in which particular combinations of variables loaded onto the seven factors raises questions as to the structure of numerical cognition in relation to the models proposed by leading theorists. The empirical work that is described in the following chapters was designed to explore these questions in greater depth. Certain concepts (for example, the relationship between complex arithmetic and working memory) have received only a small amount of experimental attention in this field. However, the results of the factor analytic study suggested that experimental work on this topic would assist in elaborating on that specific aspect of numerical cognition. This and other questions that have arisen from the evaluation of results of the factor analytic study are discussed later in this chapter.

The discussion below covers each of the seven factors that emerged from the analysis in Chapter 6, giving detailed attention to the two factors that accounted for 34.6% of the variance. Where the findings have theoretical implications for influential models of numerical cognition, these implications are examined in some depth.

7.2 Factor 1 – Access to representations

7.2.1 Magnitude comparison tasks

Factor 1 was identified in Chapter 6 as referring to access to representations. This factor accommodates a number of varied numerical and non-numerical tests including lexical decision tasks, rotation of letters, subitizing and magnitude judgement tasks. This section discusses issues that revolve around the access to representations in relation to the two magnitude comparison tasks.

The significant correlation found between the magnitude judgement of animals and numbers (each of which loaded on Factor 1) needs to be interpreted in the light of earlier work in this field. Studies of simple tasks such as deciding which of two numerals is the larger or deciding if two numerals represent the same or different quantities have shown that the time required to chose the larger of two digits decreases as the numerical difference between the digits increases (Moyer & Landauer (1967). It has also been shown that when the numerical difference remains constant, the time taken to compare two digits decreases as the minimum digit decreases (Parkman 1971). These two effects are known as the distance effect and the minimum effect (see Chapter 1). An assumption derived from these two effects is that numbers are converted to representations of magnitude that are analogueous to representations such as size or brightness. On this basis magnitude representations are compared by the processes associated with perceptual comparisons (Buckley & Gillman 1974; Foltz 1982; Moyer & Landauer 1967). Dehaene (1992) suggested that humans have an approximate representation of numerical magnitude. He suggested that this approximate representation of magnitude could be characterised as a mental number line that becomes increasingly compressed as magnitudes increase. The number line is accessed by a process, which repeatedly activates small areas of the number line in approximately the correct location. This in turn produces a distribution of activation across the line. The question arises as to whether humans have an approximate representation for the magnitude judgement of animals and is the number line able to accommodate physical size? A further possibility is that size may be encoded as a property of any object and the strength of this encoding is greater for highly familiar items, for example, low numbers and common animals.

Dehaene (1992) argued that the analogue magnitude code plays a crucial role in understanding the quantity a numeral represents, therefore producing the semantic representation of numbers. On the basis of the factor analysis reported above and of Dehaene's concept of a semantic representation of magnitude the prediction in experimental work reported in Chapter 8 was that semantic processing is required for the execution of magnitude judgement of numbers and objects varying in physical size, in this case animals.

7.2.2 Subitizing

The subitizing circles task loads onto Factor 1, even though the subitizing numbers test appears to be unrelated to the tests in Factor 1 and forms an independent factor (Factor 4). Subitizing circles may be r elated to the speed of accessing semantically related concepts and the automatic recognition of the number of items clustered together. The prediction in the experimental work relating to subitizing is that pre-semantic processing is required for the successful execution of a subitizing task.

7.2.3 Magnitude comparison, subitizing and models of numerical cognition

How do the models of numerical cognition, reviewed in Chapter 2, accommodate the findings found in Factor 1? In McCloskey et al.'s (1985) model it is proposed that numerical information, in word form or numerical form, is initially transferred into an abstract semantic representation before further calculation. However, the results of this factor analysis suggest that full calculation is not necessary for the size comparison and subitizing tasks as the tests that do require calculation load onto Factor 2 and Factor 3 and not onto Factor 1. Thus McCloskey et al.'s (1985) model does not appear to accommodate the processes necessary for magnitude judgements and subitizing. However, these processes may be conducted through the abstract semantic system or the calculation component of the model. McCloskey et al.'s (1985) model seems to be a model for basic fact retrieval based on the procedures and processes used for simple arithmetic calculations. The model provides the facility for numerical input, transferred to an abstract semantic representation followed by access to arithmetic facts in long-term memory with the relevant information then transferred to short-term memory before the required output.

Clark & Campbell's (1991) abstract network model gave most attention to arithmetic fact retrieval. It is less concrete in its composition as compared to McCloskey and Dehaene's models. Clark & Campbell (1991) considered that humans possess the ability to store numerical information in many different ways including visual, semantic, written and auditory forms. Accordingly they argued that all numerical cognition involves a complex interaction of many different format-specific codes. Campbell (1995) suggested that arithmetic facts are represented by activating physical codes in every form in which arithmetic terminology may be described, for example, visually, in written words, in imaginary number lines, in colours etc. Campbell (1995) suggested that during arithmetic problem solving not only is there activation of physical codes but activation of a magnitude code. Throughout the network physical codes and magnitudes codes are connected through a series of nodes. The physical code structures receive secondary activation based on the digits' similarity in magnitude. However, Clark & Campbell (1991) and Campbell (1995) have not explained how the various codes may interact in detail. Although the abstract network theory provides a general theoretical framework the specific processes operating within the theory remain under specified.

7.2.4 Reanalysis of data on magnitude comparison

On the basis of findings relating to Factor 1 a further analysis of the data was made in Chapter 8 of the cognitive processing of magnitude judgement of numbers and animals. The analysis utilised a repeated measures 3-factor Anova to re-examine the data received from the 100 participants who took part in the factor analytic study.

The rationale for the re-examination of the data arose from the significant correlations between the two magnitude judgement tests and their loading on Factor 1. It was necessary in the light of this finding to consider in greater depth Dehaene's concept of a number line. The number line as discussed earlier in the chapter is involved in the process of representations of numerical size. The prediction was that as the two magnitude tests load onto Factor 1 both tests may be dependent on cognitive processes supporting the representation of size. It may be that the processes surrounding the number line are involved in accessing representations of size and are not necessarily exclusive to number comparisons. It was predicted that the Anova analysis would show no significant interactions between the magnitude judgements of animals and of numbers.

7.2.5 Experimental work on magnitude comparison

Chapter 9 presents further experimental work investigating the notion that numerical size comparison is underpinned by semantic encoding. This study used dual task methodology to investigate whether numerical processing is linked to a long-term semantic system. Two empirical studies are reviewed. The effects of the auditory presentation of a lexical processing task (experimental group) and a pre-lexical processing task (control group) and the visual presentation of a magnitude comparison of numbers task were investigated in Experiment 1. It is conceivable that quantity and size information about numbers could be encoded at a relatively early stage within the processing system such that stored representations of words give access to numerical sizes. Experiment 2 investigated processing at a semantic level. The working assumption was made that, if magnitude comparison of numbers involves accessing a long-term semantic store, then requiring participants to process aurally presented words (which requires semantic processing) will interfere with the magnitude comparison of visually presented numerical stimuli.

7.2.6 Experimental work on subitizing

Chapter 10 is a report of a dual task study designed to investigate the effects of lexical and pre-lexical processing on a visually presented subitizing task. The experiment investigated the assumption that the subitizing process may be based on pre-semantic or perceptual processes. As the total number of stimuli presented to the participants was no more than 4 items in each group, the Gestalt principles (for example, grouping or closure) might apply. The items presented in the group could be transformed into a canonical shape, for example a group of three items could represent a triangle. If, as part of an experimental paradigm, lexical and pre-lexical word tasks are presented aurally at the same time as a subitizing task there will be interference with the subitizing process, leading to longer reaction times. Thus the rationale for the experimental work in Chapter 10 was that if subitizing depends on early perceptual processing, an interference effect would be found when participants are simultaneously presented with a task involving the identification of non-words in a mixed list of words and non-words. A second experiment to investigate the possibility of semantic processing in a subitizing task was not carried out due to time constraints. However, Chapter 12 identifies future experimental work that would explore outstanding theoretical issues about the concept of subitizing.

7.3 Factor 2 - Working memory

Factor 2 (working memory) is of considerable interest in relation to models of numerical cognition because it includes tests that are traditionally associated with working memory together with the complex addition and multiplication tests. This suggests that complex addition and multiplication require some of the same cognitive processes as the traditional measures of working memory. The story test in the RBMT battery involves longer-term memory processes. A possible explanation for this test loading onto the working memory factor is that it may also have required the central executive and phonological loop in order to refresh the information until recall of the story was required.

Another interesting question to emerge from a consideration of Factor 2 concerns the association between the tests generally related to the central executive component of working memory (e.g. the Stroop effect and the Trail Making Test

B) and complex problem solving. It appears that these tests have elements in common. According to Martin (1998) the central executive is thought to control input stimuli and monitor information processing.

The Stroop test requires selective attention in order to complete the task. During testing attention may easily become distracted. When participants are shown the word 'blue' written in 'red ' ink there is no difficulty in saying the word. However, naming the colour of the ink the word is written in can be more difficult due to the distracting influence of the colour-word.

The Trail Making Test B, according to Lamberty, Putnam, Chatel, Bieliauskas, & Adams (1994), requires motor and spatial tracking skills and basic sequencing abilities. Arnett & Labovitz (1995) suggested that the cognitive processes involved in the completion of this test are attention, concentration, speed of mental operations and problem solving.

In a similar way the forward and backward digit span tasks require attending to the sequence of digits to be recalled. However, the phonological loop and visuo-spatial sketchpad may be involved in these tests. According to Rudel & Denckla (1974) different cognitive processes underlie the two digit span tasks. They suggested that forward digit span requires the maintenance of the order of the digits possibly using a verbal code whereas backward digit span requires a spatial representation or mental image of the digits.

Thus the tests loading onto this factor appear to utilize similar cognitive processes. However, the precise nature of the cognitive processes involved during the tests generally associated with working memory and complex arithmetic is unclear. There is a need for further investigation which lies beyond the scope of this thesis.

7.4 Complex arithmetic and models of numerical cognition

This part of the discussion will focus on the variables loading on this factor which relate directly to numerical processing - the complex addition and multiplication tasks. It would appear that the cognitive processes required for the calculation of complex arithmetic involve the components of working memory. However, the factor analysis cannot clarify which components are involved or what role the visuo-spatial sketchpad may have in these processes.

In general the experimental work designed to validate current models of numerical cognition has focused on the calculation of simple arithmetic problems. Research into more complex problem solving in the context of these models has been limited. McCloskey et al.'s (1985) model can accommodate simple arithmetic procedures within the calculation system but how the model accounts for more complex problem solving is less clear. McCloskey considers that the comprehension subsystem converts numerals into abstract representations that become inputs for the calculation system. If it were shown that visual spatial representations of problems are necessary for calculation procedures in complex problems, that finding would indicate a limitation in McCloskey's model.

Dehaene's (1992) triple code theory does appear to accommodate the calculation of multi-digit problems. However, this model does not include a role for visual spatial processing. It does include an assumption that there will be access to different aspects of memory in as much as numbers can be represented mentally in three different codes, the verbal code, the visual Arabic code and the analogue magnitude code. Thus the model does accommodate numerical comparisons, subitizing, estimation and approximate calculation procedures. However, it may have similar limitations to McCloskey's model in relation to visual spatial processing.

Thus it would appear that these two models of numerical cognition can accommodate simple arithmetic problem solving adequately. However, it is unclear as to how these models would account for the solving of multi-digit problems.

Ashcraft (1995) proposed a speculative relationship between the various components of working memory and arithmetic problem solving (see Chapter 3, Figure 3.1). At present there is insufficient empirical evidence to support the proposal. However, Ashcraft suggested that there are at least two main functions of working memory in arithmetic problem solving.

Firstly the central executive retrieves and then manipulates number and fact information. This component may also be involved with keeping track of counted digits and digits to be counted together with keeping track of progress within the counting system. Retrieval of facts that are less familiar and of lower strength in long-term memory will utilize more of the central executive's mental resources. Therefore, retrieval of less familiar information when accompanied by a secondary task also using central executive resources should result in interference. Secondly, the articulatory loop is involved in maintaining the counting sequence and the rehearsal and holding of intermediate values during problem solving.

According to Ashcraft the 'evidence for a visuo-spatial sketchpad role in arithmetic performance is quite speculative' (p.16). Ashcraft suggested that there would be disruption of arithmetic processing during problems that require carrying operations and problems where digits are maintained in columns when accompanied by a secondary task utilizing visuo-spatial processing. The loading of complex arithmetic on Factor 2 in this study and Ashcraft's speculations on the role of the visual spatial component of working memory in complex arithmetic are the basis of the experimental work in Chapter 11.

The aim of the experimental work on complex arithmetic and working memory, which is reported in Chapter 11 was to investigate in much greater detail the relationship between complex arithmetic and the components of working memory, in particular the visuo-spatial sketchpad. It was predicted that arithmetic processing would be disrupted whilst undertaking a visual spatial task. This is because it is speculated that a visual spatial contribution is necessary for any problems that involve carrying operations. The results of the study are considered in relation to Baddeley's (1986) working memory model and the models of numerical cognition developed by McCloskey et al. and Dehaene.

7.5 Factors accounting for under 10% of the variance

Factors 3 – 7 accounted for 5 – 9% of the variance each. They were not the subject of further experimental work. So a brief commentary is provided on each factor.

7.5.1 Factor 3 – Basic number facts

The factor analysis result clearly shows that simple addition and multiplication have elements in common loading onto Factor 3 at .90 and .89 respectively. Ashcraft (1992) proposed that simple whole-number facts are stored in a mental network lexicon with numerical representation held in long-term memory. The strength of the network connections among digits and answers produced reflects the degree of learning together with the ability to access the representation via automatic processing. The suggestion is that the network contains associations to both correct and incorrect answers and again the strength of the connections vary. Zbrodoff & Logan (1986) provided evidence to suggest that the arithmetic store is accessed automatically finding slow response times to verification problems like 4 + 3 = 12 in which the suggested result is in fact the result of the wrong operations, multiplication instead of addition. The suggestion is that both multiplication and addition processes are able to commence regardless of the arithmetic problem. LeFevre, Bisanz & Mrkonjic (1988) conducted research finding that during a memory task the presentation of two digits, for example, '6 2' prompted the automatic activation of addition with the result of 8.

Campbell (1995) suggested that the network interference model simulates number fact retrieval processes for single digit multiplication and addition problems.

According to the theory, when a problem is presented, memory codes corresponding to all the addition and multiplication facts in the network are activated to some degree. Memory for arithmetic facts is considered to involve both a magnitude code and physical codes. The magnitude code represents the approximate numerical size of the answer and the physical codes are thought to be visual or verbal units consisting of the operation sign, (for example, '+' or 'x') and the answer. Retrieval of the answer within this model involves a series of processing cycles determined both by physical code and magnitude similarity of the problem. This is in conjunction with excitatory and inhibitory input across nodes within the network. Equilibrium is reached and an answer produced to the problem when one of the nodes in the network reaches a critical threshold level of activation.

On the basis of the above research and theorising it is not surprising that both the simple arithmetic tests load onto the same factor with the exclusion of the other tests in the battery. The conclusion reached is that simple arithmetic is an automatic process based on the strength of prior learning and speed of retrieval of information from long-term memory.

7.5.2 Factor 4 - Retrieval of mathematical knowledge

The two tests loading onto this factor are subitizing numbers, a computer based test analysed using reaction time data and the basic arithmetic facts test involving 10 questions scored with a maximum of ten correct answers. Subitizing numbers loaded onto this factor at .77 and it should be noted that the basic arithmetic facts test only reached the loading criteria at .50. The relationship between these two tests is very speculative. However both tests involve the manipulation of numerical information.

The basic arithmetic facts test, a production test required participants to retrieve from long-term memory previously learned information. The subitizing numbers test, a verification test, was based on the speed at which participants were able to judge whether or not the stimuli numbers presented on the computer screen were equal to the number presented in a box above the stimuli.

To speculate on the common elements that these tests may share is difficult at this stage, and it may not be important as the total variance accounted for by this factor is only 6.5%.

7.5.3 Factor 5 – Central executive (planning)

The tests loading onto this factor are the block design test at .69 and the Tower of Hanoi test at .66. Both these tests are generally associated with the planning of cognitive strategies (Martin 1998, Parkin 1996). For the successful completion of the block design test participants are required to plan and reconstruct a design using patterned blocks. This task requires forward planning or looking ahead to recreate the design. The Tower of Hanoi task involves moving discs into a given position in as few moves as possible. However, to complete the task in as few moves as possible it is necessary to plan ahead for each move in order to advance closer to the goal. It would seem reasonable to speculate that the common element shared by both the tests on this factor is the planning of cognitive strategies that is necessary to reach the goal of the task.

7.5.4 Factors 6 – visual long-term memory

The only variable loading on this factor is the Doors Test B. This test reflects visual long-term memory. The stimuli are coloured photographs of doors. There are 12 'target' doors and 36 distractors. The 12 target doors are presented individually for the participant to memorise. For the recognition set each target is presented with three distractors and the participant is required to identify the target door. The score is a maximum of 12 correct responses. It appears that this test is a pure measure of visual long-term memory and it is not unexpected that this test forms a factor on its own as no other variable in the battery reflects visual long-term memory.

7.5.5 Factor 7 - Visual spatial short-term memory

The results of the factor analytic study suggest uncertainty as to the involvement of visual spatial abilities in arithmetic problem solving. The abstract visual pictures task was expected to correlate with the complex arithmetic tests, but this result was not found. A possible explanation for this is that the abstract visual pictures task relied upon the visual encoding of shape and colour rather than the visual spatial processes that may be required for solving complex arithmetic problems.

7.6 Conclusion

This chapter has considered questions that have arisen from the factor analytic study and the rationale for the experimental work reported in the following chapters. Although the prominent questions are considered in-depth other and equally important questions arise from the study. Due to practical constraints, an in depth study of all the issues has not been possible, however, throughout the following chapters these areas are identified together with suggestions for future experimental research.

Chapter 8 – The nature of magnitude comparison

8.1 Introduction

This experimental investigation arises from the results of the factor analytic study that revealed a highly significant correlation between two separate tests, the magnitude comparisons of numbers written as digits, Variable 7, and the magnitude comparison of animals written as words, Variable 8, at $p < .01$. Both tests load onto Factor 1, access to representations, with a factor loading of .73 for the magnitude comparison of numbers and .57 for the magnitude comparison of animals. Based on the above associations between the two tests, the processes of numeric and animal magnitude comparisons were investigated.

This chapter reports an investigation of a phenomenon that has theoretical significance in relation to central issues in the field of numerical cognition. Firstly are magnitude comparisons influenced by the modality in which symbols are encoded? Secondly, do different types of stimuli affect the processes by which magnitude information is retrieved?

The numerical distance effect has been obtained for stimuli in a variety of forms, including Arabic digits (e.g. Moyer & Landauer, 1967), written number words, (Foltz, Poltrock & Potts, 1984) and patterns of dots, (Buckley & Gillman 1974). The results of these studies support the notion that regardless of the notation in which stimuli are presented, performance on the task of numerical comparison is mediated by abstract quantity representations that reflect magnitude relations among numbers. However, Campbell & Clark (1988) have argued that performance in numerical comparison tasks is influenced by the format of the stimuli. They have suggested that judgements of relative magnitude are made on the basis of codes associated with the form in which stimuli are presented, such as visual digit representations or phonological number word representations.

Trabasso & Riley (1975) proposed that individuals might associate with each stimulus a linguistic size code, and retrieve and compare these codes when comparative judgements are required.

Foltz, et al. (1984) considered the mental comparison of size and magnitude in relation to objects. This research suggested that to determine which is the larger of two animals, for example an *elephant* or a *mouse,* (words that contain no explicit size information), participants retrieve information about the size of animals from semantic memory. However, questions arise as to the nature of the stored size information and the nature of the comparison process. Writing some years earlier Moyer (1973) had suggested that participants 'first convert the animal names to analogue representations that preserve animal size and then compare these analogues by making an 'internal psychophysical judgement' (p.183).

More recently McCloskey (1993) argued that the relationship between numerical and non-numerical processing had not been adequately researched. The question arising from this concern is whether numerical processing mechanisms are incorporated within, or are separate from, the cognitive language-processing system. To elaborate on this issue the data from the magnitude comparison of digits and animals tasks are empirically analysed.

8.1.1 Theoretical perspectives relating to the magnitude comparison of numerical stimuli

Previous research has highlighted different perspectives in relation to the processing of magnitude information. Moyer & Landauer (1967) suggested that adults use an analogue representation of numbers. When participants are shown a pair of digits and asked to indicate which is the larger by pressing one of two buttons, reaction times become faster as the difference between the specified numerosities increases. For example, reaction times are quicker for numbers that are more distant in magnitude such as 2 and 8 than for numbers that are close in magnitude such as 3 and 4. It also appears that reaction time increases the larger the numerosities are. For example, the reaction time to indicate which is the larger number between 2 and 9 is faster than the time taken to decide which is the larger between 58 and 59. These two effects are commonly known as the distance effect and the minimum effect. The assumption held is that numbers are converted to representations of magnitude that are similar to representations of such physical dimensions as size, length or brightness. Magnitude representations are therefore, compared by similar processes to those responsible for perceptual comparisons (Buckley & Gillman 1974).

According to Fias, Brysbaert, Geypens & d'Ydewalle (1996) numbers can be represented in Arabic (numbers in numerical format) or in verbal form(numbers in written or spoken word format), and numbers can be related to magnitude. Magnitude is considered as the semantic element of numerical processing as numbers (in whatever format they are presented) are a symbolic representation of a magnitude or quantity. It is on this basis that the processing of magnitude information in relation to Arabic and verbal representations is of interest to researchers.

There appear to be opposing views on the processing of magnitude information. Firstly McCloskey, Caramazza & Basili (1985) and McCloskey (1992) consider that access to magnitude information is essential prior to any other numerical processing taking place. The assumption of this model is that number processing is not notation-specific. In this view, different types of input, for example Arabic digits and written or spoken number words, are converted to a common abstract quantity code and all further operations are based on this abstract code. This modular theory assumes verbal or Arabic notation-specific comprehension modules that translate numerical input into an abstract number code from which calculation procedures and number knowledge are accessed. The abstract information is then translated to a notation-specific code further processed by notation-specific modules for either verbal or Arabic written or oral output. The general assumption is that all numerical input is subjected to an abstract representation before further processing.

Other theorists have rejected the above perspective. Examples of alternative models include the triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995), the encoding complex hypothesis (Campbell & Clark, 1988); Campbell, 1994) and the preferred entry code model Noël & Seron, 1993). In the triple-code model Dehaene (1992) assumes three representational codes, the auditory verbal word frame, the visual Arabic number form and magnitude information accessed along the analogue magnitude code. The assumption of the triple code model is that neither the Arabic number form nor the verbal word frame contains any semantic information. The meaning of numbers is represented only in the third component of the model - the analogical magnitude representation. The prediction arising from this model is that numerical information is represented in a semantic representational format prior to the calculation process.

The three representational codes form the starting point for different arithmetic operations. For example multi-digit operations are assumed to begin from Arabic representations, number size comparisons are accessed from the analogue magnitude representations and retrieval of arithmetic facts from the verbal representations. However, during processing information can be exchanged between the systems without the mediation of an abstract code and not all systems need be utilised for the processing of a particular problem. The magnitude analogue scale would be necessary for only certain tasks, for example, subitizing, approximate calculation, estimation and comparison and not required for other types of tasks such as counting and arithmetical fact retrieval. The processing of magnitude information in relation to this theory is conceptualised as an oriented number line, and like McCloskey et al.'s (1985) model, the triple-code model suggests that different types of stimulus input should not interact with processes of calculation, quantity judgement and output. This is because after the input stimuli are converted to the appropriate internal representation, number processing continues independently of the input notation.

In contrast to McCloskey et al.'s (1985) abstract modular theory and Dehaene's triple code theory, Campbell & Clark (1988), Clark & Campbell (1991) and

Campbell (1994) proposed in their encoding complex hypothesis that numerical processing may vary with different types of input. This theory allocates a primary role to modality-specific processes and representations and rejects the assumption of abstract representations. Instead this theory suggests that collectively modality specific number codes form a complex associative network that mediates numerical functions. For example, different stimuli input (Arabic digits, written or spoken number words) may activate different central processes so that processing is mediated by different representations given different types of stimuli. McNeil & Warrington (1994) suggested that Arabic digits activate a visual number fact system and on the other hand auditory presentation of problems accesses a verbal number fact system. Campbell (1994) and Campbell & Clark (1992) highlighted the possibility that Arabic digits produce faster activation of number fact representations than written number words because it is in general common to perform arithmetic calculations on Arabic digits. However, Dehaene (1996) expressed the view that Arabic digits, for example 3, 5, 7, and written number words, three, five, seven, present different visual encoding conditions due to the fact that words present a wider visual angle. He suggested for that reason that encoding time might be related to word length.

Noël & Seron (1993) suggested that individuals have a unique preferred entry to access number knowledge and calculation procedures. This may be the route for access to number knowledge from Arabic representations while other routes give access to information from verbal representations. If a number is presented in a modality that is not preferred a direct transcoding to the required modality is performed from which magnitude and other semantic number knowledge is accessible.

The theories of numerical cognition offer different predictions that relate to the processing mechanisms that are used for numerical information. The particular focus of this chapter is the processing of magnitude information and the way in which the findings drawn from the present research align with the prominent theories. According to McCloskey (1992) magnitude information is represented primarily as abstract representations before further processing. This abstract semantic system would mediate all the tasks requiring an interpretation of numerals, such as magnitude comparison. Dehaene (1992) and Dehaene, Bossini & Girauz (1993) suggested an analogue format that is conceptualised as an oriented mental number line. The theory does not assume that magnitude judgement requires the activation of a semantic representation. In both of these models it is assumed that magnitude processes should operate in a similar way regardless of the stimulus notation. However, the models can accommodate a main effect of notation (stimulus input), as this could be attributed to differences in stimulus encoding. Nevertheless, the interpretations drawn from both models agree with the view that the relative difficulty of magnitude judgements should remain constant across notations. Therefore there should be no interaction between the different types of stimulus input and distance between the numerals (Dehaene & Akhavein 1995).

Campbell & Clark (1988) suggested that within the encoding complex hypothesis modality-specific number codes are the operating units of numerical functions. Noël & Seron (1993) envisaged preferred entry codes to modality specific processing from which magnitude and semantic number knowledge is accessed. These two models imply a very different perspective on the processing of numerical information. Clark & Campbell (1991) suggested that number processing may vary, dependent upon notation. The way in which the numerical information is presented is relevant to this theory. For example, Arabic digits, written or spoken number words might activate different central processes so that calculation processes are mediated by different representations given the different input notations. Noël & Seron argued against an analogue representation of numerical stimuli. They suggest that if number processing is dependent upon notation then this view is unable to accommodate how individuals deal with the internal structure of complex numerals in order to form a representation of the corresponding quantity. For example, verbal numerals are considered as lexical concepts and although 'thirteen hundred' and 'one thousand three hundred' represent the same quantity, different representations are activated as each involves different numerical processing of the different values. Noël & Seron (1995) provided experimental data showing that comparing the magnitude of two numerals is faster if both stimuli share the same notation structure. The suggestion is that the operation of magnitude comparison is influenced by the underlying representation of these numerals. This is termed 'Intermediate Semantic Representations' (ISR) and has been demonstrated by showing that, for example, spoken numerals are connected through semantic relationships. Although 'thirteen hundred' and 'one thousand three hundred' are equal to the same quantity, different representations of the quantity are activated. 'Thirteen hundred' can be represented by the relationship of <13> to <100>, whereas 'one thousand three hundred' suggests a relationship with <1000>, <3> and <100>. Noël & Seron concluded that numerical relationships do not necessarily operate on the basis of abstract representations but rather on the distinction between numeral formats, such as 'fourteen hundred' and 'one thousand four hundred'.

The experimental work by Campbell, Kanz, & Xue (1999) reviewed in Chapter 2 investigated Chinese-English bilinguals across two types of numerical notations (Arabic and mandarin). The aim of the experiment was to investigate the main effect of notation (Arabic vs. mandarin) for the purpose of considering whether or not magnitude processing is notation specific. Furthermore the effect of numerical distance between the two numbers (small distance vs. large distance) was considered. Participants were required to select either the smaller or larger of two single digit numbers by pressing a key corresponding to the left or right position of the target number. In summary, the results suggested that magnitude selection was slower and more prone to errors with mandarin than with Arabic stimuli, and that number pairs with small distances between the numbers of 1, 2 or 3 produced longer reaction times and more errors than larger distances of 4 to 7. Analysis of the results showed that magnitude processing was less efficient with mandarin than with Arabic stimuli. The interpretation of the notation by distance interaction was that the judgement relating to a contrast between numbers in which the distance was small was more difficult with mandarin characters as opposed to Arabic digits. According to Campbell et al. (1999) this result does not easily fit into either the McCloskey (1992) or Dehaene & Cohen (1995) models that assume that magnitude processing is not affected by notation. The results appear to be more consistent with the Clark & Campbell (1991) hypothesis as Arabic numbers are regularly used for calculation, judgement of quantity and magnitude retrieval and comparison processes. However, mandarin characters are not generally used to represent numbers for numerical comparison and as a result it is more difficult to activate magnitude information with mandarin input. The activation of magnitude information is considered to facilitate arithmetic retrieval by priming associated number facts. A further finding from this study is that weaker activation of magnitude information may also have contributed to the overall slower reaction times with mandarin stimuli and that the problem size effect was larger with mandarin than with Arabic input. The basis for this finding is that larger magnitudes are in general more difficult to discriminate.

8.1.2 Theoretical perspectives relating to the magnitude comparison of non-numerical stimuli

Interest in magnitude comparison has not been restricted to numbers but has extended to the comparison of object and animal size. Research by Moyer (1973) and Paivio (1975) compared the size of named animals. Participants were visually presented with the names of two animals, for example rabbit-wolf, and were required to indicate which animal was the larger of the two. The results suggested that the reaction time increased as the difference in animal size became smaller. The conclusion formed was that animal names are converted to analogue representations that preserve animal size. It was hypothesised, therefore, that small size differences between numbers and animal size are represented as smaller differences within the internal analogue scales which results in a decreased ability to discriminate between two digits or animals. This is then reflected in increased reaction times.

Foltz et al. (1984) built on the research of Paivio (1975) and considered the magnitude judgement of Arabic digits, numerical stimuli written as words and object names including animals. The mean reaction time scores for object names produced the slowest scores followed by digit names with the quickest scores in the Arabic digit condition. The results were interpreted as an interaction between physical size comparisons and conceptual size comparisons. The explanation provided assumed that similar processing stages were involved in comparing magnitudes of each stimulus type. The key consideration was how quickly participants were able to recognise that one stimulus was physically larger than the other. Therefore comparison of physical sizes of the stimuli was relatively fast and automatic, regardless of the stimulus types. Following on from this comparison stage is a decision process that identifies the physically larger stimulus.

The process of conceptual size comparison is that each stimulus is recognized and the corresponding magnitude retrieved from memory. A possible explanation for the differences in reaction times between types of stimuli is that the duration of these stages is shortest for Arabic digits and longest for object names. The comparison produces a decision process indicating the larger of the stimuli and it is the decision process that is used to select a response.

Foltz et al. (1984) also considered how the two processing stages may interact. They suggested that the pairs of stimuli are considered firstly in relation to their physical size. This comparison is made quickly and is completed before the conceptual size comparison stage begins. The conclusion is that as Arabic digits are encoded quickly, these two comparison stages may occur simultaneously and may interfere with one another. However, if the decision codes for the physical and conceptual size comparisons conflict, then additional processing time may be required to select the appropriate decision code. In the digits and digit names conditions conceptual size comparison is completed soon after response preparation begins. In the object name condition there is more time required for response preparation as object names take more time to encode and compare. This provides an explanation for the slower reaction time scores received for the digits written as words and in particular the object names.

Although the Foltz et al. (1984) suggested possible processing methods that may be employed when performing magnitude judgements, there are still unanswered questions. Firstly, to what extent do participants form an image of the number or animal name? Secondly, Logie (1995) questioned whether or not comparative judgements are based on semantic knowledge of the physical characteristics. The question also arises as to whether magnitude information is processed in a similar way, whether the stimuli are numbers or animals and whether there is the equivalent of a mental analogue of a number line for animal names. If this is the case, are the functional aspects of the lines similar?

8.2 Magnitude Comparison Task

The two magnitude comparison tasks, numbers and animals, required participants to select the larger of two stimulus numbers, written as digits, and the larger in physical size of two animals, written as animal words. Participants pressed a key corresponding to the left or right position of the target number or animal. The three independent variables were:

i. The stimuli (digits vs. animals).
ii. The size of the digits and animals, for example 4 and 8 or 56 and 59 and for animals, mouse and rabbit or giraffe and elephant, (small object vs.large object).
iii. The numerical distance between the numbers, for example, 2 and 3 for a small distance and 52 and 59 for a large distance and between animals, for example, a wolf and a sheep for a small distance and for a large distance an ostrich and a dog (small distance vs large distance).

Moyer & Landauer (1967) suggested that the difficulty of magnitude comparison increases as the difference in distance between the numbers decreases. For example reaction times are longer to decide the larger of the numbers 3 and 5 having a difference of two than to decide the largest of 3 and 8 with a difference of five.

8.3 Aim

In the study reported in Chapter 6 the two tests of magnitude judgement loaded convincingly onto Factor 1. Dehaene's notion of a number line is considered to accommodate numerical comparison, however, as the two comparison tests have loaded onto the same factor this suggests that both types of stimuli may access similar internal representations. If this is the case then similar results and patterns of results will arise from the analysis based on the speed of the decision process as to how large or small the stimuli are relative to each other. The expectation is that the patterns of results will be similar in both stimuli but amplified, producing slower reaction times in the comparison of animals as this comparison test is using stimuli that are not routinely associated with comparison of size. On the other hand, if magnitude processing is dependent upon notation, then the use of Arabic digits vs. animal stimuli may produce very different results. Although animal words may activate magnitude information, it is numbers rather than animal words that are used for calculation and numerical size comparisons. As Arabic digits are often encountered in numerical comparison it might be expected that individuals will be more experienced at making numerical size comparisons for digits than for animal words. On this basis it may be more difficult to activate or utilise magnitude information given animal words than Arabic digit input. The expectation is that no interactions will occur between the magnitude judgement of numbers and animals and that the two comparison tasks are simply accessing representations and concepts relating to the relative size of the stimuli. The assumption is that there are no other additional processes involved, for example, the accessing of magnitude information along an analogue magnitude number line as proposed by (Dehaene 1992).

On the assumption that the two types of judgement rely on the same underlying processes, then the expectation is that no interactions will be found. However, such a result would show that the processes may be the same but it would not show precisely what those processes are. It is possible that animal stimuli are converted into an analogue magnitude representation, but this could be argued as unlikely. Taking the example of an ostrich vs. a cow the question arises as to how does the mass of the animal compare against height when making these judgements? The visual image of these animals suggests that an ostrich is much taller than a cow but a cow has a greater bulk. On this basis the concept of 'bigger' or 'greater' may be considered 'fuzzy' in relation to animals. A possible way of reconciling this point is to consider Campbell's interactive model that suggests that representations are stored in varying degrees of elaboration, including a notion of size through which comparisons can be made. Therefore, more familiar and elaborated concepts are quicker to compare. This also explains the size effect, since larger numbers are less familiar. The distance effect simply reflects the fact that the differences between the sizes being compared are large and easily judged to be different. Thus a theory of size comparison based upon semantics can account for all size comparison tasks in all domains with all stimuli. This is in line with current views of the structure of semantic memory, unlike Dehaene's notion of a number line.

From the initial results obtained from the factor analytic study the experiment that is reported here tested the prediction that stimuli-specific retrieval will produce slower reaction times for judging the larger of two animals written in words than are found when participants judge which number is the larger of two numbers written as digits. A detailed analysis was performed to compare numbers (written as digits) with nouns (animal words) and the effects of size of comparison targets and distance between targets in terms of their size. It was predicted that the overall patterns produced for the comparison of two animals written as words and two digits written in Arabic would be the same. However, because there is possibly reduced precision when comparing the size of two animals, and because it is less common to compare two animals than it is to compare two digits, there might be an increase in the reaction times for animals. The reaction time data is analysed using a repeated measures 3-factor Anova to examine the following: 1) animal vs. numerical stimuli, 2) large vs. small items, 3) large vs, small distances between the items and 4) an overall interaction between 1, 2, and 3.

The expectations are that the classic number effect will be found in as much as small numbers are faster to judge than large numbers and greater distances between numbers will be faster than smaller distances. These two effects will be identical in the animal stimuli with small animals being faster to judge than large and greater distances between the animals faster to judge than small distances. However, the overall reaction times for animals will be slower due to the unfamiliar nature of the task in that it is not usual to process animal size information explicitly whereas it is more often done with numbers.

The predictions of this experimental work are that there will be a significant difference in reaction times between the magnitude comparison of numbers and the magnitude comparison of animals. The magnitude comparison of animals will produce significantly longer reaction times. Judgements related to the size of the stimuli will be quicker for smaller stimuli than larger stimuli. The pattern of the results relating to the size of the stimuli will be similar for animals and digits and produce no significant interaction. The reaction times for smaller distances between the stimuli (digits and animals) will be slower than for a larger distance between the stimuli. There will be no interaction between the stimuli types. There will be no significant interaction between the stimuli (digits and animals) and the size of the stimuli or the distance between the stimuli.

8.4 Method

The analysis of this study is a repeated measures 3-factor analysis of variance (Anova). The three within factors are animals vs. numbers, the size of the stimuli, small vs. large digits and animals, and a small vs. a large distance between the digits and animals.

8.4.1 Magnitude comparison of number

The stimuli, coding procedures and the instructions given to the participants are the same as described in Chapter 5 (see Chapter 5, Section 5.8.7, p.115).

8.4.2 Magnitude comparison of nouns

The stimuli, coding procedures and the instructions given to the participants are the same as described in Chapter 5 (see Section 5.8.9, p.117).

8.5 Results

Reaction time data for correct responses received from the left and the right hand were used in the following analysis. Mean reaction times were computed for each participant for the following effects:

- a difference in reaction times between numbers and animals,
- the effect of the size of the digit or animal dependent on reaction time
- an effect relating to the distance between the two animals or digits.

These data were entered into SPSS. A repeated measures 3-factor Anova, animals vs. numbers, large vs. small animals and numbers and the distance between animals vs. numbers (large or small) was computed.

Table 8.1 Mean and standard deviation scores for the two groups - magnitude comparison of animals and digits. N = 100

Animals and Digits	Mean	Std. Deviation
Animals	1437.86	289.65
Numbers	667.21	123.55
Animal large/small distance	1579.26	334.40
Animal large/large distance	1342.96	284.89
Animal small/small distance	1591.81	360.47
Animal small/large distance	1440.70	282.79
Number large/small distance	774.22	146.70
Number large/large distance	710.21	131.88
Number small/small distance	633.18	130.91
Number small/large distance	555.65	107.52

Table 8.1 shows the mean and standard deviation scores for animals and numbers. The mean scores clearly indicate that reaction times are slower for the judgement of animals than numbers (1437.86 vs. 667.21 msec.) with a mean difference of 770.65 msec. From the above table it can also be seen that large animals with a small distance between the pairs of animals show a mean reaction time of 1579 msec. and large animals with a large difference between the pairs show a mean reaction time of 1342 msec. The respective figures for small animals are 1591 for a small distance between stimuli and 1440 for a large distance between the stimuli. This is counter to the classic effect found with numbers.

The numerical stimuli do show all the classic effects. Large numbers with a large difference between the pairs are quicker to judge than large numbers with a small distance between the pairs. The mean scores show this effect - 710 msec. vs. 774 msec respectively. A similar effect is found with responses to small numbers recorded as faster than responses to large numbers - 555 msec vs. 633 msec respectively.

The overall pattern of mean scores produced across the three factors, animals vs. numbers, large vs. small animals and numbers, and large vs. small distance between the animals numbers indicates that the comparison judgement of animals takes a much greater length of time than the comparison judgement of numbers. Figure 8.1 below provides an illustration of the mean reaction times as a function of stimulus input, size of the stimulus and the distance between the pairs of stimuli.

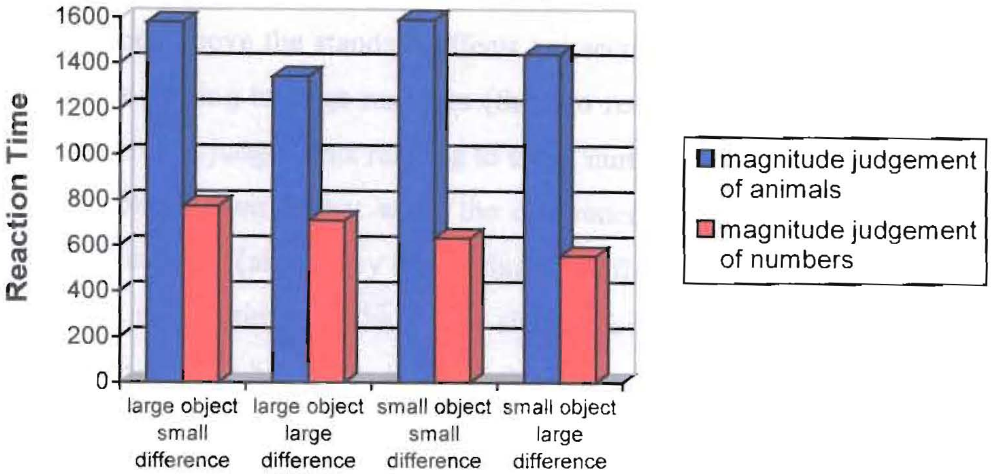


Figure 8.1 Mean and standard deviation scores for the magnitude judgement of animals and numbers

Figure 8.1 shows that the experiment produced the standard distance effect with slower reaction times for smaller distances than larger distances between pairs of stimuli regardless of whether they are pairs of animals or digits. A further standard effect is that large numerical stimuli produce slower reaction times than small numerical stimuli. This effect was not found for animals. Figure 8.1 shows that the two blue bars (representing the large animal stimuli) on the left are almost identical to the two blue bars on the right representing the small animal stimuli. The pattern for small vs. large animals appears to be the same. Figure 8.1 also clearly shows that the comparison of all numbers is quicker than the comparison of animals regardless of the distance between the stimuli.

A possible explanation for this is that none of the animal stimuli represented a sufficient shift to larger magnitudes. For example, if the difference in 2 vs. 4 is compared with the difference in 52 vs. 54, the latter represents movement along a scale of several thousand percent. That does not seem to be the case when comparing, for example, the difference in 'cat' vs. 'dog' to the difference in 'elephant' vs. 'cow'. There are few animals that could be used to represent such extremes of the scale. This experiment was not designed to yield data that would confirm or invalidate that hypothesis

As mentioned above the standard effects are seen in Figure 8.1. This shows that judgements relating to large numbers (the two red bars on the left of the figure) were slower than judgements relating to small numbers (the red bars on the right). A further observation is that when the difference between the pair is large the judgement is faster (as seen by comparing the left and right bars within each pair of large or small stimuli). The figure shows, for numbers, a very encouraging slope across all four bars from left to right, representing the combination of these two effects. However, for animals the only effect evident is the effect of large vs. small distances between stimuli.

Table 8.2 Mean and standard error for digits and animals

Stimuli conditions	Mean	Standard error
Small stimuli	1055.34	18.72
Large stimuli	1101.66	18.85
Small distance between stimuli	1144.62	19.94
Large distance between stimuli	1012.38	17.31
Animals (large)	1461.10	28.62
Animals (small)	1516.26	30.14
Numbers (large)	742.22	13.64
Numbers (small)	594.41	11.61
Animals (small distance between animals)	1585.54	31.49
Animals (large distance between animals)	1391.83	26.65
Numbers (small distance between numbers)	703.70	13.51
Numbers (large distance between numbers	632.93	11.50

The mean scores shown in the above table indicate that overall small stimuli are faster to judge than larger stimuli. However, this is probably due to the fairly strong effect shown in numbers as the opposite effect was seen for animals. Furthermore faster larger reaction times are obtained for large distances between the stimuli than that observed for small distances. The result is in the expected direction as this effect was seen to be operating in both numerical and animal stimuli. Regardless of the small or large stimuli factor the effect in animals is opposite to that for numbers. Larger animals are judged faster than small animals but for numerical stimuli small numbers are judged faster than larger numbers. The effect for small versus large distance between the stimuli is in the same direction for both animal and numerical stimuli, with participants taking longer to make a judgement when there is a smaller distance.

In the analysis of reaction time data, using a repeated measures 3-factor Anova the main effects, double interactions, and the overall interaction for the three factors were significant. The results produced a significant main effect of stimuli (animals 1437.86 msec.vs. numbers 667.21 msec.), [F (1,99) = 1240.033, p<. 001]. Slower reaction times are considerably more pronounced for animal comparisons than numerical comparisons.

There is a significant main effect of small and large digits and animals (1101.66 vs. 1055.33 msec.), [F (1,99) = 22.851, p<. 001]. Similarly the main effect of a small and large distance between the two digits or animals is significant (1144.62 vs.1012.38 msec), [F (1,99) = 228.469, p<. 001]. The interaction of small/large by digits and animals is significant [F (1,99) = 109.463 p<. 001]. This interaction can be seen in Figure 8.2.

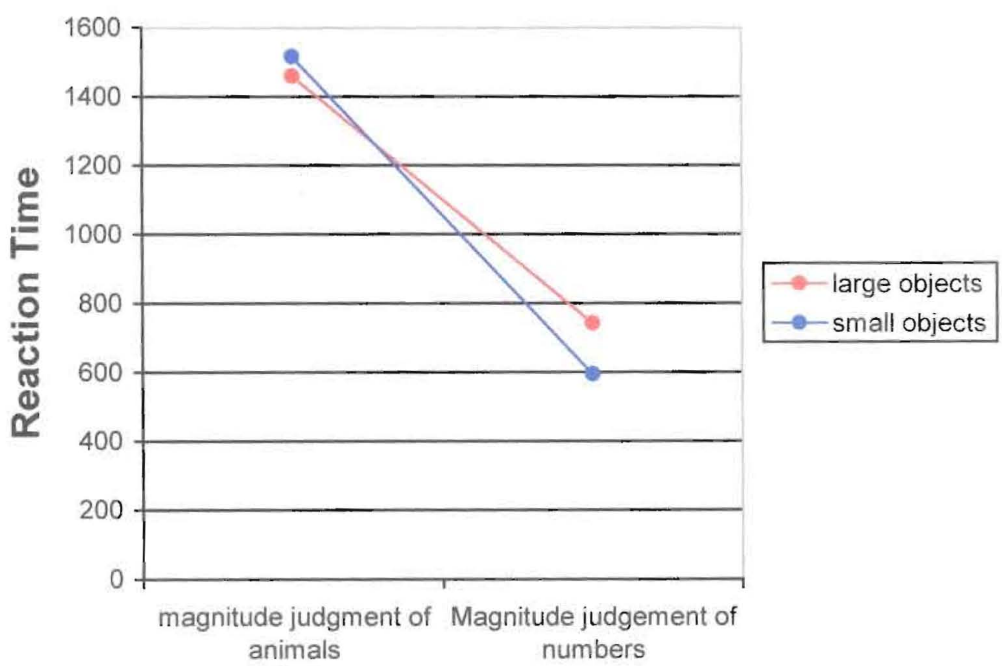


Figure 8.2 Interaction plot showing the main effect of animals and numbers by small and large objects

The interaction plot displayed in Figure 8.2 shows that the effect of large versus small stimuli is reversed across numbers and animals.

The interaction between animals/numbers by small/large distance between the stimuli is significant at [F (1.99) = 47.721, p<001]. The interaction for this result is shown in Figure 8.3.

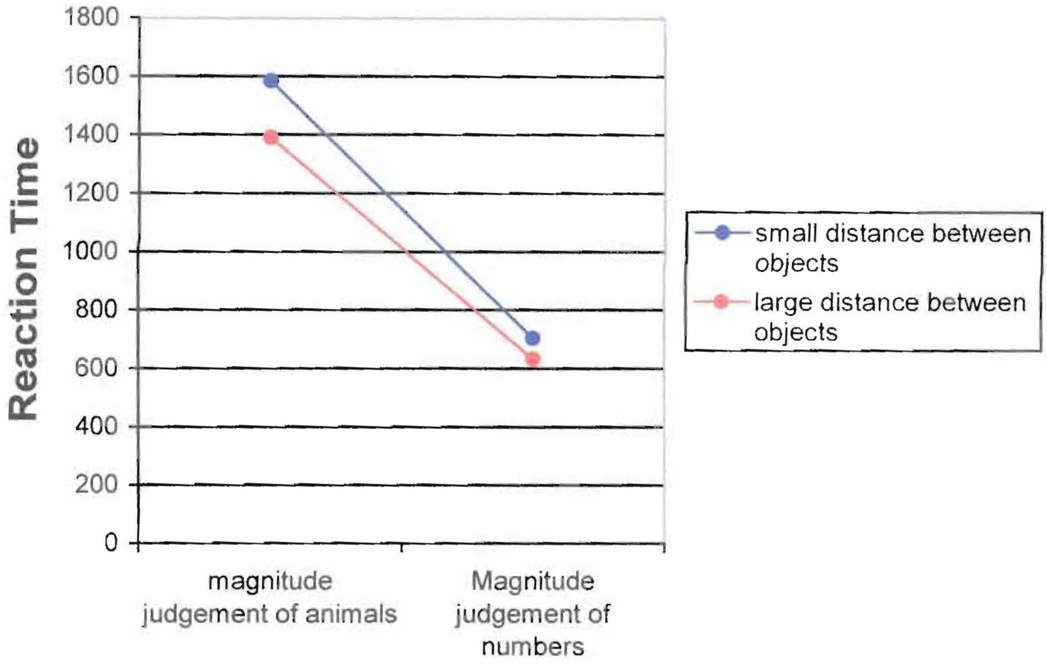


Figure 8.3 Interaction plot showing the main effect of animals and numbers by a small and large distance between the objects

The interaction shown in Figure 8.3 shows the classic distance effect in both numbers and animals. However, this is more pronounced in the comparison of animals than that of numbers. This effect indicates that the size judgement of animals is more disrupted if there is a small distance between the stimuli than a large distance. A similar effect is evident for the comparison of numbers. However, this effect is reduced with numerical stimuli.

The interaction between small and large stimuli and a small and large distance between the stimuli is significant [F (1,99) = 4.338, p<. 05]. This result, however, only just reached that level of significance as the exact p value is p.040.

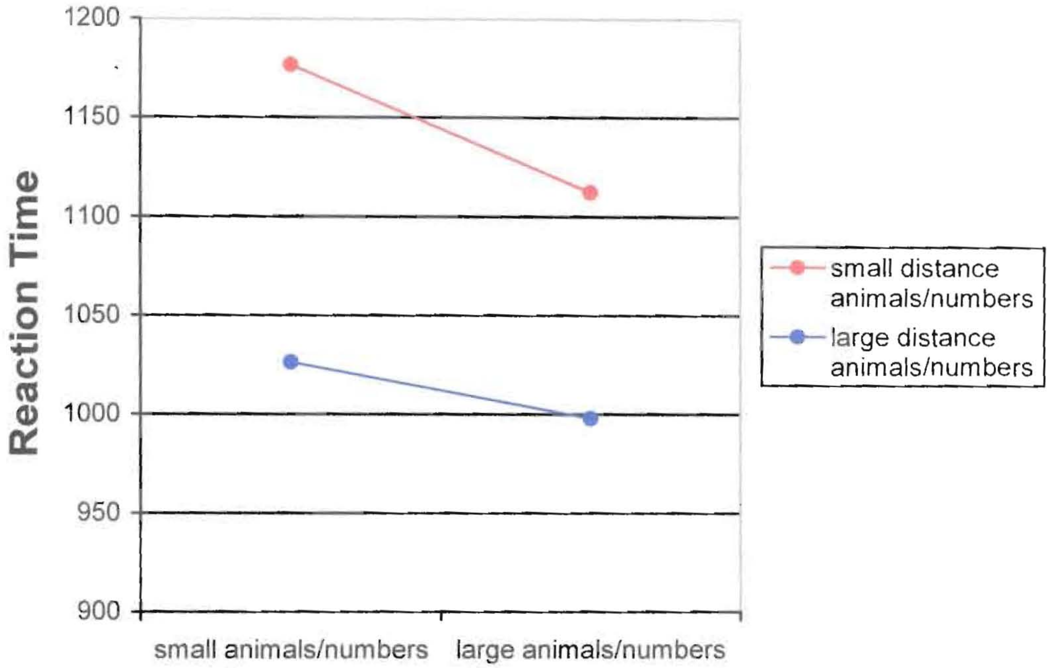


Figure 8.4 Interaction plot showing the main effect of small/large animals and numbers by a small/ large distance

Figure 8.4 shows that small stimuli with a small distance between the stimuli produce slower reaction times than larger stimuli with a small distance between pairs to be compared. The effect of size of stimuli x distance was more disrupted for small stimuli x a small distance with a slightly reduced effect shown for large stimuli by a large distance between the pairs.

The three way interaction of stimuli, small/large stimuli and small/large distance and was significant [$F(1,99) = 8.225$, $p < .01$].

8.6 Interpretation of the results

The aim of this analysis was to consider three, two way interactions between, 1) animal vs. numerical stimuli, 2) large vs. small items, 3) large vs. small distances between the items and 4) an overall interaction between all three. Each of these interactions together with the related hypothesis will be discussed in turn.

8.6.1 Animal vs. numerical stimuli

The mean scores clearly indicated that reaction times were slower for the judgement of animals than numbers (1437.86 vs. 667.21 msec.) with a mean difference of 770.65. The results produced a highly significant main effect of stimuli, $p< .001$ supporting the hypothesis that there would be a significant difference in reaction times between the magnitude comparison of numbers and the magnitude comparison of animals.

A study of this nature, using a large sample of data that compares animal words and numerical stimuli, has not previously been undertaken. However, it is interesting to note that the results are in line with the work of Campbell, Kanx, & Xue (1999), despite the fact that the stimuli used in their study were slightly different. The research by Campbell et al. (1999) compared performance across numerical and mandarin notations. The aim of the experiment was to investigate the main effect of notation (Arabic vs. mandarin) for the purpose of considering whether or not magnitude processing is notation specific. Mandarin characters, according to Campbell et al., are not generally used to represent numbers for numerical comparison and, as a result, it was considered to be more difficult to activate magnitude information with mandarin input. In a similar way in the present study, animal stimuli written as words are not generally used to activate magnitude information.

The results of the research by Campbell et al. (1999) indicated that magnitude selection was slower with mandarin than with Arabic stimuli. The present study would seem to be in line with the work of Campbell et al., with the results showing that reaction times were slower for the comparison of stimuli that are less frequently compared in terms of size. According to Campbell, et al. (1999) their finding does not easily fit into either the McCloskey (1992) or Dehaene & Cohen (1995) models that assume that magnitude processing is not affected by notations. The results would appear to be more consistent with the Clark & Campbell (1991) hypothesis as Arabic numbers are regularly used for calculation, judgement of quantity and magnitude retrieval and comparison processes. However mandarin characters are not generally used to represent numbers for numerical comparison and as a result it is more difficult to activate magnitude information with mandarin input. In the present study, too the theoretical perspective taken by Clark & Campbell (1991) appears to be an appropriate explanation for the slower reactions times with the animal stimuli on the basis that a comparison of the magnitude of animals is not made so frequently. Therefore, it is more difficult to activate magnitude comparison information with animal stimuli.

8.6.2 Large vs. small items

With respect to the second hypothesis, that judgements related to the size of the stimuli will be quicker for smaller stimuli than larger stimuli, in this study there was a significant main effect of small and large digits and animals (1101.66 vs. 1055.33 msec., $p<.001$). The results shown in Figure 8.1 indicated the expected pattern for small versus large numbers. The reaction times for the comparison of larger numerical stimuli (53, 58) are slower than those for smaller numerical stimuli (3, 8). The overall difference in reaction times between small and large animals is very small compared to a much greater difference between small and large numbers. The effect of large versus small stimuli is significant only with the numerical stimuli.

Foltz, et al. (1984) considered the difference in reaction times between different types of stimuli to be the result of an interaction between physical size comparisons and conceptual size comparisons. In the Arabic digits condition conceptual size comparison is completed soon after response preparation begins. However, in the object name condition there is more time required for response preparation as object names require more time to encode and compare. This argument can be applied to the present study, suggesting that overall animal comparisons require a longer length of time to compare before a decision is finally reached. This would account for the increased reaction times compared to those obtained for the numerical stimuli.

The inconsistent pattern observed between small and large animals and digits could be accounted for by assuming that retrieval of physical and conceptual size comparison between animals is a process that is not frequently undertaken. It is therefore less relevant whether or not the animal is large or small, as the difficulty of the task is consistent across the different sizes of the stimuli. This is compatible with Clark & Campbell's (1991) encoding complex hypothesis which allocates a primary role to modality-specific processes and representations in the form of a complex associative network which could be applied to the strength of associations between stimuli other than numerical information.

The results of this study can be evaluated with respect to the different theoretical models of numerical cognition. If a number line were considered, as Dehaene (1992) suggested, it would appear that semantic representations of the objects to be compared are formed. However, if the argument proposed by McCloskey, Caramazza & Basili (1985) is considered, then the numeric stimuli are first converted into a semantic representation followed by converting the representation into the required response.

The pattern of mean scores shown in Table 8.1 is that for the magnitude comparison of numbers, a greater length of time is taken to envisage numbers from the smaller end of the scale to the larger end. However, if a similar operation is undertaken in the magnitude comparison of animals, a shorter time is taken. This suggests a compressed scale for the speed of judgement of differences between pairs of animals. A possible explanation for this is that the visual presentation of animal words may form semantic representations at a greater speed than the semantic representations of numbers so that the processing of information for animals, be they small or large, is similar in the length of time taken.

8.6.3 Large versus small distances between the items

The interaction between small and large stimuli, and small and large distance between the stimuli, just reached the level of significance, [$F(1,99) = 4.338$, $p <$

.040]. The result indicates that the size judgement of animals is more disrupted if there is a small distance between the stimuli than a large distance. A similar effect is evident for the comparison of numbers. However, this effect is reduced with numerical stimuli. The enhanced effect of distance between small and large animals is more evident than in numbers possibly because the concept of size relationships in animals represents an abstract concept of size that is not clearly definable. A further possibility is that animals are all equally represented in terms of strength and size, and that they do not represent a compressed scale. On the other hand the process by which large numbers are represented may be dependent upon the decade number for example, 56 (Hinrichs 1981). The decade here is fifty plus six with the fifty being strongly activated followed by the six. This double process may slow down the comparison process of two numbers within the same decade. If this were the case then the comparison of 53 and 65 should be faster than the comparison of 53 and 55 because sixty plus another digit will be judged as larger than fifty plus another digit.

8.6.4 Overall interaction between the stimuli (digits and animals) and the size of the stimuli and the distance between the stimuli.

A significant 3-way interaction was found between the stimuli (digits and animals), the size of the stimuli and the distance between the stimuli at $p < .01$. This result is not in the expected direction and does not support the hypothesis, that there will be no significant interaction between the stimuli (digits and animals), the size of the stimuli or the distance between the stimuli. A possible explanation for this is based on familiarity of the tasks since the magnitude comparison of numerical stimuli is a more familiar task than the magnitude comparison of animals. According to Campbell, et al. (1999) numbers are in regular use and numbers are used for calculation and in other contexts where quantity is relevant. It would appear that Arabic digits may become associated with skilled magnitude retrieval and comparison processes. In contrast animal magnitude comparisons are not required on a regular basis and are not necessary for calculation. As a result it becomes more difficult to activate magnitude information given animal stimuli and this effect is evident in the results.

8.7 Discussion

The rationale for the further analysis of the two magnitude tests arose from the findings of the factor analysis study discussed in Chapter 6. Factor 1 referred to the access of representations and this factor a ccommodated a number o f v aried numerical and non-numerical tests including lexical decision tasks, rotation of letter, subitizing and magnitude comparison tasks.

Following the analysis of the two magnitude tests a number of interesting observations have been made. The classic number effects have been found, with small numbers faster to judge than large numbers, and a greater distance between the numbers producing decreased reaction times compared to smaller distances. With the animal stimuli reaction times were judged faster for large animals than to small animals. Furthermore large distances between pairs of animals are judged faster than small distances. From these results it does appear that size may be encoded as a property of any object and the strength of this encoding is greater for some objects than others. It is possible that familiarity is a key factor. In addition, the overall reaction times for animals were found to be slower, perhaps due to the unfamiliar nature of the task in that the processing of animal size information is not as frequent as the processing of numerical size information.

To explain the significant three-way interaction a number of perspectives can be considered. According to the modular model proposed by McCloskey et al. (1985) the Arabic and verbal number comprehension component of the model converts numerical inputs into abstract semantic representations that specify the basic quantities in a number. The semantic representation is assumed to provide the basis for internal processing of numbers, for example, in performing calculations or number comparisons. The result of this representation is that the Arabic number production component translates the internal semantic representations of numbers into Arabic or verbal form for output.

The emphasis of this model is on the semantic representation of numbers. However, if it is assumed that there is a conversion of numerical input into a semantic representation, then it could be considered that for the comparison of animals a semantic processing element is present. If McCloskey et al.'s (1985) model is assumed, the question arises as to whether semantic processing is required for the comparison of numbers? Alternatively it may be based on a shallower level of processing that simply takes into account previously learnt, familiar information relating to quantity retrieved from long-term memory. With respect to the animal stimuli judgements may be based upon the physical characteristic of size but not processed in relation of quantity.

Dehaene (1992) suggested that approximate representation of magnitude could be characterised as a mental number line that becomes increasingly compressed as magnitudes increase. The number line is accessed by a process, which repeatedly activates small areas of the number line in approximately the correct location. This in turn produces a distribution of activation across the line. Dehaene (1992) considered that the number line plays a crucial role in identifying the quantity a number represents, therefore producing the semantic representations of numbers. However, as the two magnitude judgement tests have produced a significant interaction it remains unclear as to whether the processing of representation of size relies on a number line. It is possible that the number line is exclusive to number comparisons. Yet the results of the factor analytic study suggest that both tests support a common element. The result may reflect the notion that numerical size comparison is underpinned by semantic encoding and this issue is investigated further in Chapter 9.

Clark & Campbell (1991), on the other hand, suggest that numerical information is stored in many different ways including visual, semantic, written forms and auditory. Campbell (1995) suggested that arithmetic facts are represented by activating physical codes in every form, for example, visually written words, and imaginary number lines. The view is that during arithmetic problem solving not only is there activation of physical codes but activation of a magnitude code.

According to this theory a network is made up of physical codes and magnitude codes are connected through a series of nodes. It is possible that the results of the two magnitude tests reflect this model and that the retrieval of magnitude comparison of both types of stimuli is dependent upon the strength of the association between nodes.

It appears that the results of this study cannot be explained within the framework of the modular theory proposed by McCloskey et al. (1986) and the triple code theory Dehaene (1992). If no interaction had been found then it would be possible to assume that numerical and animal size judgements were processed using a similar information processing mechanism. However, this is not the case and a more feasible explanation is that of Campbell's interactive model based on representations of varying degrees of strength and familiarity.

8.8 Conclusion

The results indicate that reaction times differed significantly between small and large numerical stimuli, and as distances changed between the numerical stimuli. This finding is in line with classic effects. Thus, small numbers are faster to judge than large numbers, and comparisons involving greater distance between stimuli are also faster to judge than those involving a smaller distance. Animals only show one of the classic effects. This is, judgements are faster if the differences between the animals being compared is greater. It is possible that the animals chosen for this study do not represent a sufficiently wide scale. Furthermore numerical stimuli are precise whereas animal stimuli tend to represent less well defined categories, therefore, making it more difficult to manipulate the small vs. large differences and object sizes.

The key implication is that the evidence does not support Dehaene's proposal of a number specific process, which represents a compressed analogical scale. As the judgement of animal stimuli follows some of the same patterns as the numerical stimuli, the question arises as to the extent to which the analogue scale is number specific. Any item that can be represented by the semantic system, which has a size property, be it numbers, animals or objects, could follow the same patterns, for example, large vs. small objects and large vs. small distances between the objects. A suggestion for future research would be to replicate this study but substitute other stimuli for the animal stimuli for judgements of size between objects. It is possible that the use of animal stimuli alone represents only a small section of the possible physical continuum. A possible consideration for future research could be the inclusion of a range of different stimuli, for example, elephant vs. bus, and giraffe vs. the Eiffel Tower.

Chapter 9 presents experimental work investigating the notion that numerical size comparison is underpinned by semantic encoding. The dual task methodology is used to investigate whether numerical processing is linked to a long-term semantic system. The assumption is that, if magnitude comparison of numbers involves accessing a long-term semantic store, then requiring participants to process aurally presented words (which requires semantic processing) will interfere with the magnitude comparison of visually presented numerical stimuli.

Chapter 9 – Magnitude judgement of numbers Experiments 1 & 2

9.1 Aims of Chapter 9

This chapter builds on the information found in Chapter 8. The key outcome of the analysis of the data in Chapter 8 is the view that the number-specific analogue scale proposed by Dehaene may not be purely exclusive to the magnitude judgement of numbers. Dehaene (1992) suggested that this number-specific process represents a compressed analogue scale. However, the analysis of the data in Chapter 8 suggests that the magnitude comparison of animals may follow similar patterns to that of numbers. The development of the argument in this chapter is to consider the cognitive processes that may account for the similar findings found between the magnitude comparison of numbers and animals.

The results of the factor analysis discussed in Chapter 6 showed Factor 1, labelled as 'access to representations' and comprising six tests, the English and French lexical decision tasks, rotations of letters, subitizing circles and the magnitude comparison of numbers and animals. The aim of this chapter is to determine the underlying cognitive processes that may account for the finding that two magnitude tests loaded onto this factor. From the analysis of the data in Chapter 8 a possible common element between the magnitude comparison of numbers and animals is that of the long-term semantic access to representations. It would seem from the results that any items which can be represented by the semantic system, be it numbers, animals or objects etc. and which has a size property could follow similar patterns. The emphasis in this chapter is to investigate, through experimental work, using dual task methodology, the notion that numerical size comparison is underpinned by semantic encoding. The assumption is that if the magnitude comparison of numbers involves accessing a long-term semantic store, then requiring participants to process aurally presented words will interfere with the magnitude comparison of visually presented numerical stimuli. It is anticipated that the results of the experimental work will contribute to a fuller understanding of the common element that may link the tests loading on Factor 1.

McCloskey (1993) observed that the relationship between numerical and non-numerical processing mechanisms has not been researched in any detail. One of the main questions he proposed concerned whether numerical processing systems are separate from or incorporated within the cognitive language processing system. The chapter begins with a review of the literature surrounding issues relating the cognitive representations of words and numbers. The literature drawn on here comes from neuropsychological and cognitive research on the involvement of language processes in numerical processing.

9.2 Introduction

The literature is reviewed relating to the recognition of spoken words and extracting the meaning according to the model proposed by Ellis & Young (1996). According to this model the first stage of auditory word recognition requires the first component, an auditory analysis system. This system identifies phonemes in the speech wave. From this analysis the outcome is transmitted to the auditory input lexicon. This stage involves finding a match between the phonemes in the speech wave and the stored characteristics of known words. If a match is found the appropriate recognition unit in the auditory input lexicon will be activated. This then activates the representation of the meaning of the heard word in the semantic system. It is the semantic system too that instigates the word production process in speaking via the speech output lexicon and phoneme level.

This model provides three routes between hearing a word and producing the word in speech. The first route is through the auditory input lexicon, the semantic system, and the speech output lexicon. The second is a more direct route by way of the auditory analysis system and the phoneme level. The third route involves the auditory analysis lexicon linking to the speech output lexicon. This route allows heard words to be directly activated in the speech output lexicons without utilizing the representation of the word meaning in the semantic system. Ellis & Young (1996) considered that, up to that date, evidence for the latter route had not been substantiated by extensive experimental work.

The above model involves a lexical system in which knowledge of lexical forms is represented in functionally independent modality-specific components. According to Caramazza & Hillis (1990) input and output lexicons could be connected through a modality-independent semantic component. Supporting evidence for this model comes from the observation that brain damage can selectively impair the knowledge of words. This word deficit can be either word comprehension or word production. Some patients have been described with a difficulty in accessing the phonological form of words for output but have shown preserved lexical-semantic processing (Kay & Ellis, 1987; Caramazza & Hillis, 1990). Furthermore there is evidence for a distinction in the representation of knowledge within the semantic, phonological and orthographic lexical components. There have been a substantial number of reports of selective impairment in use of words in the category of living things (Warrington & Shallice, 1984; Caramazza & Shelton 1998), and the opposite pattern for man-made items has also been noted (Warrington & McCarthy, 1983; Hillis & Caramazza, 1991). These semantic category-specific deficits suggest that semantic representations may be organized in living/non-living categories. Hillis & Caramazza (1995) suggested that the representation of meanings is supported by particular brain mechanisms for different semantic categories of words. For example, separate regions of the brain process the categories of plants and animals. On the other hand McCarthy & Warrington (1985) and Zingeser & Berndt (1988) showed that brain damage can selectively affect specific grammatical categories of words, for example nouns versus verbs.

Seron & Noël (1992) reported neuropsychological research suggesting the existence of separate subsystems for the processing of numerals and other words at the lexical or at the semantic level. For example, Anderson, Damasio, & Damasio (1990) have described a patient who suffered from severe alexia and agraphia. The patient showed intact oral naming and good comprehension with no sign of aphasia. Of particular interest was the observation that the patient could read digits aloud but was unable to read letters or words and that the patient was able to write down digits but could not write letters or words. The precise nature of the impairment in this case was difficult to establish according to the researchers.

Cipolotti, Butterworth, & Denes (1991) reported the case of CG who could neither produce nor understand verbal numerals beyond four. CG was unable to read aloud Arabic numerals, to write Arabic numerals to dictation, to discriminate Arabic numerals from meaningless shapes, to discriminate verbal numerals from non-words, to produce orally the numeral that follows or precedes a given auditory numeral or to judge which of two verbal numerals was the larger. Further deficits occurred with ordered sequences, such as an inability to recite the days of the week, the months of the year or the alphabet. Having been prompted with part of a sequence CG was unable to provide the section that followed. CG was unable to comprehend numerals above 4 regardless of the task and the modality in which the stimuli were presented. However, CG showed good comprehension of spoken words and a normal performance in a verbal fluency and naming task that utilised various semantic categories, such as fruits, animals and cars. The researchers concluded that CG suffered from damage to the semantic system specific to the category of numerals.

Rossor, Warrington & Cipolotti (1995) described a patient who had developed severe non-fluent aphasia. The patient was impaired in all linguistic tasks, for example, picture naming, word to picture matching and repetition, suggesting impairment at both the semantic and the phonological level. However, the patient was able to solve addition, subtraction and some multiplication problems presented in written Arabic form though he was unable to produce or comprehend spoken verbal numerals. It was concluded that the patient was impaired at the level of the semantic system, except for the category of numerals which showed preserved abilities.

Thioux, Pillon, Samson, de Partz, Noël & Seron (1998) reported the case of patient NM showing impaired performance on comprehension tasks as well as a severe anomia. NM was able to use numerals despite an impairment to the semantic system and an almost complete inability to give a single correct response on tasks such as naming and production of synonyms, verbal fluency, picture naming and picture and word classification into semantic categories, for example animal, plant, object and transport. However despite NM's severe anomia he was able to recite any numeral when given its Arabic form and was also able to recite the months of the year and the days of the week. This case study shows the opposite effect to the case study of patient CG (Cipolotti et al., 1991). According to Thioux et al. (1998) a possible explanation for this double dissociation between numerals and other words may be that the semantic relevance of numerals is processed in separate brain regions from other words. A further possibility is that numerals form a separate category in the semantic system as they share certain properties that are processed in distinct brain regions. According to Seron et al. (1992) a particular property could be that numerals are organized in an ordered sequence. Thioux et al. (1998) suggested that at the semantic level each numeral may be linked to the following one by a 'plus-one' link and to the preceding one by a 'minus-one' link.

Thioux et al. (1998) further suggested that numerals may be processed in a specific semantic sub-system stored in separable brain regions as numerals are associated with quantities and are subjected to specific semantic manipulations, such as calculation, odd-even judgements and size judgements. In conclusion Thioux et al. (1998) suggested that NM's test performance implies that numerals may constitute a distinct category at the semantic level. It was also suggested that the processing of numerals may rely on a specific neural network system that had remained intact in NM although the brain regions supporting the semantic representation of other words appeared to be damaged.

The above review provides evidence from case studies that has supported models where knowledge of lexical forms (for example, orthography and phonology) are represented in modality specific components comprising of separate input and output systems. The input and output lexicons are connected through an independent semantic component (Ellis & Young 1996; Caramazza & Hillis 1990). Support for these models has come from research into patients with brain damage with the findings that brain damage can selectively impair either word comprehension or word production at various levels within the models. Furthermore studies have reported semantic category-specific deficits between living versus non living objects. It is suggested that this dissociation represents the way in which semantic material is organized in the brain (Warrington & McCarthy 1983; Warrington & Shallice 1984).

The key issue of this chapter is an investigation into the processing of visually presented numerical stimuli and verbally presented words at the lexical and semantic level. To fully understand the complexity of this area of research the following research is reviewed.

9.3 The relationship between the processes investigated in this chapter and published models of numerical cognition

McCloskey & Macaruso (1995) suggested that for the comprehension of an Arabic numeral the digits are converted to graphemic representations that specify the digit's abstract identity. This is similar to the identification of letters that make up a word whereby the graphemic letter representation specifies the abstract identity of a letter. From the graphemic digit represention the digits' meaning is activated. The semantic representations for the individual digits are then used to form a representation for the meaning of the numeral as a whole.

McCloskey (1992) proposed that most numerical processing is connected by number-semantic representations. According to McCloskey's modular theory, representation reflecting the characteristics of external number formats (e.g.

graphemic representations of digits) are involved in converting the digit or number word to internal semantic representations.

A different perspective was adopted in the encoding complex theory, (Campbell & Clark, 1992 and Campbell 1994). The essence of this theory is that there is widespread interaction among multiple forms of numerical representation. For example, the processing of Arabic or verbal numerals may activate 'visual and written codes for digits, analogue codes for magnitude (e.g. number lines), and combined visual-motor representations (e.g. counting on fingers; using an abacus)' (Campbell & Clark, 1992, p. 459). This is in conjunction with verbal codes that include 'articulatory and auditory codes in most people, visual and number-word codes in literate individuals, and unique codes in various specific groups (e.g. sign-language codes for numbers)' (Campbell & Clark, 1992, p. 459). According to empirical reports this theory suggests that the various forms of representation are interconnected in an associative network so that activation of one representational format leads directly or indirectly to activation of other formats, therefore, producing a multi-component 'encoding complex'.

In contrast to the above perspective Dehaene (1992) suggested that number semantic representations are of less importance. The triple code theory, Dehaene (1992) assumes that each of several representational formats are involved in carrying out numerical processes. For example, Dehaene proposed that phonological number word representations are central to the retrieval of arithmetic facts from memory and that visual Arabic number form representations form the basis for odd-even judgments. Dehaene (1992) suggested that the semantic representations of numbers are characterized as analogue representations. The view is that semantic representations become necessary when the meanings of numerals are directly involved in a task for example, determining which numeral is larger in magnitude.

The above review of neuropsychological and cognitive research relating to the cognitive representations of words and numbers forms the basis for investigating whether numerical processing systems are separate or incorporated within the cognitive lanuage processing system. The results of the experimental work are interpreted using the theoretical models discussed above.

9.4 Aim of Experiment 1 and 2

The two experiments reported in this chapter will investigate the effects of lexical and semantic processing on the magnitude comparison of numbers using the dual task methodology. It is anticipated that analysis of the results from both experiments will contribute to the understanding of the common element that may link the magnitude comparison of numbers to the other language based tests found to load onto Factor 1 of the factor analytic study 'access to representations' (see Chapter 6).

Furthermore experiments 1 and 2 will build on the information found in Chapter 8. The key outcome of the analysis of the data in Chapter 8 is the view that the number specific analogue scale proposed by Dehaene may not be purely exclusive to the magnitude judgement of numbers. Dehaene (1992) suggested that this number specific process represents a compressed analogue scale. However, the analysis of the data in Chapter 8 suggests that the magnitude judgement of animals may follow a similar pattern to that of numbers. The development of the argument in this chapter is to consider the view that to judge the relative size of two numbers requires semantic processing and that the number-specific analogue scale proposed by Dehaene (1992) may not be specific to numbers

Experiment 1 (lexical processing), focuses on the effects of the auditory presentation of a lexical processing task (experimental group) and a pre-lexical processing task (control group) and the visual presentation of the magnitude comparison of numbers task. Whilst the expectation is that no significant interaction will be found it is conceivable that quantity and size information about numbers could be encoded at a relatively early stage within the processing system such that accessing stored representations of words gains access to numerical sizes. Thus it is possible to predict interferences from a lexical task (experimental group) but not from a pre-lexical task that only requires the participants' attention to the initial phoneme, and not the word (control group). This view is in line with the encoding complex theory (Campbell & Clark 1992 and Campbell 1994). In Experiment 2 (semantic processing) it is expected that the auditory presentation of the semantic processing tasks will produce a significant interaction with the magnitude comparison of numbers task. If the lexical processing tasks used in Experiment 1 do not produce an interference effect with the magnitude comparison of numbers tasks this will suggest that the two tasks are utilizing different processing systems. If the results from Experiment 2 (semantic processing) do indicate an interference effect this will suggest that the two tasks, semantic processing of words and the magnitude comparison of numbers, may require the same processing system. Following on from the above discussion of the results from the experimental work it may be that the common element linking the two tests together in Factor 1 of the factor analytic study is that both tests require semantic processing.

9.5 Rationale for Experiment 1

Experiment 1 will investigate the effects of pre lexical and lexical processing on the magnitude comparison of numbers. This study required participants in the experimental and control groups to select the larger number from a pair of visually presented single and double digits. The same numerical stimuli were used for the experimental and control groups. In the experimental group a lexical decision task (lexical processing) was presented using an audio cassette player. A list of 64 words and non-words was used and participants were required to respond with the word 'yes' each time they heard a non-word. In the control group a letter identification task (pre lexical processing) required participants to respond with the word 'yes' each time they heard a word or non-word beginning with the letters 'c' or 'p'. The same lists of words were used in the experimental and control groups.

Participants first performed a single magnitude comparison task and a lexical decision task or a letter identification task to provide baseline scores. The two tasks were then performed concurrently. The differences between the scores on the concurrent tasks and the baseline scores were analyzed to investigate for any interaction effects.

The hypotheses to be tested in this experiment are that a significant difference will be found between the single and dual task conditions. There will be a significant difference between the experimental and control groups. It is anticipated that the experimental group will experience more difficulty in the dual task condition than the control group. There will be an interaction between the single and dual task conditions and the experimental and control groups in that the experimental group will show the greater decline in the dual task condition.

9.6 Method

The data were analyzed using three repeated measures 2 factor mixed Anovas. The first Anova was used to analyse the percentage of correct responses for the verbally presented stimuli. The second analysis was of the reaction time data for the magnitude judgement task and the third Anova was used to analyse the participants' correct responses to the magnitude judgement task.

The first factor is the within-subjects factor of task with three conditions, single task conditions for the visual and verbally presented stimuli and the dual task condition. The second factor is the between groups factor, experimental and control. The dependent variables are the percentage correct scores for the verbally presented stimuli, the reaction time data and correct responses to the magnitude judgement task.

9.6.1 Experimental group - Three conditions of the independent variable:

1 Visual stimuli/single task

This condition consisted of pairs of single or two digit stimuli visually presented on the computer screen using the Superlab programme. On each trial the participants were required to make a motor response as to which was the larger (in terms of numerical value) of the two digits presented.

2 Auditory stimuli/single task

A series of taped words and non-words were presented using an audio cassette player. Participants were required to respond verbally with the word 'yes' each time they heard a non-word.

2 Dual task

This condition required participants to key a response as to which number in the pairs of numbers presented was the larger whilst simultaneously responding verbally each time they heard a non-word from the taped list of words.

9.6.2 Control group - Three conditions of the independent variable:

1 Visual stimuli/single task

Pairs of single or two digit stimuli were visually presented on the computer screen. On each trial participants were required to make a judgement as to which was the larger (in terms of numerical value) of the two presented digits. The pairs of digits used were the same as in the single task condition of the experimental group.

2 Auditory stimuli/single task

A series of taped words and non-words were verbally presented. Participants were required to respond verbally with the word 'yes' when they heard a word or a non-word beginning with the letters 'c' or 'p'. The list of words used was the same as in the single task condition of the experimental group.

3 Dual task

This condition required participants to key a response as to which number in the pairs of digits presented was the larger whilst simultaneously responding verbally each time they heard a word or a non-word beginning with the letters 'c' or 'p'. The pairs of digits and list of words used were the same as in the dual task condition of the experimental group.

Table 9.1 Table summarizing the conditions of the experimental and control groups

Conditions	Experimental group	Control group
Single task/visual stimuli	Magnitude judgement of pairs of numbers	Magnitude judgement of pairs of numbers
Single task/auditory stimuli	Identification of a non-word	Identification of a word or non-word beginning with the letters 'c' or 'p'
Dual task	Magnitude judgement of pairs of numbers and the identification of a non-word	Magnitude judgement of pairs of numbers and the identification of a word or non-word beginning with the letters 'c' or 'p'

9.6.3 Participants

Sixty participants took part in the experiment. There were thirty participants in the experimental group, 24 female and 6 male, with an age range of 18 to 47 and a mean age of 25.76 and the standard deviation was 8.84. In the control group there were also thirty participants, 27 female and 3 male, with an age range of 18 to 48 and a mean age of 23.83 and the standard deviation was 8.72.

9.6.4 Materials – Experimental Group – Visually presented stimuli

Two sets of 30 pairs of digits presented on the computer using the Superlab software. (See Appendix 1 for the pairs of digits used in the single task condition of the experimental and control groups and Appendix 2 for the pairs of digits used in the dual task condition of the experimental and control groups).

In both sets of 30 trials each stimulus consisted of two numbers (both Arabic numbers) in the range from 2 to 9 to represent low numbers and in the range of 51 to 98 to represent high numbers. Fifteen trials were constructed to represent low numbers and fifteen trials to represent high numbers. Numbers holding zeros were excluded from the trials, for example, 10, 50, 60 etc. and pairs of high numbers consisted of numbers from the same decade, such as 54 57, and 87 88. Twenty trials required the right hand response made up of ten pairs of low numbers and ten pairs of high numbers. Ten trials required a left hand response made up of five trials of low numbers and five trials of high numbers. Analysis of the data included correct responses and reaction time data using the right hand.

Table 9.2 Summary of the Superlab coding procedure for the magnitude comparison of numbers.

Code	Code title
1/response	Response with the right hand (Mm key)
2/response	Response with the left hand (Zz key)

9.6.5 Materials – Experimental Group – Verbally presented stimuli

Two taped lists of 64 words and non words were presented verbally. (See Appendix 3 for the list of words used in the single task condition for the experimental and control groups and Appendix 4 for the list of words used in the dual task condition of the experimental and control groups).

Each list consisted of thirty-two words and thirty-two non words, eight words and eight non words beginning with the letter 'c' and eight words and eight non words beginning with the letter 'p'. The remaining thirty-two words (16 words and 16 non words) began with other tetters of the alphabet. A score sheet was prepared containing the 64 words. The speed of the words presented was at intervals of approximately 2 seconds and the order of presentation of the words was randomized.

9.6.6 Materials – Control Group – visually presented stimuli

Two sets of 30 pairs of digits were presented on the computer using the Superlab software. The pairs of digits used were the same as in the experimental condition.

9.6.7 Materials – Control Group – verbally presented stimuli

Two taped lists of 64 words. Each list consisted of sixteen words and sixteen non words, eight words and eight non words beginning with the letter 'p' and eight words and eight non words beginning with the letter 'c'. The lists of words used were the same as in the experimental condition.

9.7 Procedure

9.7.1 Experimental condition – Three conditions of the independent variable.

1 Magnitude judgement of pairs of numbers – single task

Participants received 30 trials in the magnitude judgement of numbers task and selected the larger number from a pair. The numbers were presented horizontally on the computer screen and separated by 4 centimeters. The larger operand appeared in the right position in twenty trials and in the left position for the remaining ten trials.

There was an interval of 1500 milli-seconds before the presentation of the stimuli and an interval of 2500 milli seconds following each pair. The stimuli remained on the screen for 5 seconds or until the correct response was made. The stimuli were presented in random order with the order consistent for all participants

Participants read the instructions presented on the screen and then pressed any key when they were ready to commence the experiment. Participants pressed the 'M' key with their right hand if the larger of the numbers was on the right of the screen or pressed the 'Z' key with their left hand if the larger of the numbers was on the left of the screen. Reaction time and correct responses were recorded.

2 Verbally presented list of words and non words – single task

Following the presentation of the 30 trials of the magnitude judgement task participants were presented with a list of 64 words using an audio cassette player. The list comprised 32 words and 32 non-words. Participants were required to respond with the word 'yes' each time they heard a non-word. The words were presented in random order with the order of presentations consistent for all participants. Error scores were recorded on the score sheet. The error scores were subtracted from the list of 64 words and non-words and the percentage correct calculated and used in the analysis of the data.

2 Magnitude judgement of pairs of numbers and identification of non-words – dual task

A set of 30 trials for the magnitude judgement task and a list of 64 words were used in this condition. The pairs of digits and list of words and non-words used in this condition were different from those used in the single conditions in order to eliminate learning effects. Both tasks were presented simultaneously. Participants were required to press the 'M' key with their right hand if the larger of the numbers was on the right of the screen or press the 'Z' key with their left hand if the larger of the numbers was on the left of the screen. At the same time participants responded with the word 'yes' each time they heard a non-word.

As the length of time taken for each participant to complete the magnitude judgement task varied, the dual task condition was terminated when a participant had completed the magnitude judgement task. Scoring ceased at that point. The number of words and non-words heard by each participant varied dependent upon their reaction time speed in response to the visually presented magnitude judgement task. Error scores were recorded on the score sheet for the identification of a non-word up to the point where the condition remained dual task. The error scores recorded were subtracted from the list of words and non-words up to the point where the scoring ceased, and the percentage correct was calculated and used in the analysis of the data.

9.7.2 Control group – Three conditions of the independent variable.

1 Magnitude judgement of pairs of numbers – single task

The same stimuli and procedure was used as in the experimental group.

2 Verbally presented list of words – single task

Following the presentation of the 30 trials of the magnitude judgement task participants were presented with a list of 64 words using an audio cassette player. The same list of words was used as in the experimental group. Eight words and eight non-words began with the letter 'c' and eight words and eight non-words began with the letter 'p'. The remaining sixteen words and sixteen non-words began with other letters of the alphabet. Participants were required to respond with the word 'yes' each time they heard a word or a non-word beginning with the letters 'c' or 'p'. The words were presented in random order with the order of presentations consistent for all participants. Error scores were recorded on the score sheet. The error scores were subtracted from the list of 64 words and non-words and the percentage correct calculated and used in the analysis of the data.

3 Magnitude judgement of pairs of numbers and identification of the letters 'c' and 'p' – dual task

The same procedure and list of words was used as in the dual task condition for the experimental group.

9.8 Results

The results are presented in three sections. In the first section group differences for the percentage correct scores obtained from the verbally presented stimuli in the single and dual task conditions are examined. The second section presents the results for the reaction time data from the control and experimental groups received from the visually presented stimuli in the single and dual task conditions. The third section considers the analysis of the correct responses obtained from the magnitude judgement task in the single and dual task conditions. A repeated measures 2 factor mixed Anova is used to analyse the data from these three sections.

9.8.1 Section 1 Analyses of verbally presented stimuli

Table 9.3 represents the mean and standard deviation scores for the control and experimental group based on percentage correct scores. The scores are derived from the single and dual task conditions for the verbally presented stimuli.

Table 9.3 Mean and standard deviation scores for single and dual task conditions for percentage correct verbally presented stimuli.

Conditions	**Participants**	**Mean % correct**	**Std.Dev**
Control Group single task/words commencing with 'c' or 'p'	30	58.40	3.25
Control Group Dual task/ words commencing with 'c' or 'p'	30	43.27	3.92
Experimental Group single task/identification of non-words	30	50.33	4.22
Experimental Group dual task/identification of non-words	30	41.50	5.60

From Table 9.3 it can be seen that there is a difference in percentage correct responses between the single and dual task conditions in the control group of 15.13%. In the experimental condition the difference between the single and dual task conditions is not as great at 8.83%. Figure 9.1 below illustrates this difference between the groups and the single and dual task conditions.

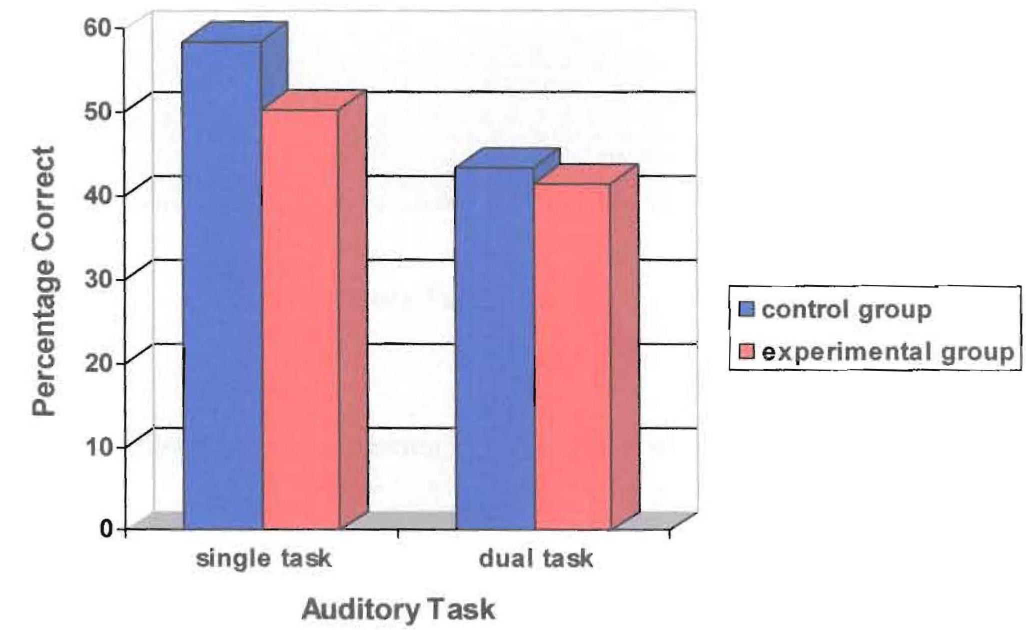


Figure 9.1 Bar Chart showing % correct responses in the single and dual task conditions and between the groups.

The results of the repeated measures 2 factor mixed Anova show that there is a significant difference between the single and dual task conditions [F, (1,58) = 239.15 p< .01]. The result for between groups is also significant [F, (1,58) 37.144, p< .01]. The interaction between the control and experimental groups and the percentage correct scores was statistically significant [F, (1,58) = 16.53 p < .01].

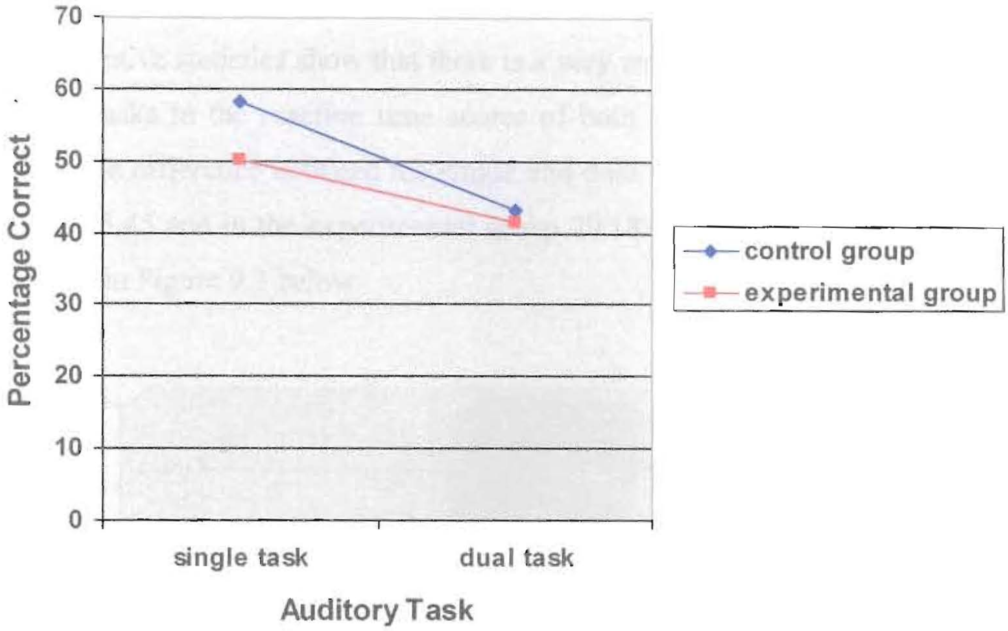


Figure 9.2 Interaction plot showing the main effect of single and dual task on group

An examination of the simple main effects revealed a significant simple main effect of group on the single task ($p < .01$). Groups did not significantly differ in the performance on the dual task.

9.8.2 Section 2 Analysis of reaction time data for the magnitude judgement task

The mean and standard deviation scores for reaction time data for the single and dual tasks and the control and experimental groups are shown in Table 9.4 below. The reaction time data is based on correct responses received from the participants responding with the right hand.

Table 9.4 Reaction time scores for the magnitude judgement task

Conditions	Participants	Mean	Std.Dev
Control group Single task/magnitude comparison	30	683.27	152.91
Control group Dual task/ magnitude comparison	30	718.72	144.83
Experimental group Single task/magnitude comparison	30	689.05	118.41
Experimental group Dual task/magnitude comparison	30	718.23	149.92

The descriptive statistics show that there is a very small difference between single and dual tasks in the reaction time scores of both the control and experimental groups. The difference between the single and dual task conditions in the control group is 35.45 and in the experimental group 29.18. The descriptive statistics are illustrated in Figure 9.3 below.

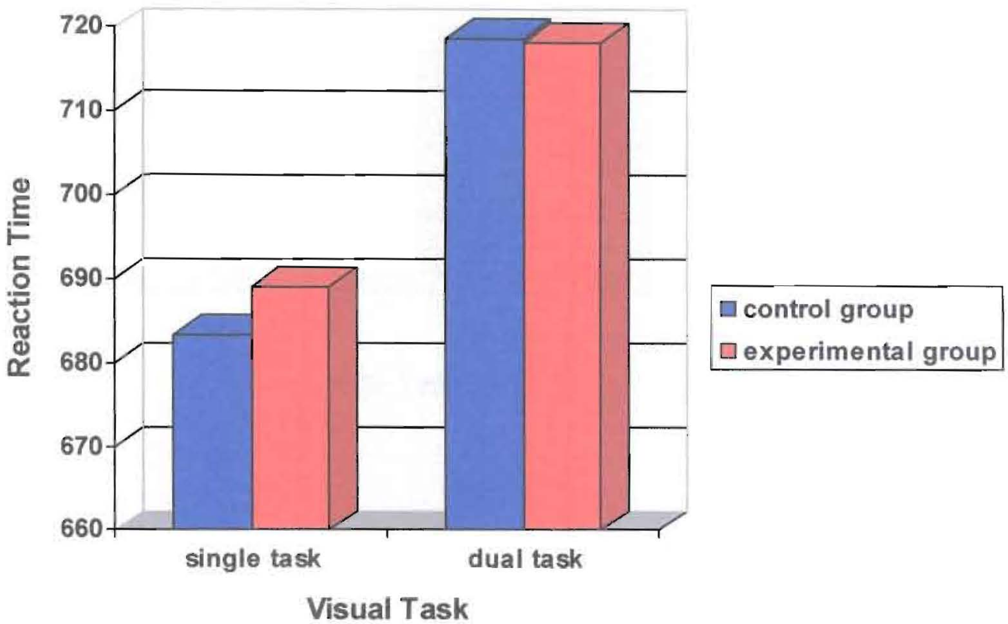


Figure 9.3 Reaction time scores for the control and experimental group across the single and dual task conditions

It is interesting to note that in both groups the reaction times are slower in the dual task condition and that the mean scores in the control group dual task condition are almost identical to the mean scores in the experimental group dual task condition.

The results of the repeated measures 2 factor mixed Anova show that there is no significant main effect for the single and dual conditions [F, (1,58) = 2.63 p .11].

The between groups factor is not significant [F, (1,58) = .007 p .93], and there is no significant interaction of task on group [F, (1,58) = .02 p .87].

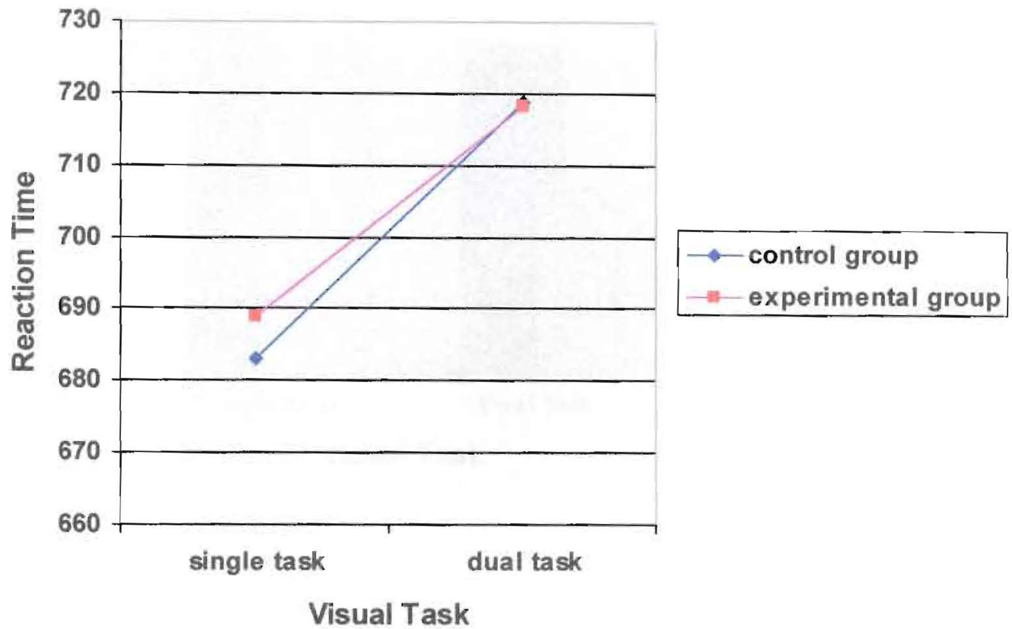


Figure 9.4 Interaction plot for the visually presented stimuli.

9.8.3 Section 3 Analysis of correct responses in the magnitude judgement task

Mean and standard deviation scores of correct responses in the single and dual task conditions for each group are shown in Table 9.5.

Table 9.5 Mean and standard deviation scores for the magnitude judgement task

Conditions	Participants	Mean correct responses	Std.Dev
Control Group single task	30	19.63	.808
Control Group dual task	30	19.36	.808
Experimental Group single task	30	19.63	.808
Experimental Group dual task	30	19.1	5.60

Table 9.5 shows that across the control and experimental groups and the single and dual task conditions the responses are very similar. In the experimental and control group single task the mean scores are the same. The level of accuracy within the groups and conditions was high.

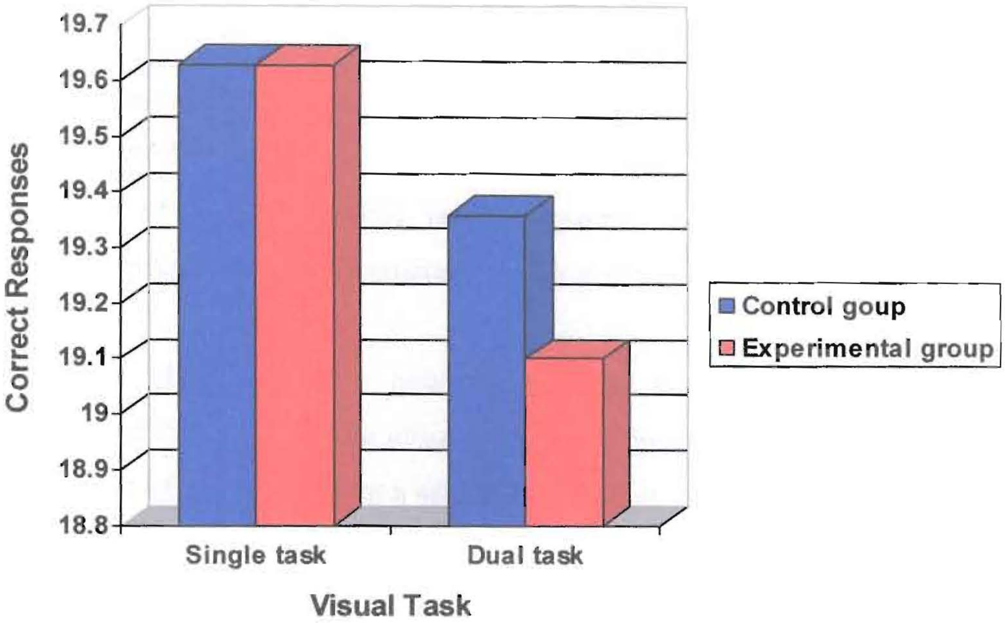


Figure 9.5 Correct scores for the control and experimental group across the single and dual task conditions.

The results of the repeated measures 2 factor mixed Anova show that there is a significant difference between the single and dual task conditions [F, (1,58) = 4.44 p<.03]. The result for between groups is not significant [F, (1,58) = .49 p .48] and there is no significant interaction of single and dual task by group [F, (1,58) = .49 p .48].

9.9 Discussion – Experiment 1

The results of this experiment are in line with the prediction that there will be no significant interaction of the lexical processing tasks on the visual presentation of the magnitude judgement task. Analysis of the accuracy data for lexical processing did show significant differences between the groups by conditions. Examination of the simple main effects showed a significant simple main effect of group which was only evident in the single task. The effect of group on dual task performance did not significantly differ. The results of the analysis of the reaction time data did not produce significant results of group by conditions. This suggests that the magnitude judgement task does not impair performance on the lexical processing tasks. The results of the correct responses for the magnitude judgement task did show a significant difference between the single and dual task conditions. However, the between groups result is not significant, and there is no significant interaction of single and dual task by group. This supports the view that magnitude judgement is not impaired by lexical processing.

The results of Experiment 1 have not revealed an interference effect, which suggests that the two tasks are utilizing different processing systems. To build on the outcome of this experiment a second experiment was conducted to investigate the effects of *semantic* processing on magnitude comparison.

9.10 Rationale for Magnitude judgement of numbers – Experiment 2

In order to examine the effects of semantic processing on the magnitude judgement of numbers a second experiment was conducted with alterations to the aurally presented stimuli used in Experiment 1. The rationale of the present study is to investigate, using the dual task methodology, the notion that numerical size comparison is underpinned by semantic encoding. The prediction is that if the magnitude comparison of numbers involves accessing a long-term semantic store, then requiring participants to process aurally presented words will interfere with the magnitude comparison of visually presented numerical stimuli. It is anticipated that the results of the experimental work will contribute to the understanding of the common element that may link the tests loading on Factor 1.

It is anticipated that a significant difference will be found between the single and dual task conditions in the experimental and control groups. The single tasks for the experimental group are a visually presented numerical comparison task and the judgement of the size of objects from a list of verbally presented words. Dual task is the concurrent performance of the two tasks. The single condition for the control group is the visual presentation of the numerical comparison task and the identification of a living or man-made object from a list of verbally presented objects. The dual task is the concurrent performance of the two tasks. It is also thought that there will be a significant difference between the experimental and control groups. It is anticipated that the experimental group will experience more difficulty in the dual task condition than the control group. It is expected that there will be an interaction between the single and dual task conditions and the experimental and control groups such that the experimental group will show the greatest decline in the dual task condition.

9.11 Method

The data were analyzed using three repeated measures 2 factor mixed Anovas. The first Anova was used to analyse the percentage of correct responses for the verbally presented stimuli. The second analysis was of the reaction time data for the magnitude judgement task and the third Anova was used to analyse the participants' correct responses to the magnitude judgement task.

The first factor is the within-subjects factor of task with three conditions, single task conditions for the visually and verbally presented stimuli and the dual task condition. The second factor is the between groups factor, experimental and control. The dependent variables are the percentage correct scores for the verbally presented stimuli, the reaction time data and scores for correct responses to the magnitude judgement task.

9.11.1 Experimental group – Three conditions of the independent variable:

1 Visual stimuli/single task

This condition consisted of pairs of single or two digit stimuli visually presented on the computer screen. On each trial the participants were required to make a judgement as to which was the larger (in terms of numerical value) of the two digits presented. The stimuli used were the same as for the experimental group, single task in Experiment 1.

2 Auditory stimuli/single task

A series of taped words consisting of living and man-made objects were verbally presented. Participants were required to make a size judgement and respond verbally as to whether the object they heard from the taped list of words was smaller or larger in size as compared to the size of a 'sheep'.

3 Dual task

This condition required participants to key a response as to which number in the pairs of digits presented was the larger whilst simultaneously responding verbally with the words 'smaller' or 'larger' each time they heard from the list of words the name of a living or man-made object that was smaller or larger than a 'cow'. The stimuli used in the magnitude judgement of numbers task were the same as in the dual task condition experimental group in Experiment 1.

9.11.2 Rationale for the auditory stimuli

The rationale for using these two particular animals, the sheep and the cow, was based on their relative size consistency across the species. Both the visually presented stimuli and the verbally presented stimuli required the participants to make judgements as to the size of numerals, in terms of numerical value, and the size of objects as compared with the baseline animals. It is considered that both these tasks will utilize semantic processing systems.

9.11.3 Control group – Three conditions of the independent variable:

1 Visual stimuli/single task

This condition consisted of a pairs of single or two digit stimuli visually presented on the computer screen. On each trial the participants were required to make a judgement as to which was the larger (in terms of numerical value) of the two digits presented. The stimuli used were the same as for the control group in Experiment 1.

2 Auditory stimuli/single task

A series of taped words made up of living and man-made objects were presented verbally. Participants were required to respond with the word 'living' each time they heard a word representing a living object and 'man' each time they heard a man-made object.

3 Dual task

This condition required participants to key a response as to which number in the pairs of digits presented was the larger whilst simultaneously responding verbally each time they heard a living or man-made object. The stimuli used in the magnitude judgement of numbers task were the same as in the dual task condition control group in Experiment 1.

9.11.4 Rationale for the auditory stimuli

In the control condition participants were required to decide whether the object they heard was a living or man-made object. This did not require a judgement as to the size of the object.

Table 9.6 Table summarizing the conditions of the experimental and control groups

Conditions	Experimental group	Control group
Single task/visual stimuli	Magnitude judgement of pairs of numbers	Magnitude judgement of pairs of numbers
Single task/auditory stimuli	Size comparison task	Identification of a living or man-made object
Dual task	Magnitude judgement of pairs of numbers and the size comparison task	Magnitude judgement of pairs of numbers and the identification of a living or man-made object

9.11.5 Participants

Forty participants took part in the experiment. There were twenty participants in the experimental group, 19 female and 1 male, with an age range of 18 to 41 and a mean age of 20.9 and a standard deviation of 5.15. In the control group, there were also twenty participants, 15 female and 5 male, with an age range of 19 to 42 and a mean age of 25.25 and a standard deviation of 6.63.

9.11.6 Materials – Experimental Group – visually presented stimuli

Two sets of 30 pairs of digits presented on the computer using the Superlab software. The two sets of 30 pairs of digits were the same as used in Experiment 1.

9.11.7 Materials – Experimental Group – verbally presented stimuli

Two taped lists of 48 words (See Appendix 5 for the list of living and man-made objects used in the single task condition of the experimental and control groups and Appendix 6 for the list of objects used in the dual task condition of the experimental and control groups).

Each list consisted of 24 living objects and 24 man-made objects. Twelve living objects were smaller than the target animals and twelve were larger. The 24 man-made objects consisted of twelve objects smaller and twelve objects larger than the target animal. The words were presented at intervals of approximately 3 seconds.

9.11.8 Materials – Control Group – visually presented stimuli

Two sets of 30 pairs of digits presented on the computer using Superlab software. The pairs of digits used were the same as in the experimental condition.

9.11.9 Materials – Controp Group – verbally presented stimuli

Two taped lists of 48 words. Each list consisted of 24 living objects and 24 man-made objects. The lists used were the same as in the experimental condition.

9.12 Procedure

9.12.1 Experimental condition – Three conditions of the independent variable.

1 Magnitude judgement of pairs of numbers – single task

Participants received 30 trials in the magnitude judgement of numbers task and selected the larger number from a pair. The numbers were presented horizontally on the computer screen and separated by 4 centimeters. The larger operand appeared in the right position in twenty trials and in the left position for the remaining ten trials.

Prior to the presentation of each trial the word 'ready' appeared to cue the participants for the upcoming pair of numbers. There was an interval of 1500 milli-seconds before the presentation of the stimuli and an interval of 2500 milli seconds following each pair. The stimuli remained on the screen for 5 seconds or until the correct respose was made. The stimuli were presented in random order with the order consistent for all participants

Participants read the instructions presented on the screen and then pressed any key when they were ready to commence the experiment. Participants pressed the 'M' key with their right hand if the larger of the numbers was on the right of the screen or pressed the 'Z' key with their left hand if the larger of the numbers was on the left of the screen. Reaction time and correct responses were recorded.

2 Verbally presented size comparison task – single task

Following the presentation of the 30 trials of the magnitude judgement task participants were presented with a list of 48 words using an audio cassette player. The list comprised 24 living objects and 24 man-made objects. Participants were required to respond with the word 'larger' or 'smaller' each time they heard the name of a living or man-made object that was larger or smaller than a 'sheep'. The words were presented in random order with the order of presentations consistent for all participants. Error scores were recorded on the score sheet. The error scores were subtracted from the list of 48 living and man-made objects and the percentage correct calculated and used in the analysis of the data.

3 Magnitude judegement of pairs of numbers and verbally presented size comparison task – dual task

A set of 30 trials for the magnitude judgement task and a list of 48 living and man-made objects was used in this condition. The pairs of digits and list of living and man-made objects used in this condition were different from those used in the single conditions in order to eliminate learning effects. Both tasks were presented simultaneously. Participants were required to press the 'M' key with their right hand if the larger of the numbers was on the right of the screen and to press the 'Z' key with their left hand if the larger of the numbers was on the left of the screen. At the same time participants compared the size of objects and responded with the word 'larger' or 'smaller' each time they heard a living or man-made object that was 'larger' or 'smaller' than a 'cow'

.

As the length of time taken for each participant to complete the magnitude judgement task varied, the dual task condition was terminated when each participant had completed the magnitude judgement task. Scoring ceased at that point. The number of living or man-made objects that were smaller or larger than a 'cow' heard by each participant varied dependent upon their reaction time speed in response to the visually presented magnitude judgement task The error scores recorded were subtracted from the list of objects up to the point where scoring ceased and, the percentage correct was calculated and used in the analysis of the data.

9.12.2 Control group – Three conditions of the independent variable.

1 Magnitude judgement of pairs of numbers – single task

The same stimuli and procedure were used as in the experimental group.

2 Identification of living and man-made objects – single task

Following the presentation of the 30 trials of the magnitude judgement task participants were presented with a list of words made up of 48 living and man-made objects using an audio cassette player. The same list of words was used as in the experimental group. Twenty-four of the words were referred to living objects and twenty-four to man-made objects. Participants were required to respond with the word 'living' each time they heard a living object and the word 'man' each time they heard a man-made object. The words were presented in random order with the order of presentations consistent for all participants. Error scores were recorded on the score sheet. The error scores were subtracted from the list of 48 objects and the percentage correct calculated and used in the analysis of the data.

3 Magnitude judgement of pairs of numbers and identification of living and man-made objects – dual task

A set of 30 trials for the magnitude judgement task and a list of 48 living and man-made objects was used in this condition. The pairs of digits and list of living and man-made objects used in this condition were different from those used in the single conditions in order to eliminate learning effects. Both tasks were presented simultaneously. Participants were required to press the 'M' key with their right hand if the larger of the numbers was on the right of the screen and to press the 'Z' key with their left hand if the larger of the numbers was on the left of the screen. Participants were required to respond with the word 'living' each time they heard the name of a living object and the word 'man' each time they heard the name of a man-made object. The same scoring procedure was used as in the experimental condition.

9.13 Results

The results are presented in three sections. In the first section group differences for the percentage correct scores obtained from the verbally presented stimuli in the single and dual task conditions are examined. The second section presents the reaction time results for the control and experimental groups in the single and dual task conditions. The third section presents the analysis of the correct responses obtained from the magnitude judgement task in the single and dual task conditions. A repeated measures 2 factor mixed Anova is used to analyse the data from these three sections.

9.13.1 Section 1 Analyses of the verbally presented stimuli

Table 9.7 represents the mean and standard deviation scores for the control and experimental groups showing the percentage correct scores. The scores are derived from the single and dual task conditions for the verbally presented stimuli.

Table 9.7 Mean and standard deviation scores for the verbal stimuli

Conditions	Participants	Mean	Std.Dev
Control Group Single task/living/man-made	20	86.72	8.68
Control Group Dual task/living/man-made	20	87.11	10.51
Experimental Group Single task/object size	20	84.57	10.98
Experimental Group Dual task/object size	20	74.03	10.92

Table 9.7 above shows that the scores in both conditions of the control group were consistent with a small difference between conditions of .39. However, in the experimental group a large number of correct responses were received in the single task with a fall of 10.54 to 74.03 in the dual task condition. Figure 9.6 below illustrates the differences between conditions.

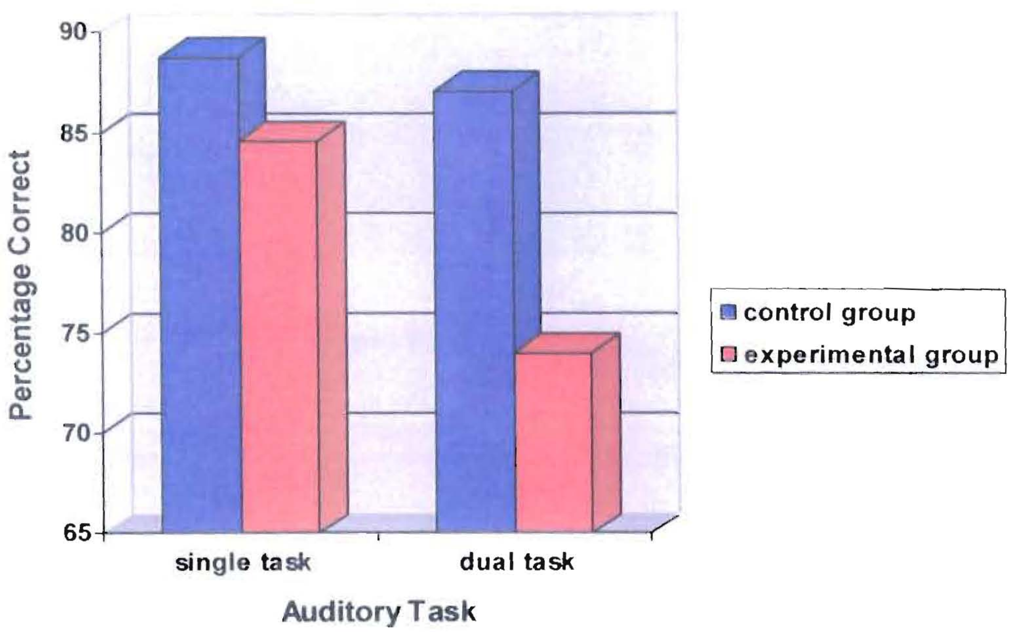


Figure 9.6 Percentage correct scores for groups in the single and dual task conditions

The results of the 2 factor mixed Anova of percentage correct responses yielded significant main effects for the within-subjects factor, single and dual task, [F

(1,38) = 14.98, p < .01]. The between-subjects factor of group, control and experimental is significant at [F (1,38) = 9.08, p < .01]. The interaction of the two factors is significant [F (1,38) = 8.085, p < .01]. This interaction is displayed in the Figure 9.7 below, showing that a judgement relating to the size of an object is more disrupted in the experimental group, dual task condition than in the single task condition. This is not the case in the control group where distinguishing between living and man-made items is not disrupted in the dual task condition.

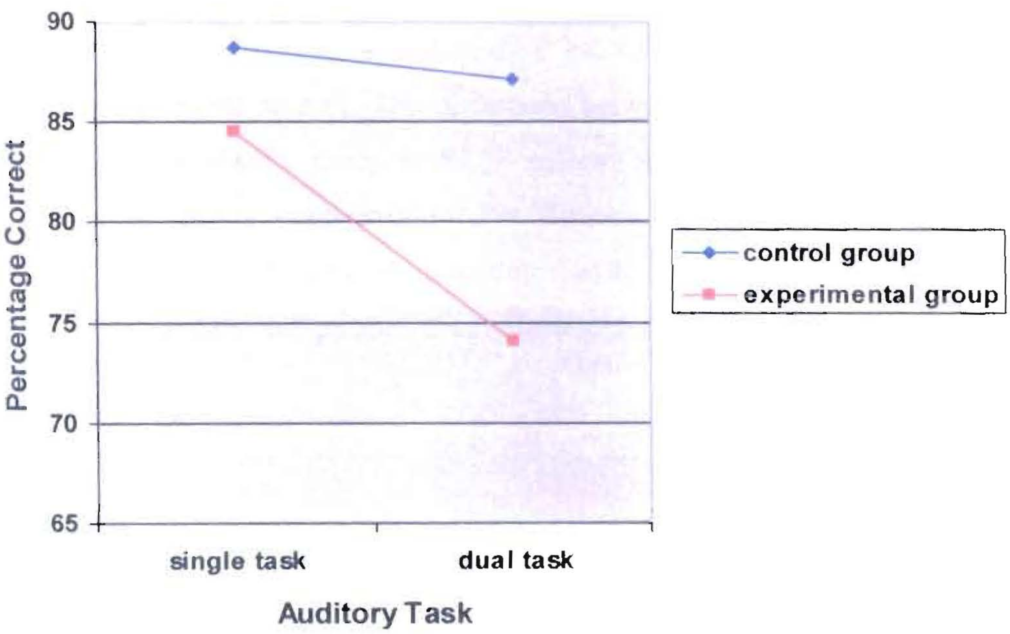


Figure 9.7 Interaction showing the main effect of auditory single and dual task on group

The form of the interaction has produced a significant simple main effect between the control and experimental group in the dual task condition at $p < .01$. There is no significant simple main effect between the groups in the single task. The paired t comparison between the single and dual task conditions in the experimental group has produced a result of [t = 5.51, p <. 01]. The result of the paired samples t-test for the control group between the single and dual task condition was not significant at [t = .65, p .52].

9.13.2 Results Section 2 Analysis of Reaction Time Data for the magnitude judgement task

Table 9.8 Mean and standard deviation reaction time scores for the visually presented stimuli.

Conditions	Participants	Mean	Std.Dev
Control Group Single task/magnitude comparison	20	787.73	309.70
Control Group Dual task/magnitude comparison	20	865.32	324.56
Experimental Group Single task/magnitude comparison	20	720.20	195.01
Experimental Group Dual task/magnitude comparison	20	988.21	499.66

Table 9.8 above shows the reaction time scores for the visually presented magnitude judgement stimuli. The difference between the single and dual task conditions in the control group is 77.59 msecs. However in the experimental group the difference is much greater - 268.01msecs. In both groups the dual task condition produced increased reaction times with the most noticeable increase shown in the experimental group. This is illustrated in Figure 9.8 below.

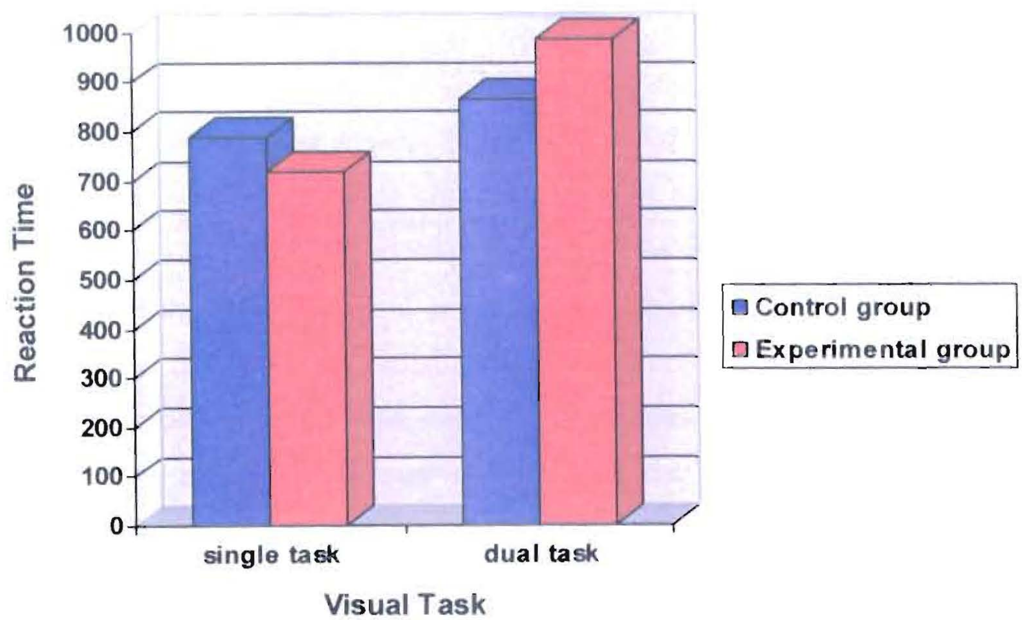


Figure 9.8 Reaction time scores for the single and dual task condions

The 2 factor mixed Anova produced a significant main effect within the single and dual task conditions [F (1,38) = 12.14 p < .01]. The between group factor is not significant [F, (1,38) = .078 p .781]. The interaction of the two factors fails to reach the level of significance [F (1,38) = 3.68 p .062]. The form of the interaction is shown in Figure 9.9.

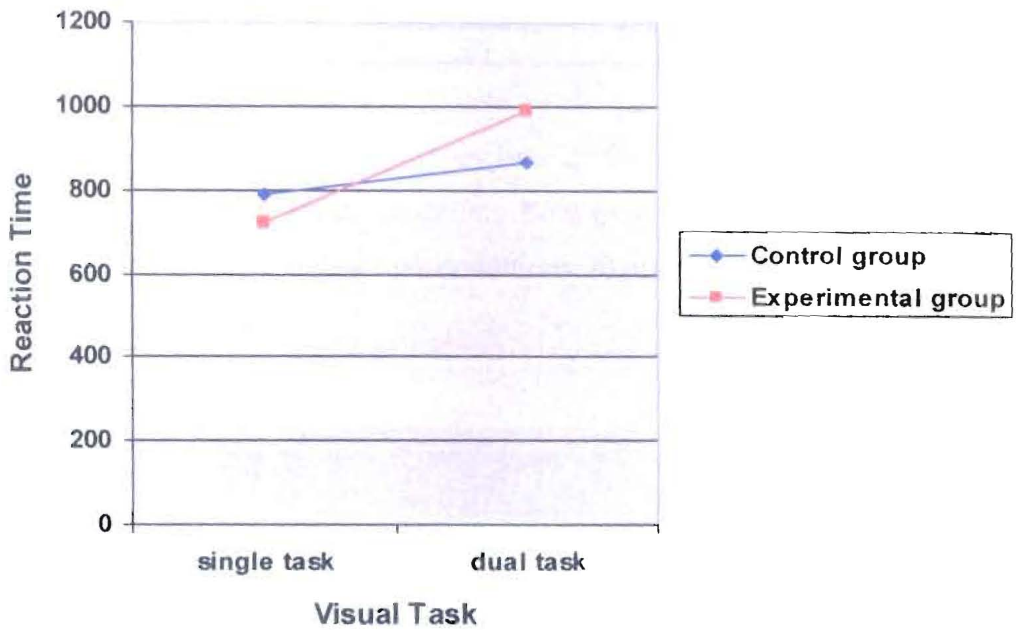


Figure 9.9 Interaction plot showing the main effect of visual stimuli on group

Although there is no significant interaction, the plot shown in Figure 9.9 clearly indicates that there is an increase in the reaction time scores for the experimental group within the dual task condition. The control group remains relatively consistent across the single and dual task conditions. The results of the paired samples *t*-test show a significant difference in the reaction time data in the experimental group between the single and dual task conditions [t = 3.10, p <. 01]. No significant difference was found in the control condition between tasks [t = 1.59, p .12].

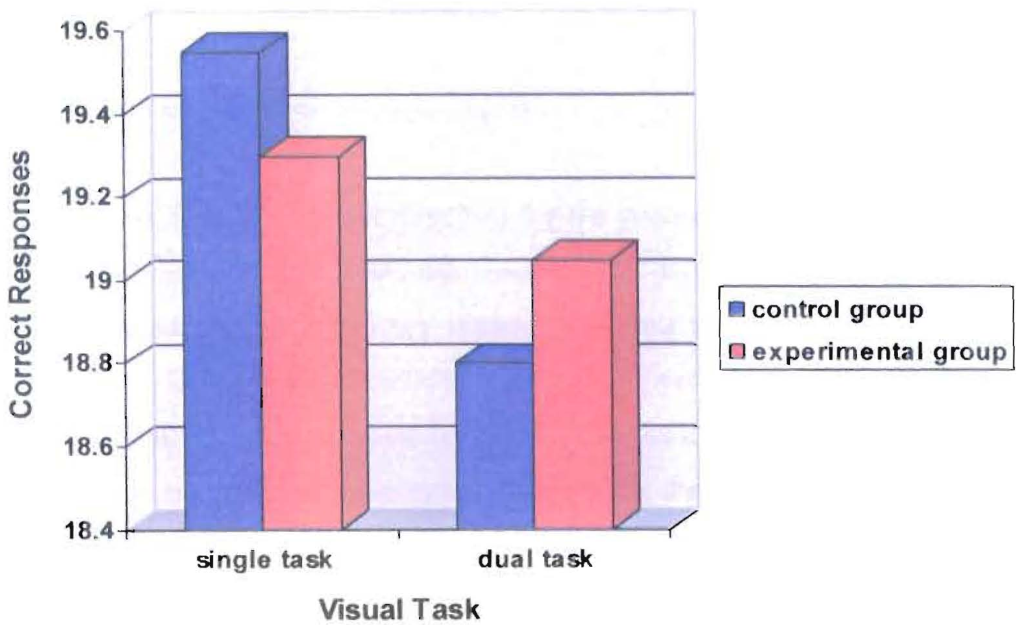
9.13.3 Section 3 Analysis of correct responses in the magnitude judgement task

Mean and standard deviation scores of correct responses in the single and dual task conditions and between groups are shown in Table 9.9.

Table 9.9 Mean and standard deviation scores for the magnitude judgement task

Conditions	Participants	Mean correct responses	Std.Dev
Control Group single task	20	19.55	.60
Control Group dual task	20	18.80	1.88
Experimental Group single task	20	19.30	1.30
Experimental Group dual task	20	19.05	1.63

The above table shows that there is very little difference between the groups across the single and dual task conditions. Both groups indicate a slight decrease in correct responses in the dual task conditions. Figure 9.10 below illustrates the mean correct responses.



Figurre 9.10 Correct responses to the magnitude judgement task

The repeated measures 2 factor mixed Anova did not produce a significant main effect within the single and dual task conditions [F (1,38) = 3.53 p .06]. The between group factor is not significant [F, (1,38) = .00 p 1.0]. The interaction of the two factors fails to reach the level of significance [F (1,38) = .88 p .35]. The form of the interaction is shown in Figure 9.11

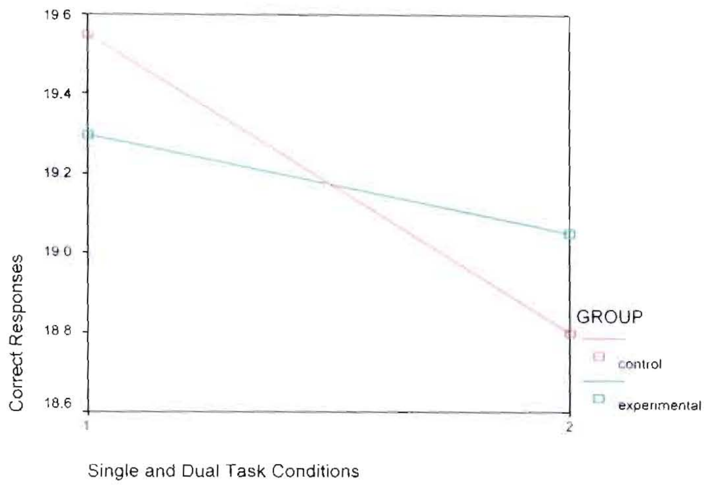


Figure 9.11 Interaction Plot of correct responses for the magnitude judgement task

9.14 Discussion – Experiment 2

The results are in the predicted direction for the percentage correct responses to the verbally presented stimuli and the reaction time data to the visually presented stimuli. The results of the verbally presented stimuli analysed using percentage correct data yielded significant results for the within and between-subjects factors. The interaction of group by task was also significant. The simple main effect between the control and experimental groups in the dual task condition was also significant but no simple main effect was found between the groups in the single task. The interaction of group by task suggests that a judgement relating to the size of an object is more disrupted in the experimental group, dual task condition than in the single condition. This is not the case in the control condition where distinguishing between living and man-made objects is not disrupted in the dual task condition. The results of the paired t comparison produced a significant result in the experimental group between the single and dual task condition. However no significant difference was found in the control condition.

The pattern of results is similar for the reaction time data with a significant difference found between the single and dual task conditions. The between groups factor is not significant and the interaction of the two factors just fails to reach the level of significance. The results of the paired *t* comparison suggest that the trend of the reaction time results is similar to that of the percentage correct response. The paired samples *t* test for reaction time data in the experimental group was significant with no significant results within the control group.

The results are in the predicted direction showing effects on reaction time and accuracy data from interference produced by the size comparison tasks. This experiment investigated semantic interference with two conditions either directly related to size as in the experimental condition or a decision process made between living and man-made objects and a numerical magnitude judgement task in the control group. The experimental group showed considerable interference with a reduction in accuracy and increased reaction time in the dual task condition. The control group showed a sight difference in reaction time, but very little change is evident in the accuracy data for the aurally presented stimuli.

The results of the two experiments indicate that firstly, lexical tasks do not interfere with numerical size judgement and secondly that the magnitude judgement of numbers can be disrupted by performing a concurrent semantic magnitude judgement task. This disruption appears to be at the semantic level of processing.

9.15 General Discussion

McCloskey (1993) suggested that work relating to the relationship between numerical and non-numerical processing mechanisms had not been researched in any detail. He raised questions as to whether numerical processing systems were separate or incorporated within the cognitive language processing system. From the results of the experiments reported in this chapter there seems to be some support for the view that the magnitude judgement of numbers may utilize the language processing system. Furthermore Warrington & Shallice (1984) reported findings from patients suffering from brain injury that suggested a separation of how knowledge is represented within the semantic and lexical components of the language system.

Research investigating intact and impaired performance incorporating the processing of words and numerals has produced mixed results. Patient CG reported by Cipolotti, Butterworth & Denes (1991) was unable to produce or understand verbal numerals beyond four regardless of the task and the modality in which the stimuli were presented. CG did show good comprehension of spoken words. The conclusion was that CG suffered from damage to the semantic system, specific to the category of numerals. The opposite effect was reported by Thioux, Pillon, Samson, de Partz & Noel (1998) in patient NM who showed preserved numerical performance and impairment on language tasks. Thioux et al. (1998) suggested that this double dissociation between numerals and other words may be caused because the semantic relevance of numerals is processed in separate brain regions from other words. A further suggestion was that numerals form a specific category within the semantic system.

Although previous research using patients with brain injury has produced inconclusive results the experimental work reported in this chapter has examined predictions with a 'normal' population of participants with the aim of assisting to resolve the controversy. This experimental work has taken a very specific numerical task, size comparison, and shown that it is disrupted when participants are required to undertake a semantic object size comparison task at the same time finding that there is disruption of the numerical task. This suggests that the two semantic tasks are competing for similar processing facilities. Whether there is a specific numerical category within the semantic system or the semantic relevance of numerals is processed in separate brain regions from other words is a consideration for future research and beyond the scope of this thesis. In terms of this research using a 'normal' population of participants there seems to be support for semantic interference between the two tasks used in Experiment 2.

9.16 Conclusion

The analysis of the data in this chapter has provided interesting results in relation to the understanding of the common element that may link the magnitude comparison of numbers to Factor 1 of the factor analytic study 'access to representations.' The results of Experiment 1 have not revealed an interference effect, which suggests that the lexical and pre-lexical tasks are utilizing different processing systems. The results of Experiment 2 are in the predicted direction showing effects on reaction time and accuracy data from interference produced by the size comparison tasks. It seems possible that the results of Experiment 2 lend support to the view that the common element is associated with the semantic processing system required for the successful performance of the two tests.

Furthermore this chapter aimed to build on the information found in Chapter 8. The key outcome of the analysis of the data in Chapter 8 is that the number specific analogue scale proposed by Dehaene (1992) may not be exclusive to the magnitude judgement of numbers. Dehaene (1992) proposed a number specific process that was represented by a compressed scale exclusive to numbers. The analysis of the data in Chapter 8 suggested that the magnitude judgement of animals might follow similar patterns to that of numbers. This chapter developed the argument on the basis of the findings from Chapter 8 that to judge the relative size of two numbers requires semantic processing. The results of Experiment 2 suggest that judging the relative size of objects utilizes similar processing systems used for the magnitude comparison of numbers. The number specific analogue scale proposed by Dehaene may not be purely exclusive to the magnitude judgement of numbers. Dehaene (1992) suggested that this number specific process represents a compressed analogue scale. However, the analysis of the data in Experiment 2 suggests that the judgement of object sizes may follow a similar pattern to that of numbers. This potentially important conclusion will be discussed in more detail in Chapter 12.

In conclusion, this study has reported two experiments carried out to explore the nature of magnitude representation using the dual task methodology. Experiment 1 investigated the effect of interference tasks at the lexical level of processing whereas Experiment 2 considered possible interference at the semantic level. Accuracy and speed of making judgements relating to numerical magnitude were only impaired where the interference task involved making magnitude judgements with regard to a range of objects. It is suggested that magnitude judgements are represented at the level of semantic processing and may not be specific to numbers.

Chapter 10 – Subitizing

10.1 Aims of Chapter 10

From the results of the factor analysis study discussed in Chapter 6 the subitizing circles task was found to load onto Factor 1, 'access to representations'. This factor comprised six tests, the English and French lexical decision tests, rotations of letters, subitizing circles and the magnitude judgement of numbers and animals. The subitizing numbers test appears to be unrelated to the tests in Factor 1 and loads onto Factor 4 (retrieval of mathematical knowledge) along with the basic arithmetic facts test. Subitizing numbers may contain different components from that of subitizing circles and in terms of the meaning of numbers rather than representations of shape. The focus of this chapter is to explore the subitzing process at its simplest, in terms of shape and representations rather than the meaning of numbers. It is speculated that performance in subitizing circles may depend on speed of access to stored representations and the automatic recognition of the number of items clustered together to form canonical configurations of visual items. A further hypothesis is that subitizing circles is not necessarily closely related to the processing of arithmetic information but more closely related to the analysis of shape (Mandler & Shebo (1982). On this basis, there may be the activation of representations in the visual image system where relative shape and size are represented in long-term memory (a non-verbal representation system). Another possibility is basic perceptual processes such as grouping and proximity (Gestalt theory) may activate canonical shape representations. The analysis of shape and size may involve access to pre-semantic, processes. These pre-semantic processes may involve the object-centred representational system, a component of the model for object recognition proposed by Ellis & Young (1996).

On the other hand, non-geometrical models of subitizing have been suggested based on abstract concepts rather than the concrete concept of shape (Trick & Pylyshyn 1993; 1994a;b). Research studies by Butterworth (1999) and Dehaene

(1997) have raised the possibility that enumeration tasks rely on fast processing mechanisms that do not involve sequential counting of each item displayed. This is considered a preattentional process (Sathian, Simon, Peterson, Patel, Hoffman & Grafton, 1999). In contrast, according to Gallistel & Gelman (1992) subitizing may involve a preverbal counting mechanism. Experimental work into the subitizing process has recently been developed to include different methods of investigation to assess brain activity using positron emission tomography (PET) and functional magnetic resonance imaging (fMRI). For example, experimental work by Fink, Marshall, Gurd, Weiss, Zafiris, Shah & Zilles (2000) have investigated the distinction between participants assessment of the number of items in a display and the assessment of the shape of the display measured using functional magnetic resonance imaging (fMRI). Piazza, Mechelli, Butterworth & Price (2002) have investigated the dissociable processes of subitizing and counting using positron emission tomography (PET). These investigative techniques focus on visible activity in relation to particular brain regions when activated by specific stimuli. This addresses the question of 'where' the activity occurs whereas this chapter addresses the question of 'how' the subitizing processes may occur.

The aim of this chapter is to report an experimental investigation, using dual task methodology, of the notion that the ability to subitize small displays of circles (1 to 4 circles) that are visually presented requires access to stored representations and utilizes pre-semantic processes.

10.2 Introduction

According to Trick (1992) there are two types of enumeration. The first is subitizing that is considered to be effortless and accurate. However, the number of items that can be subitized is limited to approximately 4 items although this figure of 4 items is variable dependent upon the experimental conditions. The second process of enumeration is counting which requires effort and is subject to errors. It is the process of subitizing that is the focus of this chapter.

Trick & Pylyshyn (1993; 1994a;b) suggested that subitizing is the process of enumeration when there are 4 or fewer items in a visual display. This concept has a long history as discussed in Chapter 1. Subitizing seems to be the fast and accurate ability to judge the number of items in a visual display providing the display holds 4 or fewer items. Research using reaction time data by Akin & Chase (1978) and Klahr & Wallace (1976) indicated slower reaction times and more errors when the visual display was above 4 items. The process to accommodate larger displays is 'counting' which is a slow process, requires a degree of effort and is subject to errors. However, the precise cut off point for the subitizing range is open to debate with the 'elbow' in the reaction time curve usually taken to be the boundary between subitizing and counting ranges. The subitizing range has been shown to vary between 1 – 3 and 1 – 7. Pavese & Umilta (1998) suggested that accuracy is very close to perfect for numerosities within the range of 1 – 3 with more errors found with increased numerosities. This variability in range appears to be dependent upon the nature of the experimental task undertaken by the participants and the data analysis procedures (Mandler & Shebo 1982).

Mandler & Shebo (1982) outlined a model for subitizing based on the recognition of the spatial configurations of the visual items to be enumerated. According to this view participants use the 'canonical pattern' of the items to understand 'cardinality' when there are small numbers of items in the display. For example, the visual system may recognise three items as a triangular configuration. Whether items are animals, different types of shapes or circles, three items can be transferred into a triangular shape very quickly. Four items can also be visualised as forming a shape, for example, a square. It is hypothesised that the ability to enumerate decreases as the number of items increases because the likelihood of producing a visual configuration at a glance is eliminated. It seems that this model of subitizing relies on a person having prior knowledge of shapes and geometric patterns. However, research into adult subitizing abilities yields similar results, to some extent, to studies of infants (Gallistel & Gelman, 1991). Yet very young infants will have no prior knowledge of canonical patterns. Gallistel & Gelman

(1991) suggested that infants may use an approximate (as yet not fully explained) counting mechanism. This view is supported by the research of Starkey, Spelke & Gelman (1990) who investigated the auditory and visual presentation of stimuli to infants of 6 to 8 months. It was found that when infants heard three drum-beats they looked longer at a visual display of three objects than at a display of two objects. The findings were similar with the sound of two drum-beats and a display of two objects. They concluded that infants are able to associate a correspondence between items in the visual and auditory modalities.

Research by Starkey & Cooper (1980) and van Loosbroek & Smitsman (1990) has shown that infants of one year of age are able to discriminate between different sets of objects that are simultaneously presented in terms of the number in each set. It seems that the results are in agreement with the results found for adults for the speed of recognition of 3 to 4 items. According to Bijeljac-Babic, Bertoncine & Mehler (1991), infants of four days old were able to discriminate 1 object versus 2 object displays and 2 objects versus 3 object displays. The infants were inconsistent in discriminating 3 versus 4 and 4 versus 5 object displays and were unsuccessful for 4 versus 6 objects (Bijeljac-Babic et al., 1991).

10.3 Object recognition

A model for object recognition proposed by Ellis & Young (1996) suggested that there are three levels of representation of the visual input of a visual object prior to object recognition and processing in the semantic system. The three levels are initial representation, viewer-centred representation and object-centred representation. They consider that recognition of items is a process that compares viewer-centred and object-centred representations to stored descriptions of known objects. These stages provide a link between the visual input of an item and the semantic system. Although Ellis & Young (1996) do not relate their model of object recognition to the processes involved in subitizing, it could be hypothesised that the identification of a visual array of three or four objects forming, for example, a triangle or a square may represent pre-semantic processing and utilise the processes involved in the viewer-centred and object-centred recognition components of the model for object recognition proposed by Ellis & Young (1996).

10.4 Visual analysis

Trick & Pylyshyn (1993, 1994a;b) considered that visual analysis plays a large part in the enumeration of items based on two stages. Firstly, there is a preliminary visual analysis mechanism termed the spatially parallel pre-attentive stage. During this stage the visual images are scanned for 'features' such as colour, line orientation and brightness. The features are derived from each of the items or objects in the display simultaneously. In other words, all items to be enumerated are identified by their features. For example, if an item in the display has different features from the others, it can be easily and quickly identified (Wolfe, Cave & Franzel 1989). The second stage is the spatial serial, attentive stage where analysis occurs only at one location at a time. Here the attentional focus can be moved from one location to another in the image. However the focus of attention is the place in the image where the majority of attentional resources are concentrated. According to Trick & Pylyshyn (1993), visual processing of an array of items either occurs all at once or one item at a time. This, therefore, makes an explanation of subitizing very difficult. The suggestion made by Trick & Pylyshyn (1993) is that subitizing occurs as the result of an intermediate stage between the pre-attentive and attentive stages. This stage is considered to perform 'individuation' which is necessary to distinguish between items or 'tokens' of a similar type. For example, to subitize four green dots in a display it is necessary to identify that one green dot is a separate item from another green dot and so on. In effect all the dots in the display are separate entities. Individuation appears to be a necessary processing mechanism that allows the attentional focus to move from visual item to visual item. This serial shifting of attention where attentional focus is driven from one item to another has been termed "indexing" by Ullman (1984).

Pylyshyn (1989) suggested that individuals have only a small number of mental reference tokens termed FINSTs (fingers of instantiation). This concept is likened to the pointing of a human finger to identify an item and provide information about where the item is. Research by Wolfe, Cave & Franzel (1989) has shown that in guided search experimental work participants will search for a red O among red Ns and green Os by checking only red items. This, according to Pylyshyn (1993), supports the view that FINSTs is a selective mechanism for determining the differences between the targets and distractors and that the information required to assess which items to FINST is available before the stage of individuation.

According to Pylyshyn (1993) humans are only able to subitize a small number of items b ecause t he s ystem o f individuation that binds features of items together with mental reference tokens is a system that has a limited capacity. Subitizing, therefore, occurs when the number of items in the display is less than the number of internal reference tokens of FINSTs. Counting an array of items begins after all the reference tokens have been assigned to items and on this basis the display is too large to be subitized. Attentional processing is then in operation with attentional resources scanning the visual array section by section. Dehaene (1992) suggested that this model of subitizing does not explain why subitizing seems to be limited to 3 – 4 items and further argued that the number of FINSTs or reference tokens provides an unspecified boundary. Furthermore researchers, such as Mandler & Shebo (1982) suggested that subitizing is not an independent procedure but reflects an estimation process to small sets of numbers. This view is supported by reaction time data showing that enumeration time increases slowly for 1 up to 4 items in a display followed by a more dramatic increase in reaction time for a larger number of items in a display (Akin & Chase 1978, Mandler & Shebo 1982). The apparently quick reaction time data found for subitzing small sets of items, (for example, 1 to 4 items) Mandler & Shebo (1982) attribute to the displays forming a particular shape, for example, a square, triangle or straight line.

Limited capacity working memory has also been proposed as a concept to explain the limits placed on subitizing. It was suggested by Trick & Pylyshyn (1994a;b) that there is only a limited amount of information that can be held in short-term memory and if the number of items in the display exceed the capacity limit of short-term memory then the display has to be revisited in order to identify the number of items presented. The short-term memory account for subitizing seems able to explain spatial enumeration with objects presented in a visual display and temporal enumeration for a series of tones or flashes of light. Logie & Baddeley (1987) investigated these phenomena using an articulatory suppression task by asking participants to pronounce 'the, the, the' while simultaneously enumerating a visually presented series of dots. In another experiment using the same articulatory suppression task, participants were required to enumerate successive flashes of light at one location. The results showed that the distracter task had no effect on the visually presented dots until the display was of seven or more items. In the temporal enumeration task for successive flashes of light, articulatory suppression produced an effect for one flash. These results suggest that the range of enumeration abilities varies for displays of items and successive flashes of light.

Recent developments, such as, fMRI and PET have offered a new way of investigating the processes of visual analysis. Fink et al., (2000) investigated subitizing processes under two conditions using fMRI. In one condition, participants were required to judge whether or not the number of dots in a display equalled four. In the second condition, participants responded as to whether or not the display was represented as a square. In both conditions, reaction time data were recorded. The results showed that the temporo-occipital cortex was involved in both conditions, suggesting that visual pattern recognition was implicated in both conditions. The involvement of canonical representations is shown from the significantly faster reaction times to form the judgement '4' when the dots formed a square rather than a quadrilateral. The superior and inferior parietal cortex were also shown to be involved in the execution of both tasks, suggesting that both tasks required the identification of the items in a spatial context. Other brain regions appeared to show differentiation between the two tasks. For example, the dorsolateral prefrontal cortex and temporoparietal cortex bilaterally appeared associated with the condition requiring the identification of shape. For the judgement of numerosity, the striate and extra-striate visual processing areas seemed to be implicated. The conclusion from this research was that, as different neural activation was observed, additional processes must be utilised in different ways for the judgement of numerosities and the judgement of spatial representations.

In a study using PET, Piazza et al. (2002) also investigated spatial representations to determine whether or not subitizing and counting are separate or interlinked processes and if canonical representations at a neural level are associated with pattern recognition. In this study, participants were required to enumerate visually presented displays of dots varying from 1 to 4 dots and 6 to 9 dots and in spatial arrangements that were either canonical or random representations. Reaction time for participants verbal responses as to the number of dots presented was recorded.

The results of the PET scan under the task conditions showed little evidence for the counting and subitizing tasks utilizing different brain regions. This particular result excluded the spatial arrangement of the displays. It appeared that the brain regions implicated in subitizing and counting are the extrastriate middle occipital and intraparietal areas. The results indicated that activation increased as the number of items on the display increased.

The region associated with the spatial arrangement of the visual displays was the bilateral occipitotemporal cortex which showed activation irrespective of the arrangement of the displays with no region specifically associated with canonical representations. They concluded that there was no evidence to confirm the existence of a neural system specific to subitizing that does not involve counting.

10.5 Subitizing and models of numerical cognition

Dehaene & Cohen (1995) proposed an elaboration of the triple-code model by Dehaene (1992) which maps the cognitive processes considered in the model to anatomical regions. The anatomical model was supported by neuropsychological evidence and data from normal participants. The three components of the triple-code model are the visual Arabic number form, the verbal word component and the analogue scale. It is the analogue scale that Dehaene (1992) and Dehaene & Cohen (1994) suggested is implicated in subitizing and estimation processes. Dehaene & Cohen (1995) further suggested that both hemispheres are involved in visual identification of stimuli and both hemispheres have an anological representation of numerical quantities. The latter point here is interesting as they argued that the representation of quantities utilises the 'parieto-occipito-temporal junction of both hemispheres' (p.88). A decade earlier Ungerleider & Minshkin (1982) had studied the 'what' and 'where' anatomical pathways in the brain utilised for visual object recognition. They suggested that for the identification of 'what' an object is there is increased activation in the occipito-temporal regions and for the identification of 'where' the object is in space the parieto-temporal pathway is utilised.

Research by Dehaene & Cohen (1995) and Piazza et al. (2002) further clarified the involvement of the parietal areas in relation to numerical processing. Dehaene & Cohen (1995) argued that the right inferior parietal regions support estimation and comparison processes, represented by the analogue scale or number line. The left parietal area provides a link between the representation of quantities and the verbal representation of numbers. This view seems consistent with the research by Piazza et al. (2002) which suggested that the right parietal region may be involved in subitizing but particularly in counting large arrays of stimuli and the left parietal region does not show increased activation for subitizing. It is speculated that this region may be involved in linguistic processes, for example, sub-vocal counting of number sequences.

From both the cognitive and neuropsychological perspectives, it remains unclear as to whether there are two separate systems for subitizing and counting. From a cognitive perspective, research by Mandler & Shebo (1982) provided some evidence for two separate systems. They showed for the identification of 1–3 items, reaction time increases slowly and then much more quickly with an increased number of items in the display. According to Trick & Pylyshyn (1993) limited capacity pre-attentive visual process are involved in subitizing. This process is limited and able only to enumerate to a maximum of four items. However, counting relies upon shifts in spatial attention. The research studies using Pet and fMRI techniques have provided some support for the existence of two separate systems but this remains inconclusive to date.

In McCloskey et al's (1985) abstract modular theory it is proposed that numerical information, in word form or numerical form, is initially transferred into an abstract semantic representation before further calculation. The subitizing processes may be conducted through the abstract semantic system or the calculation component of the model. McCloskey et al's model seems to be a model for basic fact retrieval based on the procedures and processes used for simple arithmetic calculations. The model provides the facility for numerical input, transferred to an abstract semantic representation followed by access to arithmetic facts in long-term memory with the relevant information then transferred to short-term memory before the required output. However, as discussed earlier in the chapter subitizing appears to be a process of fast and accurate enumeration of small sets of objects to the exclusions of counting and calculation processes. It appears that within the framework of McCloskey et al's (1985) abstract modular theory the subitizing process is not accommodated. On the other hand Dehaene (1992) proposed that within the triple code theory number processing involves an analogue magnitude representation supporting approximate calculation, numerical-size comparisons and subitizing tasks.

Clark & Campbell's (1991) abstract network model gives most attention to arithmetic fact retrieval. It is less concrete in its composition than McCloskey and Dehaene's models. Clark & Campbell highlighted the fact that humans possess the ability to store numerical information in many different ways including visual, semantic, written forms and auditory. Accordingly they argued that all numerical cognition involves a complex interaction of many different format-specific codes. Campbell (1995) suggested that arithmetic facts are represented by activating physical codes in every form in which arithmetic terminology may be described, for example, visually, written words, imaginary number lines, colours etc. He further suggested that during arithmetic problem solving not only is there activation of physical codes but activation of a magnitude code. Throughout the network physical codes and magnitudes codes are connected through a series of nodes. The physical code structures receive secondary activation based on the digits' similarity in magnitude. However, Clark & Campbell (1991) and Campbell (1995) did not explain how the various codes might interact in detail. Although the abstract network theory provides a general theoretical framework the specific processes operating within the theory remain under-specified.

It appears from the theories of numerical cognition discussed above that the subitizing process is not within the scope of the models proposed by McCloskey et al. (1985) and Clark & Campbell (1991). The triple code theory proposed by Dehaene (1992) does appear to accommodate the subitizing process.

10.6 Aim and rationale of the present study

The experiment reported in this chapter focuses on the effects of lexical and pre-lexical processing on a visually presented subitizing task using the dual task methodology. The visually presented stimuli consisted of a series of groups of circles presented on the computer screen. Participants in both the experimental and control groups were required to determine the number of circles presented in each group. On the screen above each group a number was shown in a box. Participants were required to identify if the number in the box above the circles represented the exact number of circles presented on the screen. The same visual stimuli were used for the experimental and control groups. In the experimental group a lexical decision task was presented using an audio cassette player. A list of 64 words and non-words was used and participants were required to respond with the word 'yes' each time they heard a non-word. In the control group a letter identification task, pre-lexical processing required participants to respond with the word 'yes' each time they heard a word or non-word beginning with the letters 'c' or 'p'. The same lists of words were used in the experimental and control groups.

Participants first performed a single subitizing task and then a lexical decision task or a letter identification task to provide baseline scores. The two tasks were then performed concurrently. The differences between the scores on the concurrent tasks and the baseline scores were analysed to investigate any interference effects.

If the auditory presentation of the lexical processing tasks conducted in conjunction with visual presentation of the subitizing tasks produces an interference effect, it could be speculated that both tasks are utilizing pre-semantic processing mechanisms in an early representation system prior to the semantic system. It is anticipated that analysis of the results from this experiment will contribute to the understanding of the common element that may link the subitizing task to the other language based tests found to load onto Factor 1 of the factor analytic study, 'access to representations'.

The following hypotheses are addressed in this experimental work. Firstly, it was predicted that a significant difference would be found between the single and dual task conditions in the experimental and control groups. Secondly, it was predicted that there would be a significant difference between the experimental and control groups. It was anticipated that the experimental group would experience more difficulty in the dual task condition than the control group. Thirdly, it was expected that there would be an interaction between the single and dual task conditions and the experimental and control groups in as much that the experimental group would show the greatest decline in the dual task condition.

10.7 Method

The data were analysed using three repeated measures 2 factor mixed Anovas. The first Anova was used to analyse the percentage of correct responses for the verbally presented stimuli. The second analysis was of the reaction time data for the subitizing task and the third Anova was used to analyse the participants' correct responses to the subitizing circles task.

The first factor is the within-subjects factor of task with three conditions, single task conditions for the visually and verbally presented stimuli and the dual task condition. The second factor is the between groups factor, experimental and control. The dependent variables are the percentage correct scores for the verbally presented stimuli, the reaction time data and correct responses to the subitizing task.

10.7.1 Experimental group – Three conditions of the independent variable:

1 Visual stimuli/single task

This condition consisted of a series of groups of circles presented on the computer screen. On each trial participants were required to add up the number of circles presented in each group. On the screen above each group a number was shown in a box. Participants were required to identify if the number in the box above the circles represented the exact number of circles presented on the screen.

2 Auditory stimuli/single lexical task

A series of taped words and non-words were presented using an audio-cassette player. Participants were required to respond verbally with the word 'yes' each time they heard a non-word.

3 Dual task

This condition required participants to key a response as to whether the number shown in the box above the circles represented the exact number of circles presented on the screen. This was performed simultaneously with participants responding verbally each time they heard a non-word from the taped list of words.

10.7.2 Control Group – Three conditions of the independent variable:

1 Visual stimuli/single task

This condition consisted of a series of groups of circles presented on the computer screen. On each trial participants were required to add up the number of circles presented in each group. On the screen above each group a number was shown in a box. Participants were required to identify if the number in the box above the circles represented the exact number of circles presented on the screen.

2 Auditory stimuli/single pre-lexical task

A series of taped words and non-words were verbally presented. Participants were required to respond verbally with the word 'yes' when they heard a word or a non-word beginning with the letters 'c' or 'p'.

3 Dual task

This condition required participants to key a response as to whether the number shown in the box above the circles represented the exact number of circles presented on the screen. This was performed simultaneously with participants responding verbally each time they heard a word or a non-word beginning with the letters 'c' or 'p'.

Table 10.1 Summary the conditions of the experimental and control groups

Conditions	Experimental group	Control group
Single task/visual stimuli	Identification of the number of circles presented on the screen	Identification of the number of circles presented on the screen
Single task/auditory stimuli	Identification of a non-word	Identification of a word or non-word beginning with the letters 'c' or 'p'
Dual task	Identification of the number of circles presented on the screen and the identification of a non-word	Identification of the number of circles presented on the screen and the identification of a word or non-word beginning with the letters 'c' or 'p'

10.7.3 Participants

Sixty participants took part in the experiment. There were thirty participants in the experimental group, 25 female and 5 male, with an age range of 18 to 43 and a mean age 22.86, standard deviation of 5.74. In the control group there were also thirty participants, 20 female and 10 male, with an age range of 18 to 50 and a mean age of 25.53, standard deviation 8.38.

10.7.4 Materials – Experimental Group – visually presented stimuli

Two sets of 30 trials with each trial made up of a group of circles were presented on the computer using the Superlab software. (See Appendix 7 for and examples of the stimuli used in the single and dual task conditions of the experimental and control groups).

In both sets of 30 trials participants were shown a series of groups of circles. Each circle was 0.25 inches in diameter. The number of circles used in each group ranged from two to four and the spatial organization of the groups varied across trials. The stimuli remained on the screen for 5 seconds or until the participants made the correct response. Participants were asked to total the number of circles presented in each group. On the screen above each group a number was shown in a box. Participants were required to identify if the number in the box above the circles represented the exact number of circles presented on the screen. Twenty trials required the right hand response and ten trials required a left hand response. Twenty trials were correct and ten stimuli were incorrect. The presentation of the stimuli was randomized.

Table 10.2 Superlab coding procedure for the subitizing circles task

Code	Code title
1/response	Response with the right hand (Mm key)
2/response	Response with the left hand (Zz key)

10.7.4 Materials – Experimental Group – verbally presented stimuli

Two taped lists of 64 words were verbally presented. Each list consisted of thirty-two words and thirty-two non-words, eight words and eight non-words beginning with the letter 'c' and eight words and eight non-words beginning with the letter 'p'. The remaining thirty-two words began with other letters of the alphabet. The speed of the words presented was at intervals of approximately 2 seconds. The two lists of words were the same as used in Chapter 9, Experiment 1 (See Appendices 3 and 4).

10.7.5 Materials – Control Group – visually presented stimuli

Two sets of 30 trials with each trial made up of a group of circles were presented on the computer using the Superlab software. The groups of circles used were the same as in the experimental condition.

10.7.6 Materials – Control Group – verbally presented stimuli

Two taped lists of 64 words. Each list consisted to thirty two words and thirty two non-words, eight words and eight non-words beginning with the letter 'c' and eight words and eight non-words beginning with the letter 'p'. The remaining thirty two words and non-words began with other letters of the alphabet. The lists of words used were the same as in the experimental condition.

10.8 Procedure

10.8.1 Experimental condition – Three conditions of the independent variable.

1 Subitizing circles task – single

Participants received 30 trials in the subitizing circles task. The number of circles used in each group ranged from two to four and the spatial organization of the groups varied across trials. The size of the circles remained consistent for each trial and where there was more than one circle presented each circle was separated by 3 centimeters. On the screen above each group a number was shown in a box. Participants were required to identify if the number in the box above the circles represented the exact number of circles presented on the screen. Prior to the presentation of each trial the word 'ready' appeared to cue the participants for the upcoming stimuli. There was an interval of 1500 milli-seconds before the presentation of the stimuli and an interval of 3000 milli-seconds following each trial. The stimuli remained on the screen for 5 seconds or until the correct response was given. The stimuli were presented in random order with the order consistent for all participants.

Participants read the instruction presented on the screen and then pressed any key when they were ready to commence the experiment. If the number in the box represented the exact number of circles participants pressed the 'M' key or if the number in the box did not equal the total number of circles participants were required to press the 'Z' key. Twenty trials were correct and ten stimuli were incorrect. Reaction time and correct responses were recorded.

2 Visually presented list of words and non-words – single lexical task

Following the presentation of the 30 trials of the subitizing circles task participants were presented with a list of 64 words using an audio cassette player. The list comprised 32 words and 32 non-words. Participants were required to respond with the word 'yes' each time they heard a non-word. The words were presented in random order with the order of presentations consistent for all participants. Error scores were recorded on the score sheet. The error scores were subtracted from the list of 64 words and non-words, and the percentage correct was calculated and used in the analysis of the data.

3 Subitizing circles task and identification of non-words – dual task

A set of 30 trials for the subitizing circles task and a list of 64 words were used in this condition. The subitizing circles task and list of words and non-words used in this condition were different from those used in the single conditions in order to eliminate learning effects. Both tasks were presented simultaneously. Participants were required to press the 'M' key with their right hand if the number in the box above the circles represented the exact number of circles presented on the screen or press the 'Z' key if the number in the box above the circles did not represent the exact number of circles. At the same time participants responded with the word 'yes' each time they heard a non-word.

As the length of time taken for each participant to complete the subitizing circles task varied, the dual task condition was terminated when a participant had completed the subitizing task. Scoring ceased at that point. The number of words and non-words heard by each participant varied dependent upon their reaction time speed in response to the visually presented subitizing task. Error scores were recorded on the score sheet for the identification of a non-word up to the point where the condition remained dual task. The error scores recorded were subtracted from the list of words and non-words up to the point where the scoring ceased, and the percentage correct was calculated and used in the analysis of the data.

10.8.2 Control group – Three conditions of the independent variable.

1 Subitizing circles task – single task

The same stimuli and procedure were used as with the experimental group.

2 Verbally presented list of words – single pre-lexical task

Following the presentation of the 30 trials of the subitizing circles task participants were presented with a list of 64 words using an audio cassette player. The same list of words was used as in the experimental group. Eight words and eight non-words began with the letter 'c' and eight words and eight non-words began with the letter 'p'. The remaining sixteen words and sixteen non-words began with other letters of the alphabet. Participants were required to respond with the word 'yes' each time they heard a word or non-word beginning with the letters 'c' or 'p'. The words were presented in random order with the order of presentations consistent for all participants. Error scores were recorded on the score sheet. The error scores were subtracted from the list of 64 words, and non-words and the percentage correct was calculated and used in the analysis of the data.

3 Subitizing circles task and identification of the letters 'c' and 'p'

The same procedure and list of words were used as in the dual task condition for the experimental group.

10.9 Results

The results are presented in three sections. In the first section group differences for the percentage correct scores obtained from the verbally presented stimuli in the single and dual task conditions are examined. The second section presents the results for the reaction time data from the control and experimental groups obtained from the visually presented stimuli in the single and dual task conditions.

The third section presents the analysis of the correct responses obtained from the magnitude judgement task in the single and dual task conditions. A repeated measures 2 factor mixed Anova is used to analyse the data from these three sections.

10.9.1 Section 1 Analysis of verbally presented stimuli

Table 10.3 represents the mean and standard deviation scores for the control and experimental groups based on percentage correct scores. The scores are derived from the single and dual task conditions for the verbally presented stimuli.

Table 10.3 Mean and standard deviations scores for single and dual task conditions for percentage correct verbally presented stimuli.

Conditions	Participants	Mean % correct	Std.Dev
Control Group Single task/words beginning with 'c' and 'p'	30	92.24	5.24
Control Group Dual task/words beginning with 'c' and 'p'	30	87.76	10.38
Experimental Group Single task/identification of a non-word	30	77.97	7.19
Experimental Group Dual task/identification of a non-word	30	78.40	7.20

The table shows that in the control conditions single task there were more correct responses recorded than in the control dual task condition. The difference in scores between the conditions for the control group is 4.48%. In the experimental condition the difference between the two conditions is very small 0.43%. Figure 10.1 below illustrates this difference between the groups and the single and dual task conditions.

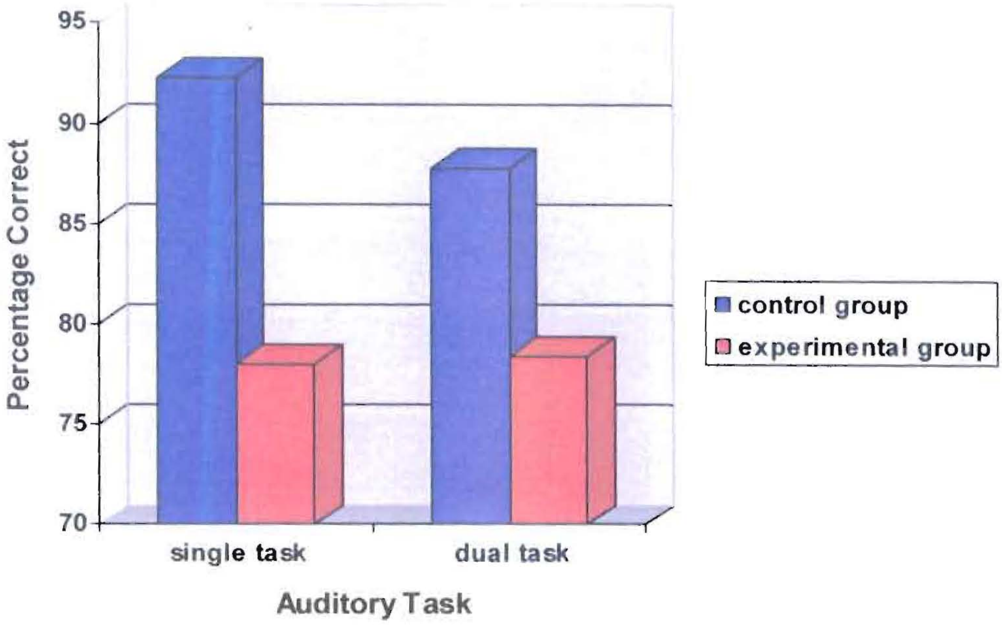


Figure 10.1 Bar chart showing the % correct responses in the single and dual task conditions and between the experimental and control groups.

The results of the repeated measures 2 factor mixed Anova yielded a significant main effect for within subjects factor single and dual task conditions $F, (1,58) = 5.16, p < .05$. The between groups factor is also significant at $F, (1,58) = 43.80, p < .01$. The interaction of the two factors, control and experimental groups and the percentage correct scores in the single and dual task conditions was significant, $F, (1,58) = 7.57, p < .01$. Figure 10.2 shows the interaction and simple main effect between the control and experimental conditions in both the single and dual task conditions at $p < .01$.

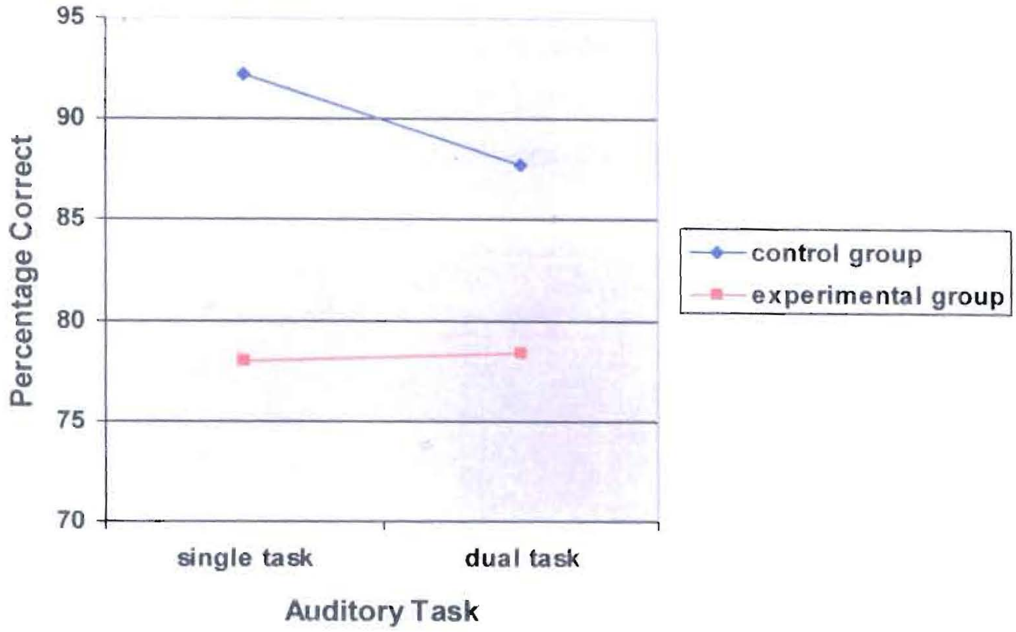


Figure 10.2 Interaction between groups and conditions

10.9.2 Section 2 Analysis of reaction time data for the subitizing circles task

The mean and standard deviation scores for the reaction time data for the single and dual tasks and the control and experimental groups are shown in Table 10.4 below. The reaction time data is based on the correct responses received from the participants responding with the right hand.

Table 10.4 Mean and standard deviation reaction time scores for the groups and conditions.

Conditions	Participants	Mean Reaction time	Std.Dev
Control Group Single task/words beginning with 'c' and 'p'	30	940	178
Control Group Dual task/words beginning with 'c' and 'p'	30	1003	233
Experimental Group Single task/identification of a non-word	30	894	159
Experimental Group Dual task/identification of a non-word	30	971	214

In both the control and the experimental groups the dual task condition showed slower reaction times. The difference in reaction time between the control group single and dual task is 62 milli-seconds and in the experimental group 78 milli-seconds. The descriptive statistics are illustrated in Figure 10.3 below.

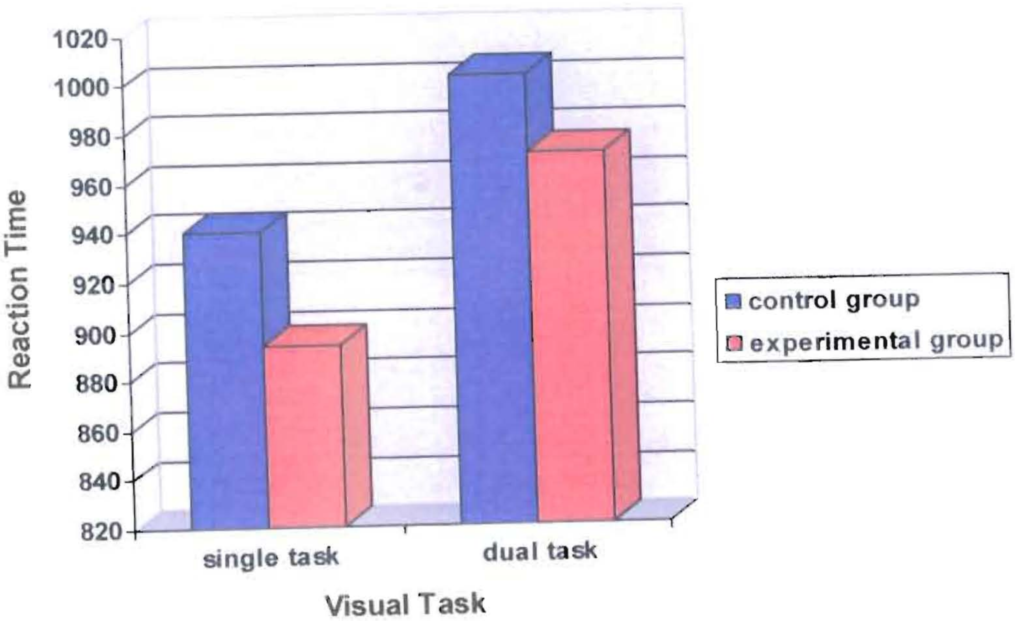


Figure 10.3 Bar chart showing mean reaction time scores for groups across conditions.

The results of the repeated measures 2 factor mixed Anova showed a significant main effect for the single and dual tasks $F, (1,58) = 8.370, p < .01$, There is no significant main effect of groups, $F, (1,58) = .391$ and there is no significant interaction of task on group $F, (1,58) = .101, p\ .751$ as shown in Figure 10.4.

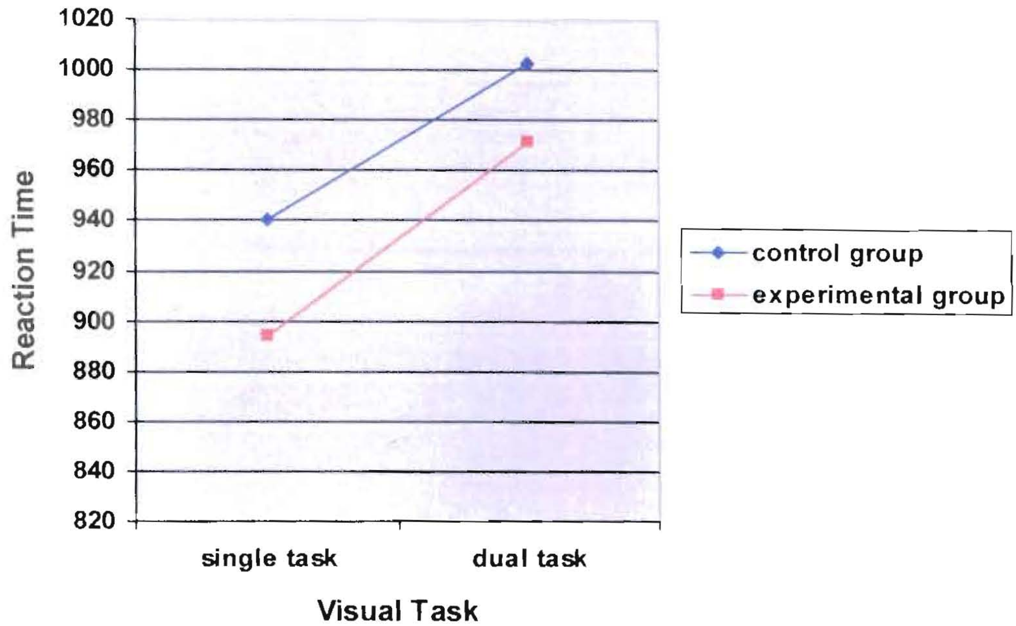


Figure 10.4 Interaction plot for the visually presented subitizing task

10.9.3 Section 3 Analysis of correct responses in the subitizing circles task

Mean and standard deviation scores of correct responses in the single and dual task condition for each group are shown in Table 10.5.

Table 10.5 Mean and standard deviation scores for the subitizing task

Conditions	**Participants**	**Mean correct responses**	**Std.Dev**
Control Group single task	30	19.4	.81
Control Group dual task	30	17.73	2.08
Experimental Group single task	30	19.43	1.00
Experimental Group dual task	30	17.83	2.10

The above table indicates that there is very little difference between the control and experimental groups. In both groups there is a noticeable difference in correct responses between the single and dual task conditions. A difference of 2.67 is shown in the control condition and 2.60 in the experimental condition. The bar chart below illustrates the mean scores.

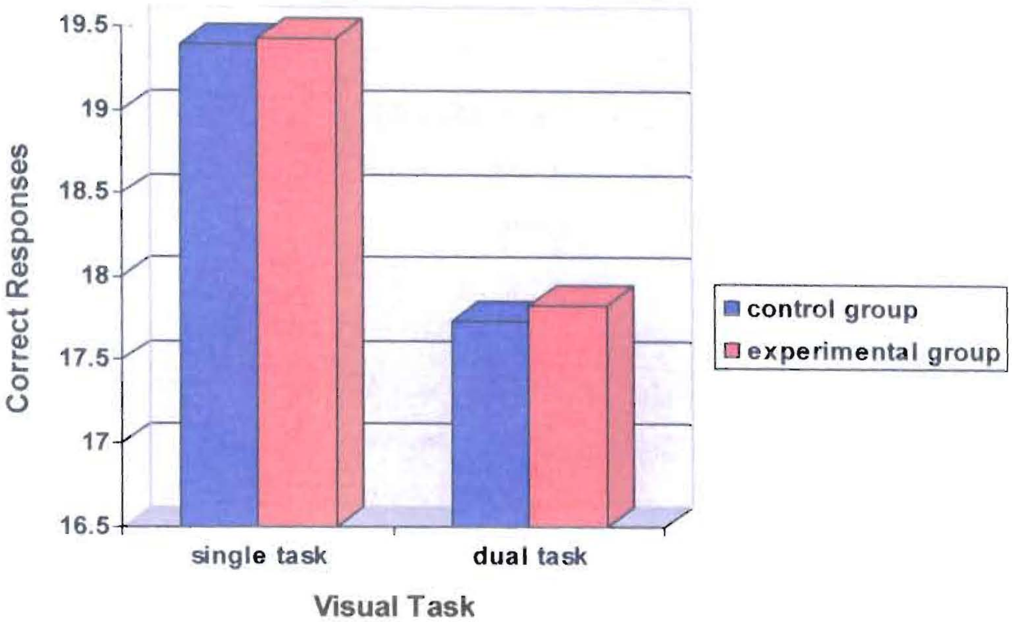


Figure 10.5 Bar chart showing the correct responses in the single and dual task conditions and between the groups

The results of the repeated measures 2 factor mixed Anova show that there is a significant difference between the single and dual task conditions [F, (1,58) = 43.00 p < .01]. The result of the between groups analysis not significant [F, (1, 58) = .04 p .84], and there is no significant interaction of single and dual task by group [F, (1,58) = .01 p .89].

10.10 Discussion

The results of the percentage correct responses to the verbally presented stimuli yielded significant differences for the within and between-subjects factors. The interaction of group by task was also significant. The simple main effect between the control and experimental conditions in both the single and dual task conditions was also significant. The pattern of results suggests that the control condition, where participants were required to identify words and non-words beginning with the letters 'c' and 'p' (pre-lexical processing) produced a greater number of correct responses than the experimental condition (lexical processing). Although the tasks in the experimental condition overall showed less correct responses to the verbally presented stimuli, the dual task condition showed a very slight increase in correct responses. It seems that there was greater disruption in the control condition (pre-lexical processing) than was found in the experimental condition (lexical processing.

The pattern of results for the reaction time data showed a significant main effect for the single and dual task conditions but no significant results were evident for the main effect of groups and the interaction of task on groups. In both the control and experimental conditions reaction times were quicker in the single condition than in the dual task condition.

The analysis of the number of correct responses produced from the visually presented subitizing task showed a significant difference between the single and dual task conditions. The results for the between groups analysis and the interaction of single and dual task by group were not significant. Across both groups the number of correct responses declined in the dual task conditions. The pattern of results shown between the groups in single and dual task conditions was very similar with slightly less correct responses recorded in the dual task conditions than found in the single task conditions.

In general the pattern of the results showed that there was disruption when participants were required to undertake a subitizing task in conjunction with simultaneously presented pre-lexical and lexical processing tasks. There is a great amount of speculation and debate with many unanswered questions relating to the processes involved in subitizing. The results of the present study support the view that early pre-lexical processes play a part in subitizing tasks. To investigate further the process involved in subitizing it would be beneficial to elaborate on this study to include a verbally presented semantic task with the visual presentation of arrays of circles displayed consistently in pattern arrangements, for example, a square, a triangle, or a diamond. This would then allow for a comparison study between lexical and semantic processing with possible findings suggesting a lack of interference from a semantic task given that subitizing is seen as an early perceptual process.

The results of the present study suggest that early processing stages may be involved in subitizing. Research by Ellis & Young (1996) suggested that there are three levels of representation of the visual input prior to object recognition and semantic processing. It is possible to speculate that the ability to subitize small sets of circles involves an early processing stage that compares the object-centred representations to stored descriptions of known objects. For example, if the display of circles represented a triangle, accessing stored knowledge would retrieve the immediate answer as 'three'. Trick & Pylyshyn (1993; 1994a;b) proposed that subitizing involves a preliminary visual analysis which they described as the spatially parallel pre-attentive stage and spatial serial attentive stage. Their suggestion was that subitizing occurs as an intermediate process between the above two stages. Although this theory is a theory of subitizing it in part parallels the theory of object recognition proposed by Ellis & Young (1996) in that subitizing is seen as an early processing phenomenon. The results of the present study are in line with the views of Trick & Pylyshyn.

Dehaene (1992) has suggested that the analogue scale that is envisaged in his model is implicated in subitizing and estimation processes. This theory hypothesises that the visual identification of stimuli is followed by an anological representation of numerical quantities. The triple-code theory assumes three representational codes, verbal, Arabic and analogue magnitude code. Each of these codes provides a starting point for different arithmetic operations. For example, a multi-digit addition problem would begin from Arabic representations, retrieval of arithmetic facts such as the meaning of division and subtraction would commence from verbal representations and magnitude representations from the analogue scale. Throughout processing information can be exchanged between the representational codes for further processing if necessary. On this basis, when a set of objects is visually presented, as in the present study, the number of objects in the display is represented on the analogue scale.

In contrast the abstract modular theory proposed by McCloskey et al. (1985) does not seem to accommodate subitizing processes. This model assumes that encoding, calculation and production are modular in as much that the processing within each module is independent of the processing that may occur in other modules. These modules are notation-specific, verbal or Arabic that translate numerical input into an abstract semantic representation prior to calculation procedures and the accessing of number knowledge. From the abstract semantic representation the information is then translated to verbal or Arabic code. The notation-specific modules that are specific to either verbal or Arabic, and become available for written or oral output, then process this information. Within this theory all numerical input is translated into an abstract semantic representation. This appears to be a model for basic fact retrieval based on the procedures and processes used for simple arithmetic calculations and does not easily accommodate subitizing.

The model for the recognition of spoken words proposed by Ellis & Young (1996) (reviewed in Chapter 10) comprises a number of stages. The first stage of auditory word recognition utilises the first component, an auditory analysis system. This system identifies phonemes in the speech wave. From this analysis the outcome is transmitted to the auditory input lexicon. This stage involves finding a match between the phonemes in the speech wave and the stored characteristics of known words. If a match is found, the appropriate recognition unit in the auditory input lexicon will be activated. This then activates the representation of the meaning of the heard word in the semantic system. From the results of the present experiment it appears that both the aurally presented word tasks produced an interference effect on the visually presented subitizing task. This lends support for the view that both the auditory word task and the visually presented subitizing tasks utilize early pre-semantic processing mechanisms. It is possible that the canonical forms of the visually presented stimuli may be processed at an early stage. According to

Dehaene & Cohen (1995) the triple-code theory provides a direct route that links the Arabic and verbal codes without passing through an intermediate representational stage as identified in the abstract modular theory McCloskey et al. (1985) as the abstract semantic representation. Dehaene & Cohen (1995) consider that the triple-code model may be similar to models of word recognition and reading in as much that to have an understanding of numbers it is not necessary to process the information through a semantic representation of the quantities. The results of this study support the triple-code theory in that the subitizing processes may not utilise semantic processing systems but represent early pre-semantic processes.

This study conducted on a normal population of participants enhances the existing knowledge and suggests an interesting link between linguistic and subitizing processes. The subitizing circles test was found to load onto Factor 1 'access to representations'. This factor comprised five tests that appear to utilize linguistic processes. In the present experiment consideration has been given to the view that there may be activation of representations in the visual image system with the concept of shape and size utilising pre-semantic lexical processes. The results indicate that there is interference when a subitizing task is performed in conjunction with pre-lexical and lexical tasks. It appears from the results that the verbally presented tasks in the control and experimental groups produced interference with the subitizing task. As the subitizing task seems to be affected by verbally presented stimuli it could be said that these verbal tasks represent early processing stages. This lends support for the view that subitizing is a reflection of processing at an early stage. Further support for the view that subitizing is based on early pre-lexical processing comes from the data received from the control group that showed greater interference for accuracy on the verbal, pre-lexical task. In conclusion this study lends support for the notion that subitizing is an early pre-lexical perceptual process possibly based on canonical representations of the stimuli. This processing may be reflected in the speed of responding to unfamiliar perceptual representations that seems to underlie the other tasks found to load onto Factor 1, access to representations.

Chapter 11 – Visual spatial involvement in the addition of multi-digit problems

11.1 Aims of Chapter 11

The previous three chapters have considered in great depth concepts arising from Factor 1 (access to representations). This chapter considers a specific aspect of Factor 2 – (working memory) that accounted for 13.21% of the variance. The tests loading onto this Factor are associated with the working memory model (Baddeley 1986), for example, the Stroop effect, Trails B, forward and backward digit span, the story taken from the Rivermead Behavioural Memory Test (RMBT).

The Stroop effect is associated with speed of processing and automaticity. This test also reflects executive control. The Trail Making Test comprises Part A and Part B. Part B was selected for the factor analytic study as it requires control of attention for the ability to move between numbers and letters yet maintaining them in the ascending sequence. It is considered that this test requires visual-spatial processing together with central executive input. The prediction was that this task would correlate with complex arithmetic problems. From the results of the factor analytic study the prediction is supported. The forward and backward digit span tasks were included to assess working memory capacity. These tests required the maintenance of information in short-term memory with the backward digit span test in particular requiring central executive control. The RMBT was designed to assess impairment of everyday memory functioning. For the purposes of the factor analytic study a short story was selected from the RMBT to assess retrieval of information from memory.

The most interesting aspect of this factor is the inclusion of complex addition and multiplication. Aschraft (1995) speculated that the central executive is involved in carrying and borrowing procedures with the phonological loop holding intermediate values and the visuo-spatial sketchpad responsible for the visual characteristics of the problems. The key objective of this chapter is to consider the relationship between complex arithmetic and working memory with particular emphasis on the involvement of the visuo-spatial sketchpad component. This chapter focuses on:

1. The principal models and theories that have been developed on the basis of previous research into the concept of working memory.
2. Working memory and numerical processing
3. Experimental work that focuses on the relationship between working memory and how the representation and manipulation of spatial information affects the impact of dual-task performance on verbally presented addition problems.

11.2 Introduction

Working memory model

Memory processes have been considered in detail in Chapter 3, but for the purpose of this chapter a brief overview of working memory (short-term memory) is provided. Working memory is considered to have a central role in the execution of mental arithmetic problems. For solving arithmetic problems individuals may use paper and pencil or calculator to externalise storage of intermediate products and to aid in the production of the final answer. However, by restricting access to external storage and encouraging individuals to carry out the calculation mentally it is possible to create interesting experimental work through which both the cognitive processes and demands on working memory can be investigated. Hitch (1978) found that participants adding numbers, such as 463 + 37, often made errors. The errors were not random but could be predicted by the length of time participants had to hold the digits in short-term memory before reporting the answer. It was found that participants kept intermediate answers to calculation problems and the final result in transient short-term memory that decays rapidly unless the information is rehearsed. Much of the research linking arithmetic problem solving with short-term memory has been influenced by the working memory model originally formulated by Baddeley & Hitch (1974). This model has been viewed as a useful way to conceptualise the short-term storage of information. (See Chapter 3 for a review of the literature on working memory).

The working memory model aims to explain the processes and structures of short-term memory and to study the holding and manipulation of information during the performance of cognitive tasks, such as comprehension, learning, reasoning and problems solving. The model includes two modality-specific systems generally referred to as 'slave systems'. These systems are able to deal with modality specific information and are controlled by a central executive. The central executive is considered to be the controller of working memory, co-ordinating the actions of the two modality specific systems, the phonological loop and visuo-spatial sketchpad. The working memory model has generated extensive research into the phonological processes in working memory. However, the visuo-spatial component has only recently undergone extensive research. According to Logie (1995) visual and spatial working memory are linked in the generation, retention and manipulation of visual images. Despite the renewed interested in the visuo-spatial component research has focused on a narrow range of imagery tasks with little research specifying in detail the relationship between working memory and mental imagery.

11.3 Components of the working memory model (Baddeley 1986)

11.3.1 Central executive

The Supervisory Attentional System (SAS) model of attentional control proposed by Shallice (1982) and Normal & Shallice (1986) influenced the concept of the central executive. This model involves two sources of action control, the first dealing with well-learned habitual patterns and the second an attentional controller, capable of overriding habitual response patterns to initiate new behaviour. Functions proposed as being under the control of executive functioning included complex strategy selection and planning and control of the recovery of stored information sin long-term memory (Duncan 1986). The executive system is considered to direct response sequences, thus allowing the achievement of behavioural goals. The co-ordination of separate tasks, switching strategies of information recall, selective attention and the temporary holding and manipulation of information from long-term memory have been considered as central executive functions (Baddeley 1996).

A memory updating task considered to specifically measure the central executive (Morris & Jones 1990) was used by Lehto (1995), who studied Finnish children aged 15 to 16 years old. In this task, participants were required to recall the last four or six items of a string of unrelated consonants. The length of the lists varied and was two, four or six items longer than the participants were asked to recall. This unexpected varying of the length of the lists required participants to drop earlier items in order to recall the last items. This task, therefore, required a shift in the participant's rehearsal strategy used for the retention of the information from the first item to a later item. This central executive, memory up-dating task was reported to be related to mathematical performance.

11.3.2 Phonological loop

The phonological loop is used for the storage of short-term verbal information. This subsystem is thought to comprise a passive phonological store and an active subvocal phonological rehearsal loop. Subvocal rehearsal allows information to be refreshed and maintained until retrieved or required for further processing. Rehearsal of items held in the phonological store enables decaying traces to be refreshed and recalled more successfully. This store has been extensively researched using a variety of methods, for example, phonological similarity, articulatory suppression, word length and irrelevant speech effects, (Baddeley & Logie 1992). It also appears to be involved in counting (Logie & Baddeley 1987) and in mental arithmetic (Dehaene 1992; Logie, Gilhooly & Wynn 1994).

11.3.3 Visuo-spatial sketchpad

The key component of working memory that specifically relates to the experimental work reported in this chapter is the concept of the visuo-spatial sketchpad. This component is less well developed than the central executive and the phonological loop both in terms of theory and application. The visuo-spatial sketchpad is assumed to have two subsystems, one being a passive visual component retaining material such as colour and shape and the other a spatial system responsible for retaining information about movement and information about spatial relationships between objects. (Logie 1991; Quinn 1991; Quinn & McConnell, 1996). Research on this component of working memory is discussed in more detail because of its relevance to the study reported later in the chapter.

11.4 Published research relating to the visuo-spatial sketchpad

Phillips & Christie (1997a;b) investigated in two studies visual recognition and visual recognition with mental arithmetic. In their first study (1997a) participants were presented with a sequence of 4 x 4 matrix patterns with half of the cells of the matrix filled at random. The arrangement of the matrix patterns was to be remembered by the participants. Following the presentation of the first sequence participants were presented with a second sequence with the patterns either identical to patterns previously seen or where there had been some change to the patterns. Participants were required to indicate whether or not they had previously seen the pattern. The recognition sequence was presented in reverse serial order beginning with the most recently seen matrix. Phillips & Christie (1997a) reported a visual recency effect in that the last matrix pattern presented was recognised correctly more often than patterns presented earlier in the sequence. The single item recency effect is considered to reflect short-term visual storage with the retention of earlier items in the sequence reflecting long-term memory.

Phillips & Christie (1997b) extended this work by introducing a secondary mental arithmetic task interpolated between the presentation of the last matrix and the presentation of the sequence to be recalled. The main finding was that the visual recency effect found in the previous study was removed. Phillips & Christie (1997b) considered that mental arithmetic does not rely on visualising; therefore visualisation and memory of the last matrix pattern in the series required general cognitive resources rather than a visual short-term memory system. Logie, Zucco

& Baddeley (1990) suggested that Phillips & Christie (1997a;b) had underestimated the importance of a visual short-term memory system. They interpreted the results as memory for a verbal cue associated with the last matrix rather than reflecting the capacity of a visual short-term memory system. They also argued that the precise nature of the interpolated task could have been given more consideration.

Logie, et al. (1990) extended the earlier work of Phillips & Christie (1997a;b). The aim was to investigate whether specialised storage or general cognitive resources are responsible for visual short-term function. Further consideration was given to the interpolated task as it was thought that mental arithmetic may involve general cognitive processes, verbal short-term memory and possibly visual imagery, in conjunction with accessing information from long-term memory (Ashcraft 1992, Dehaene 1992; Logie, Gilhooly & Wynn 1994).

Logie, et al. (1990) presented participants with a single 4 x 4 matrix pattern with half of the cells filled at random. Following the presentation of an individual pattern participants were shown the same pattern again but with one of the previously filled cells changed to a blank cell. The participants were required to identify the cell that had been changed. In subsequent trials the total number of cells in the matrix gradually increased. However, only one of the cells was changed on any one trial. This was considered to be a visual memory span task. Two secondary tasks were included. The first involved simple mental arithmetic where participants heard a series of numbers which they had to add up. The second task involved participants visualising a three by five square matrix. In this task participants responded to a series of verbal instructions to fill in or to leave blank each of the cells of the imagined matrix. The result produced from following the instructions was the shape of a number between 0 and 9. Participants were required to report the visualised number.

The prediction of the experiment was that a secondary task that involves the use of the same cognitive resources as the visual span task would interfere with visual span. However, a secondary task involving the use of different processing systems from those required for visual span will have very little disruptive effect. An important consideration pointed out by Logie et al. (1990) is that one of the secondary tasks may be more difficult than the other and to consider this possibility a primary letter span task was included. The procedure for this task involved participants viewing a series of letters on the screen one at a time. Following a delay period the same letters were shown again, but one of the letters was changed. The participants were required to detect which of the letters had been changed from the original series. The number of letters in the series increased over the trials.

The results produced a highly significant disruption of visual span by concurrent visualising of a number matrix and disruption of letter span by concurrent arithmetic. The results also indicated a significant disruption of visual span performance by concurrent mental arithmetic. There was also a significant disruptive effect of letter span performance by the number matrix task. Logie et al. (1990) suggested that the results were consistent with the concept of a specialised verbal short-term store for the letter span task with the verbal store involved in maintaining verbal sequences and mental arithmetic. They considered that the results also suggested the presence of a specialised visual short-term store to accommodate the visual span task.

Quinn (1991) investigated the concept that interference in the visuo-spatial sketchpad by a spatial movement task is related to an active encoding process rather than to the processes involved in maintenance of information. This task was designed to minimise central executive involvement. The results indicated that interference was limited to the active encoding stage and that there was no interference effect during maintenance of the information.

The interference effects of visual and spatial information have been further investigated by Logie & Marchetti (1991) who conducted an experiment were participants where required to retain information from one of two kinds of visually presented stimuli. One set of stimuli, the visual stimuli, was based on the memory for a particular shade of colour taken from a single colour, blue, green or purple. The second set of stimuli, spatial stimuli, required the memory for the sequential order for squares that were presented at different places on the computer screen. The memory for the shade of colour was intended to represent a visual temporary memory task, and the sequential squares task involved the retention of spatial information with the possibility of participants mentally rehearsing the sequence of presentation of the squares. Between the presentation and retrieval of the stimuli, the retention interval, a concurrent secondary task was introduced. This was either a movement task introduced in the condition using visual stimuli or the presentation of irrelevant pictures used in the spatial condition. Logie & Marchetti (1991) considered that if the disruptive effects found by Quinn (1991) were due to disruption during the encoding of stimuli then there would be no disruptive effects of either of the secondary tasks during the retention interval. However, if separate systems were involved in holding spatial and visual material it was predicted that an intervening movement task would disrupt memory for the presentation of the spatial stimuli and that the irrelevant pictures would disrupt the retention of shades of colour, in the visual stimuli.

The data obtained from the experiment indicated that the spatial task was disrupted by the secondary movement task and memory for a series of shades of colour was disrupted by the irrelevant pictures task. The results support the view that retention of visual and spatial material is a function of separate systems, one responsible for temporary retention of visual material and one responsible for spatial material. The findings of the above research indicated that modification to the visuo-spatial sketchpad component of working memory were necessary.

11.5 Modifications to the visuo-spatial sketchpad, Logie (1995)

The above findings prompted a change in the working memory model. Logie (1995) extended his research and suggested a modified version of the working memory model (Baddeley 1986). Here it was considered that visuo-spatial memory comprises a visual temporary store or 'cache' memory for information, which may be retrieved from long-term memory as a conscious image for manipulation and a spatial temporary store. The visual input to both stores is via long-term memory representations of the visual form of objects or the spatial information about a scene. The representations stored in long-term memory can be activated and the information then enters the visual or the spatial component of the working memory system. The nature of the information determines which system it enters. The spatial store is considered a system that is able to plan movement. However, it can also be used to rehearse the contents of the visual store. The spatial and visual components are considered to provide temporary storage of information from which the central executive can utilise the material that is relevant to the task being undertaken.

11.6 Theoretical perspectives relating to imagery and working memory

According to Logie (1995), this relationship between imagery and working memory reflects the earlier work of Paivio (1971, 1986) identified in the Dual Coding Theory. This theory suggests that pictures and words activate independent imagery and verbal codes. Paivio stated that pictures and visual objects are coded together as an image or coded as a verbal unit with the result that there are two ways from which to access information. Paivio (1986) considered that the verbal system refers to language and the imagery system refers to non-verbal images, such as the analysis of scenes and the generation of mental images. Paivio suggested that the two systems are functionally and structurally distinct and can work independently of one another. They are, however, interconnected so that they can also function in parallel.

Despite the apparent similarity between the Dual Coding Theory and the modified version of the working memory model, Logie (1995) identified distinct differences. Firstly, in Logie's model the phonological loop and the visuo-spatial components were considered to be stores rather than processors of information. Paivio included both processing and storage functions in his dual codes. Secondly within Paivio's model the formation of an image from a presented word was considered to require access to the word's semantics and not to its phonology. However, in the working memory model semantic information is associated with central executive processing and not with the phonological loop. Paivio's model does not identify a central executive and a phonological loop as separate elements.

Another researcher in the area of visual imagery is Kosslyn (1980) who described the concept of a visual buffer, which acts as 'host' for the image that is consciously experienced. Images from information stored in long-term memory create a more precise image in the visual buffer. From here it is possible to view the image in order to recognise shapes and other characteristics of the object and make decisions based on the scanning of the image. The visual buffer contains the conscious image that includes the visual properties of an object and the information about the relative location of objects to one another. The buffer also has semantic information associated with the image. Logie criticised this model arguing that the visual buffer could be likened more to a 'workspace for visual imagery' than a buffer. This concept can be related to the working memory model with a division in the visuo-spatial sketchpad between the visual store and the spatial store. Logie suggested that the visual store contains more visual information than the conscious image stored in long-term memory and the information about the object is exchanged between the stores. A further suggestion was that the processes acting on the contents of Kosslyn's visual buffer could be seen as procedures activated from long-term memory that are available to the central executive for further processing. Nevertheless, Kosslyn (1991) argued that working memory is involved in the transfer of information to and from long-term memory representations and the visual buffer. This is consistent with the role of the visual and spatial temporary stores. It is these theoretical perspectives of the visuo-spatial component of working memory that are considered when numerical processing is integrated into the concept of working memory. It is from this integration that the experimental work of this chapter has evolved.

11.7 Working memory and numerical processing

The relationship between the working memory model (Baddeley 1986) and the structure of numerical processing has only recently begun to be studied. The literature suggests that working memory plays an important role in the performance of arithmetic operations, in conjunction with stored knowledge in long-term memory (Logie, Zucco & Baddeley 1990; Dehaene, 1992; Logie, Gilhooly & Wynn 1994; Heathcote 1994; Ashcraft 1995). A review of the literature surrounding working memory and numerical cognition was given in Chapter 3. The focus of this chapter is the relationship between the visual spatial component of working memory and complex addition problems. This area remains under-researched and the following experimental work seeks to investigate the visuo-spatial sketchpad in more detail.

11.8 Aim of the present study

The present study will focus primarily on the relationship between the working memory model (Baddeley 1986) and how the representation and manipulation of spatial information affects the impact of dual-task performance on verbally presented addition problems. In particular this study focuses on the visual spatial component of working memory, investigated using a visual spatial task and a concurrent auditory addition task. This study also aims to relate the findings of the present experimental work to the speculative view held by Aschraft (1995) that the visuo-spatial sketchpad may play a role in arithmetic problem solving. Aschraft suggested that there would be disruption of arithmetic processing during problems that require carrying operations and problems where digits are maintained in columns when accompanied by a secondary task utilizing visuo-spatial processing.

11.9 Rationale of the present study

The study reported here required participants to retain information from one of two kinds of visually presented stimuli, either matrix patterns, experimental condition or shades of a basic colour, control condition. The memory for matrix patterns was hypothesised to involve retention of visual spatial material while the memory for shades of colour was designed as a purely visual memory task. The retention interval between presentation and recognition was occupied by working on two-digit addition problems. The rationale was that the study might lend support for the distinction between visual spatial memory and visual memory within the visuo-spatial sketchpad component of working memory. It is also suggested that visual spatial memory is required for working on multi-digit addition problems (Ashcraft 1995). If separate systems are involved in retaining spatial and visual material it was expected that the calculation of multi-digit addition problems would produce disruptive effects. It was expected that the calculation of multi-digit problems would cause the most disruption in the experimental condition, memory for visual spatial matrix patterns. Furthermore it could be expected that the addition of multi-digit problems would disrupt the memory for matrix patterns but with no disruption recorded for the memory of shades of colour, control condition.

In the experimental condition, a visual spatial recall task incorporated 4 x 4 matrix patterns similar to those used by Logie, et al. (1990). However the presentation of the matrix patterns was in pairs. The control condition, a visual memory task used 4 x 4 matrices with some squares filled with a shade of a single basic colour while others were left blank. The use of shades of colour was similar to that of Logie & Marchetti (1991). However a wider range of basic colours was used.

Participants first performed a single visual task and a single complex addition task using multi-digit problems to provide baseline scores. The two tasks were then performed concurrently. The differences between the scores on the concurrent tasks and the baseline scores were analyzed to investigate whether neither, both or one of the two tasks was affected by having to perform the tasks simultaneously.

The aim of the experimental work on complex addition and working memory is to investigate in much greater detail the relationship between complex addition and the components of working memory, in particular the visuo-spatial sketchpad. The prediction is that the calculation of two digit addition problems will be disrupted by a concurrent visual spatial task. This is because it is speculated that a visual spatial contribution is necessary for solving any problems that involve carrying operations. The present study aims to research the following hypotheses:

1) Between the single and dual task conditions in the experimental and control groups a significant difference will be found. The single tasks for the experimental group are a visual spatial recall task of 4 x 4 matrix patterns and the addition of multi-digit problems. The dual task is the concurrent performance of the two tasks. The single tasks for the control group are a visual memory task using 4 x 4 matrices with some squares filled with a shade of a single basic colour while others are left blank and the addition of multi-digit problems. The dual task is the concurrent performance of the two tasks.

2) The experimental and control groups will show significantly different performance results. The experimental group is required to carry out a visual spatial task together with the addition of multi-digit problems whereas the control group is required to undertake a visual task and the addition of multi-digit problems. It is anticipated that the experimental group will experience more difficulty in the dual task condition than the control group.

3) It is predicted that there will be a significant interaction between the single and dual task conditions and the experimental and control groups in as much that the experimental group will show the greatest decline in the dual task condition.

11.10 Method

The experimental method used was dual task. Data were analyzed using three, repeated measures 2 factor mixed Anovas. The first Anova was used to analyse the correct responses for addition problems. The second Anova was used to analyse correct responses for the matrix patterns and the third Anova to analyse the reaction time data recorded for correct responses to the matrix patterns. In each case, the first factor was the within-subjects factor of single and dual task, and the second factor was the between subjects factor of group, with participants either in the experimental group or the control group. Participants in both the control and the experimental groups took part in the three conditions of the independent variable.

The dependent variables were the number of correct responses made by the participants in the auditory presentation of the addition problems and the number of correct responses and reaction time data recorded for the visual presentation of the matrix patterns.

11.10.1 Experimental group – Three conditions of the independent variable:

1 4 x 4 Matrix patterns/single task – Memory for the visual spatial layout of the matrix. Some of the cells are filled in black the remaining cells are white. The matrix patterns are different for each trial. (See Appendix 8 for examples of the matrix patterns used in the single and dual task conditions in the experimental group).

2 Addition problems/single task – Mental addition of the verbally presented 2 digit addition problems.

3 Dual task – Memory for the spatial layout of the matrix patterns and mentally calculating the answer to the addition problems. The calculation of the addition problems is interpolated between the presentation of the first grid in the pair and the presentation of the second grid.

11.10.2 Control Group - Three conditions of the independent variable:

1 4 x 4 Matrix patterns/single task – Memory for the colour in the cells. Only one colour is used in each pair of matrix patterns. The cells are randomly filled. (See Appendix 11 for examples of coloured matrices used in the single and dual task conditions in the control group).

2 Addition problems/single task – Mental addition of the verbally presented 2 digit addition problems.

3 Dual task – Memory for the colour of the matrix and mentally calculating the answer to the addition problems.

Table 11.1 Summary of the conditions of the experimental and control groups

Condition	Experimental group	Control group
Single task	Memory for the visual spatial layout of the matrix.	Memory for the colour of the cells in the matrix.
Single task	Mental addition of verbally presented 2 digit addition problems.	Mental addition of verbally presented 2 digit addition problems.
Dual task	Memory for the spatial layout of the matrix patterns and mentally calculating the 2 digit addition problems.	Memory for the colour of the matrix and mentally calculating the 2 digit addition problems.

11.10.3 Participants

Sixty participants took part in the experiment. There were thirty participants in the experimental group, 21 female and 9 male, with an age range of 18 to 50 and a mean age 25.43, standard deviation of 8.4. In the control group there were also thirty participants, 24 female and 6 male, with an age range of 18 to 43 and a mean age of 23, standard deviation 5.7.

11.10.4 Materials – Experimental Group – visually presented stimuli

Two sets of 20 pairs of black and white 4 x 4 matrix patterns presented on the computer using the Superlab software. (See Appendix 8 for examples of the matrix patterns used in the single and dual task conditions in the experimental group).

11.10.5 Materials – Experimental Group – verbally presented stimuli within each set.

Two sets of 20 two-digit addition problems. Ten of the addition problems required no carrying procedures (for example, 27 + 31), while the remaining ten did require carrying operations (for example, 36 + 49 and 26 + 67). The addition problems were presented randomly. The randomization of the arithmetic problems was consistent across all the participants. (See Appendix 9 for addition problems used in the single condition in the experimental and control groups and Appendix 10 for the addition problems used in the dual task condition in the experimental and control groups).

11.10.6 Materials - Control Group – visually presented stimuli

Two sets of 20 pairs of coloured 4 x 4 matrix patterns. The patterns are based on a single colour. White squares were included in all the patterns. (See Appendix 11 for examples of coloured matrices used in the single and dual task conditions in the control group).

11.10.7 Materials – Control Group – verbally presented stimuli

Two sets of 20 two-digit addition problems. The addition problems used in the control condition were the same as in the experimental condition.

In both the experimental and control groups and in the single and dual task conditions two practice stimuli for addition problems and matrices were included. The results of the practice trials were not included in the analysis.

11.11 Procedure

11.11.1 Experimental Group – Three conditions of the independent variable.

1 Spatial layout of 20 pairs of matrice – single task.

Twenty pairs of 4 x 4 matrix patterns were used in this condition. The matrices were in black and white, (see Appendix 8). Participants were required to remember the spatial layout of the first matrix of a pair presented on the computer screen. The matrix remained on the screen for 5 seconds. There then followed a blank screen for 5 seconds. The second matrix was then displayed on the screen and participants were required to press the 'M' key on the keyboard if the second matrix was an exact match to the first grid presented. If the spatial layout of the second matrix was different from the first matrix, participants were required to press the 'Z' key. There was no time limit set for the length of time the second matrix was presented on the computer screen. Participants were, therefore, not pressured by time constraints. The remaining nineteen matrices were presented using the same format. Correct scores were automatically recorded in Excel and used in the analysis.

2 Verbally presented addition problems – single task

Following the presentation of all 20 pairs of matrix patterns participants were required to calculate the answer to 20 two-digit addition problems. The addition problems were presented verbally one at a time. Participants were required to give their answer verbally and without the use of paper and pencil for calculations. Correct scores were recorded.

3 Spatial layout of the matrices and addition problems – dual task

A set of 20 pairs of matrices and 20 arithmetic problems was used in this condition. The pairs of matrices and the addition problems were different from those used in the single conditions in order to eliminate learning effects. The mental arithmetic task was interpolated between the presentation and recall of the spatial layout of the matrices as follows.

Participants were required to remember the spatial layout of the first matrix from a pair presented on the computer screen. The patterns were in black and white and the matrix remained on the screen for 5 seconds. There then followed a blank screen for 5 seconds. During this period the participants were required to calculate the answer to a two-digit addition problem. The problems were verbally presented and a verbal response was required.

Following the completion of the addition problem participants were required look at the second matrix displayed on the screen. If the second matrix was exactly the same as the first matrix participants were required to press the 'M' key on the keyboard. However, if the spatial layout of the second matrix was different from that of the first matrix presented participants were required to press the 'Z' key. Correct scores were automatically recorded in Excel and used in the analysis.

The second matrix automatically appeared on the screen after 5 seconds. This followed the same procedure as in the single condition. However, during the calculation of the addition problems participants were asked to look away from the screen (so as not to see the second matrix until the completion of the addition problem) and only returned to the screen after they had given their answer to the addition problem. The same procedure was followed for the remaining nineteen trials.

11.11.2 Control group – Three conditions of the independent variable.

Coloured matrix patterns – single task

Twenty pairs of 4 x 4 matrices were used in this condition. The spatial layout of the patterns in this condition was not relevant, as participants were required to remember only the shade of the colour used. In each of the matrices presented some of the squares remained white with the remaining squares filled in using the same shade of a particular colour, for example blue. Participants were required to remember the shade of the colour of the first matrix of a pair presented on the computer screen. The matrix remained on the screen for 5 seconds. There then followed a blank screen for 5 seconds. The second matrix was displayed on the screen and participants were required to press the 'M' key on the keyboard if the second grid was an exact match of the shade of colour used in the first matrix presented. If the shade of the colour used in the second matrix was different from the first matrix presented participants were required to press the 'Z' key. Correct scores were automatically recorded and used in the analysis.

Verbally presented addition problems – single task
The same procedure was used as in the experimental group.

Coloured Matrices and addition problems – dual task
The same procedure was used as in the experimental group.

11.12 Results

Before a full examination of the data an independent t-test was conducted on the two different sets of 20 addition problems used in the single and dual task conditions. The results were not significant at $p > .05$. This result indicates that the two sets of addition problems were equally difficult.

The results are presented in three sections and each section includes descriptive and inferential statistics. The first section examines the results of the addition problems across single and dual task conditions and the control and experimental group using a repeated measures 2 factor mixed Anova. Participants' correct responses to the matrix patterns are analysed in the second section using two repeated measures 2 factor mixed Anovas. The same analysis is used in the third section to examine participants' reaction times to matrix patterns.

11.12.1 Section 1 Analyses of addition problems

The mean and standard deviation of the number of correct responses for the single and dual task conditions and group for addition problems are presented in Table 11.2.

Table 11.2 Mean and standard deviation scores of correct responses for group (experimental and control) and single and dual task conditions for addition problems.

Conditions	Participants	Mean	Std.Dev
Control Group Single task/addition	30	15.87	3.16
Control Group Dual task/addition	30	15.53	2.98
Experimental Group Single task/addition	30	15.6	2.63
Experimental Group Dual task/addition	30	15	3.22

An examination of the mean scores for the addition problems suggests that there are very small differences between the single and dual tasks and between the groups. Figure 11.1 below illustrates the descriptive statistics.

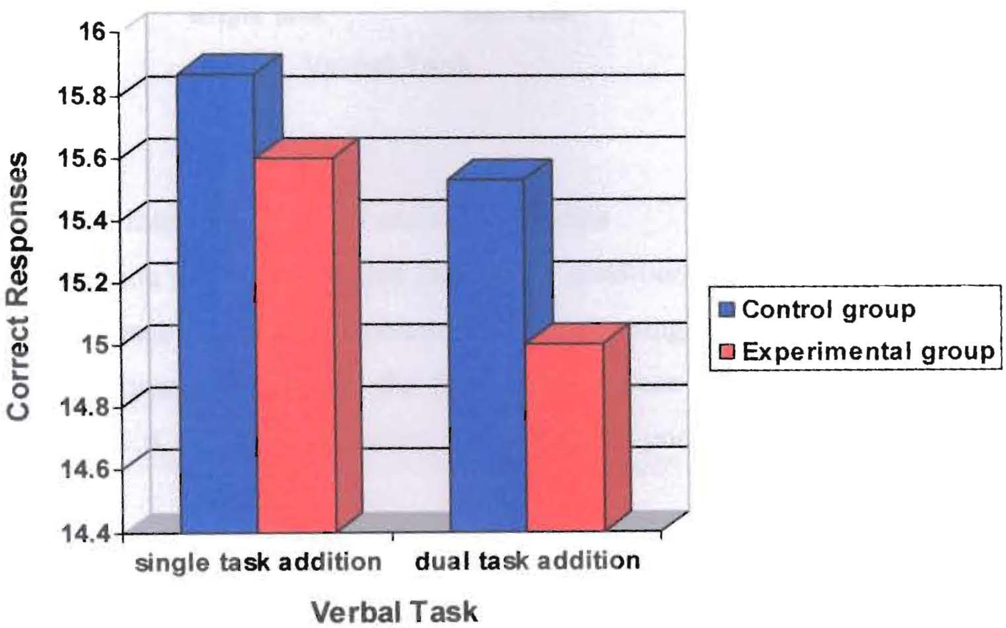


Figure 11.1 Mean scores for the control and experimental groups in the single and dual task conditions for addition problems

The results of the repeated measures 2 factor mixed Anova show that there is no significant difference between the single and dual task conditions [F, (1,58) = 2.0 p .16]. The result for between groups is also not significant [F, (1,58) = .32 p .57] and there is no significant interaction of single and dual task by group [F, (1,58) = .16 p .69].

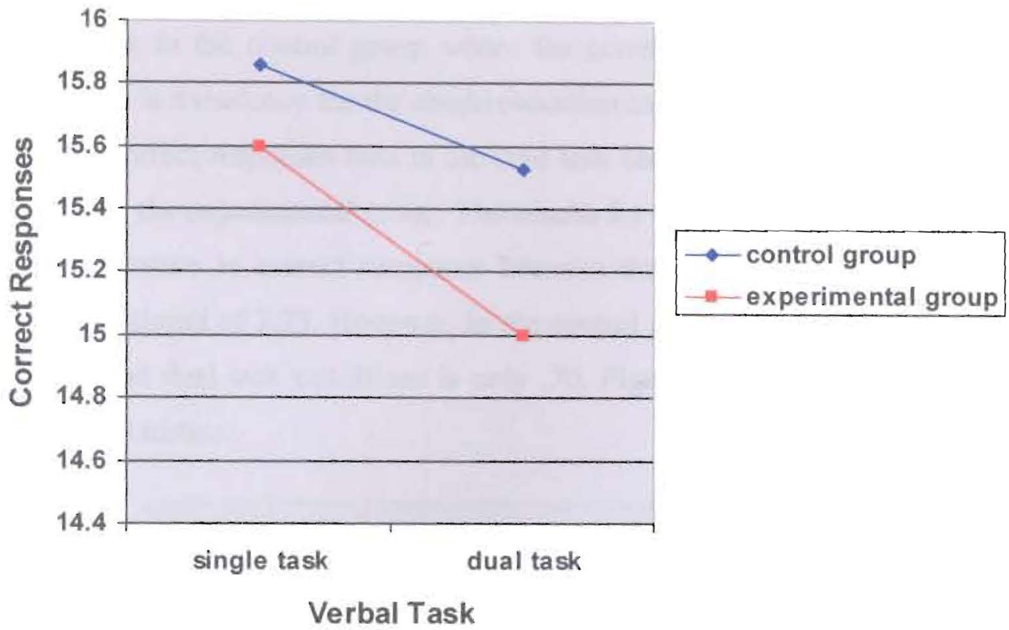


Figure 11.2 Interaction plot for addition problems

The interaction plot indicates that the control condition shows little difference in the performance on addition problems between the single and dual task condition. In the experimental condition the difference between the single and dual task conditions is .6, a very small decline in the dual task condition.

11.12.2 Section 2 Analysis of Matrix patterns

The mean and standard deviation of the number of correct responses for the single and dual task conditions and group for matrix patterns are presented in Table 11.3.

Table 11.3 Mean and standard deviation scores of correct responses for group (experimental and control) and single and dual task conditions for matrices.

Conditions	Participants	Mean	Std.Dev
Control Group Single task/matrix colour	30	15.87	3.16
Control Group Dual task/matrix colour	30	15.17	2.98
Experimental Group Single task/matrix patterns	30	17.73	2.63
Experimental Group Dual task/matrix patterns	30	15	3.22

From the table it can be seen that the experimental group performed highly with 17.73 correct responses in the single matrices task in comparison to the single matrices task in the control group where the correct number of responses was 15.87. There is a tendency for the single condition in both groups to be performed with more correct responses than in the dual task condition although this is more marked with the experimental group. The results for the experimental group show a large difference in correct responses between the single and dual task using matrices as stimuli of 2.73. However, in the control group the difference between the single and dual task conditions is only .70. Figure 11.3 below illustrates the descriptive statistics.

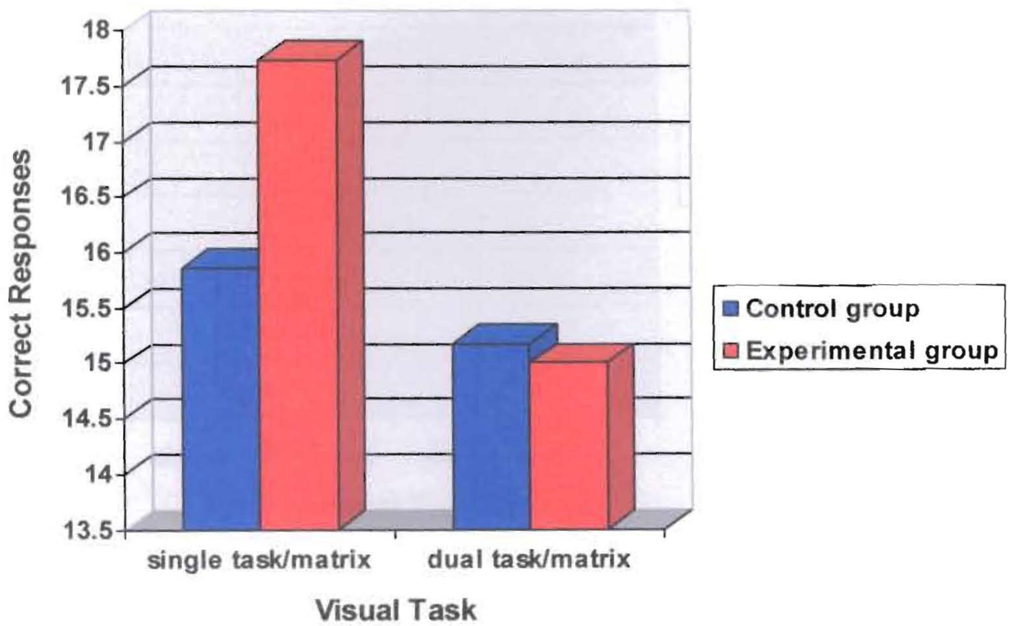


Figure 11.3 Mean scores for the control and experimental groups in the single and dual task conditions for matrix patterns

Figure 11.3 shows correct responses on the vertical axis and the control and experimental groups on the horizontal axis. There is a small difference in the control group between the single and dual task condition. There is, however, a much more noticeable difference between the single and dual task conditions in the experimental condition.

The results of the repeated measures 2 factor mixed Anova show a significant effect of single versus dual task [F, (1,58) = 22.99, p <.01]. The difference between the experimental and control groups is close to significance [F, (1,58) = 3.34, p .07]. There is a significant interaction for single and dual task by group [F, (1,58) = 8.06, p < .01]. This interaction is illustrated in Figure 11.4 below.

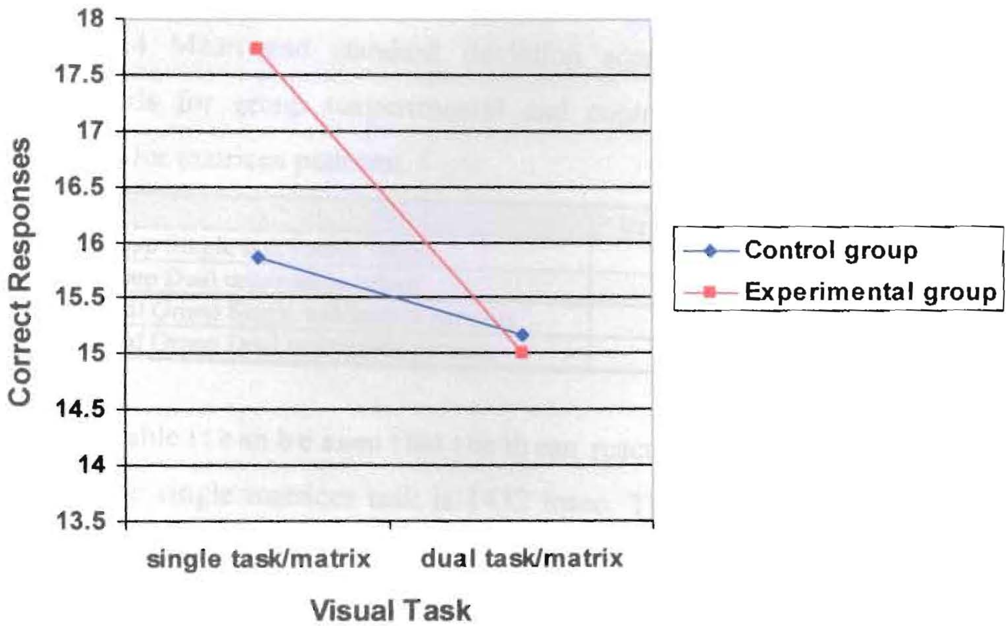


Figure 11.4 The interaction of group by single and dual task conditions for matrices

The significant interaction indicates that the effect is within the experimental group. This is supported by the results of a t-test comparing the single versus dual task in the experimental condition t (29) = 5.16, p < .01. There no significant difference within the control condition between the single and dual task conditions t (29) = 1.45, p .07.

The results suggest that the necessity to remember the layout of the matrices and to calculate the addition problems has caused an interference effect. This effect is more marked in the experimental group, dual task condition than in the control group dual task condition.

11.12.3 Section 3 Analysis of reaction time data to matrices.

The mean and standard deviation of reaction time data for the single, dual task and group for matrix patterns are presented in Table 11.4.

Table 11.4 Mean and standard deviation scores of reaction time data in milleseconds for group (experimental and control) and single and dual task conditions for matrices patterns.

Conditions	Participants	Mean	Std.Dev
Control Group Single task/matrix colour	30	1343	329
Control Group Dual task/matrix colour	30	3773	2633
Experimental Group Single task/matrix patterns	30	1432	420
Experimental Group Dual task/matrix patterns	30	3129	2105

From the table it can be seen that the mean reaction time for the experimental group in the single matrices task is 1432 msec. There is a lower mean reaction time in the single matrices task for the control group of 1342 msec. In the single condition in both groups the reaction times are considerably lower than in the dual task condition. The difference in the reaction times between the single and dual task conditions in the control group is 2430 msec. compared to 1697 msec. in the experimental condition. Figure 11.5 below illustrates the descriptive statistics.

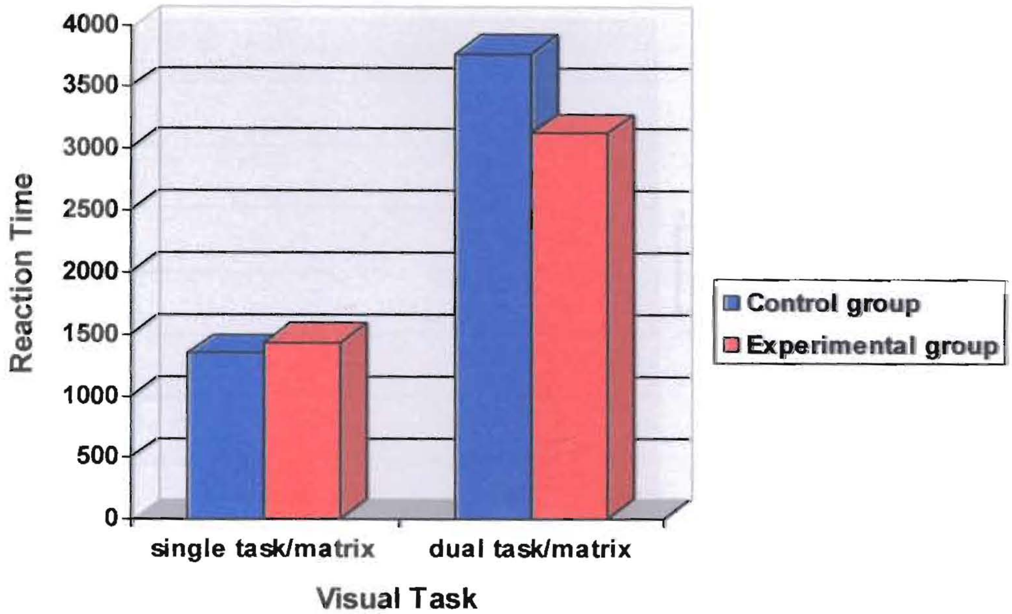


Figure 11.5 Bar chart showing the mean reaction time score between the control and experimental groups

Figure 11.5 shows that both the control and experimental groups performed the matrices task with only a small difference in reaction time data of 89 msec. In the dual task conditions across groups there is a difference of 645 msec. The reaction times are shown to be slower in the control group than in the experimental group.

The results of the repeated measures 2 factor mixed Anova show a significant difference between the single and dual task conditions $F, (1,58) = 43.88, p < .01$. Group differences were not significant at $F, (1,58) = .79$, p .37 and no significant interaction of task by group was found $F, (1,58) = 1.38$, p .24. This is illustrated in Figure 11.6 below.

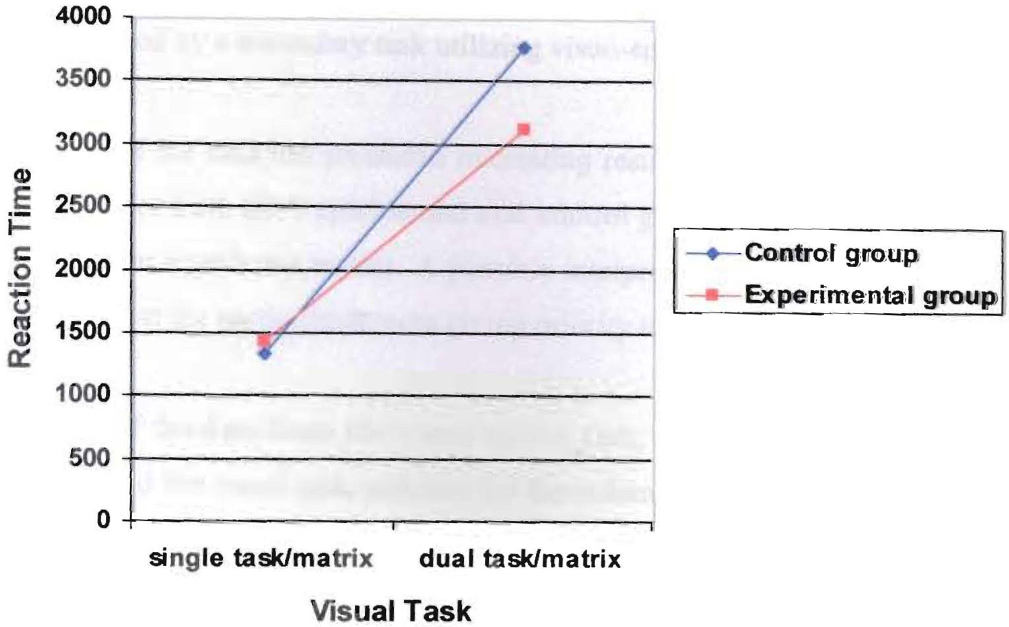


Figure 11.6 Interaction plot for reaction time data recorded in milleseconds.

The interaction plot shows that there is no significant interaction and the control condition had the larger difference in reaction times between the single and dual task condition for the coloured matrix patterns. In the experimental condition the difference between the single and dual task conditions is less.

11.13 Discussion

The present experimental work has focused on the relationship between the working memory model (Baddeley 1986) and on how the representation and manipulation of spatial information affects the impact of dual-task performance on verbally presented addition problems. In particular the visuo-spatial sketchpad component of working memory has been investigated using a visual spatial task, a visual task and a concurrent auditory addition task. This study also considers the speculative view held by Aschraft (1995) that the visuo-spatial sketchpad may play a role in arithmetic problem solving. Aschraft suggests that there would be disruption of arithmetic processing during problems that require carrying operations and problems where digits are maintained in columns when accompanied by a secondary task utilizing visuo-spatial processing.

Analysis of the data has produced interesting results. The results of the addition problems for both the experimental and control groups by single and dual task produced no significant results. A possible interpretation of these non significant results is that the participants were giving priority to this task.

Analysis of the data from the visual spatial task, memory for the 4 x 4 matrix patterns, and the visual task, memory for the colour of the 4 x 4 matrices, is close to the conventional level of significance of $p < .05$. The between groups result is p .07. There is a highly significant effect of single versus dual task with the significant interaction indicating that the majority of the effect is within the experimental group. This view is supported from the results of a t-test showing a highly significant result between the single and dual task conditions for matrices in the experimental group. There is no significant difference in the control group by single and dual task. The difference is clearly shown in these results of the matrix task in the experimental group where the performance in the dual task condition is reduced compared to the performance in the single condition.

The results of the reaction time data for the matrix tasks in both the experimental and control groups by single and dual task show that there is a significant difference between the single and dual task conditions. However group differences were not significant and no interaction of task by group was present. It appears from the results that the control group produced slower reaction times than the experimental group. The overall difference in reaction time score for performance in the single and dual tasks is more pronounced in the control group than in the experimental group.

The above experiment provides evidence of the involvement of the visual spatial component of working memory in the mental addition of multi-digit addition problems. There is a very significant dual task effect for the experimental group on the matrix scores indicating that the ability to remember 4 x 4 matrix patterns is significantly impaired by the concurrent addition of two digit arithmetic problems. It is interesting to note that participants appear to have attempted to preserve their performance on the addition problems across the single and dual conditions at the expense of their performance on the matrices task.

The results of this experiment are in line with the results obtained by Logie & Marchetti (1991) on the systems involved in retaining visual and spatial information. The control group in the present experiment was required to retain visual information with stimuli based on the memory for a particular shade of colour while the experimental group was required to memorize the spatial layout of matrix patterns by concurrent addition problems. The results suggested a relatively consistent performance from the control group in both the single and dual task conditions. In the experimental group, however, there is a significant difference between the single and dual task conditions with a decline in performance in the dual task condition. The analysis of the data obtained from the experiment indicates that the memory for the visual spatial layout of matrix patterns was disrupted by the concurrent arithmetic task. The result lends tentative support to the view that retention of visual and spatial material is a function of separate systems, one responsible for temporary retention of visual material and one responsible for spatial material.

The results also lend support to the proposal made by Logie (1995) for a modified version of the working memory model (Baddeley 1986). The modified version of the model proposed by Logie (1995) divides the visuo-spatial sketchpad into two components considered to store information. The first is a temporary visual store for information and the second a temporary spatial store. The visual input to both stores is via long-term memory representations of the visual form of objects or the spatial information of a scene. In the present experiment the visual stimuli were the memory for the shade of colour and the spatial stimuli were the memory for the layout of the matrix patterns. According to the model proposed by Logie (1995) the visual and spatial components are considered to provide temporary storage of information from which the central executive can utilize the material that is relevant to the task being undertaken.

Research by Heathcote (1994) examined the role of visual spatial working memory in the solving of three digit mental addition problems. The carry operations varied across the problems, and some of the problems did not require participants to carry data from one column to another. There were three secondary tasks used. Firstly, there was a visual interference task using matrix patterns, and secondly there was a spatial interference task using the Moar Box. This task requires participants to press keys along one column and then reverse along the next column and continue until all the keys are pressed in the correct spatial sequence. Thirdly there was an articulatory suppression task where participants were required to repeatedly vocalize the word 'the' at a rate of 2 – 3 times per second. The final control condition did not include a secondary task. The results showed that all secondary tasks produced disruption of mental addition when compared to performance in the control condition. It was also found that the disruptive effect of spatial and visual interference was greatest in problems involving carry operations.

The results of the research by Heathcote (1994) appear to suggest a reverse effect to the results found in the experimented reported in this chapter. The results from the present study have shown that the disruptive effect is evident in the reduced number of correct responses received on the matrix patterns task with the answers to the addition problems showing little reduction.

Ashcraft (1995) speculated that there would be a disruption of arithmetic problems solving for problems involving the addition of problems that required carrying operations. It would seem from the results presented here that there was a disruption involving the concurrent processing of two tasks that involved spatial processing. The result lends support to the prediction that addition of multi-digit problems requires spatial processing.

The experimental study has explored in depth the involvement of the visual spatial component of working memory in relation to the addition of two digit problems. The design of this study was not intended to exclude the possible contribution of the other two components, the central executive and articulatory loop. The central executive according to Aschraft (1995) may be the component that retrieves basic numerical information from long-term memory. It is also considered to be the component that is involved in the manipulation of numbers and facts following the retrieval of information required for solving a particular problem. Ashcraft (1995) suggested that the central executive may also play a role in maintaining the correct position within a counting sequence and that the phonological loop is involved in the holding of numerals to be counted. Heathcote (1994) suggested that the articulatory loop assists in the retaining of the initial problem information and the partial results by rehearsal of the information. It is also considered to be necessary in refreshing images stored in the visuo-spatial sketchpad. Seitz & Schumann-Hengsteler (2000) suggested that solving complex multiplication problems utilizes phonological loop and central executive processes. However, no evidence was found for the involvement of the visuo-spatial sketchpad in solvng multiplication problems. The results of this study appear to support the prediction that visual spatial memory is involved in solving complex addition problems

In summary the results indicate that the there is very little difference in the control group between the single matrices task and in the dual task. However, there is a significant difference between the single and dual task in the experimental group. This suggests that to remember the layout of the grids and to calculate answers to addition problems the same processing system is utilized.

There does appear to be support for the prediction that visual spatial memory is involved in the calculation of 2 digit addition problems. More specifically the results lend support to the modified version of the working memory model proposed by Logie (1995) where visual spatial memory comprises a visual temporary store and a spatial temporary store. Furthermore the results provide support for the view held by Aschraft (1995) of the involvement of the visual spatial component of working memory in the calculation of multi-digit problems.

Chapter 12 – Overview and Conclusions

12.1 Introduction

The aim of this thesis was to expand upon the existing knowledge of the structure of numerical cognition. Chapters 1, 2 and 3, provided an overview of the historical development of the study of numerical cognition, together with an in depth review of the current models of numerical cognition and the relationship between numerical cognition and working memory (Baddeley 1986).

Chapters 4, 5, 6 and 7 examined in detail, using a factor analytic study, the structure of numerical cognition. The rationale for the study was covered, its design and results. Chapter 7 provided the rationale, based on the outcomes of the factor analytic study, for the experimental work reported in the remaining chapters. It was from the interesting and illuminating results that the experimental work arose in Chapters 8, 9, 10, and 11.

This chapter will provide a short overview of the study draw out the implications of the results for current models of numerical cognition and for future research.

12.2 Overview of the background to the research study

The recent resurgence of research interest in numerical cognition dates from Groen & Parkman's (1972) empirical studies. Their results indicated that as the problem became more difficult, the time taken to give the answers increased as did the number of errors made (problem size effect). The aim of their work was to provide a theoretical model to account for the problem size effect. Interest in the area of numerical cognition, until the work of Groen & Parkman (1972), had been intermittent with little methodological continuity to enable theoretical frameworks to emerge. Over the past thirty years, theoretical frameworks have been developed which provided the advantage of furthering existing knowledge of numerical cognition. With the exception of Aschraft (1995), they did not elucidate the role of memory processes in numerical cognition. His work provided a starting point for the thinking on the role of the components of working memory, proposed by Baddeley (1986), in numerical cognition in this research programme.

12.3 Theoretical models of numerical cognition

The numerals to be processed may be written as '1' or 'one' or spoken. Proposals have been made relating to the organisation of number processing which in turn lead to various conjectures about the effects of numerical input stimuli on the performance of arithmetic calculations. The models of numerical cognition have sought to explain ways in which individuals comprehend, represent, analyse, calculate and estimate numerical information. The various models have provided explanations to account for the various possible functions that exist in a calculation process before the final production of an answer.

12.3.1 Abstract modular theory (McCloskey, Caramazza & Basili 1985)

This theory of numerical cognition assumes the concept of an abstract internal representation mechanism and three independent systems. The independent systems are considered to be a number comprehension system, a calculation system and a response system. The comprehension system is for encoding number input, for example, digits, written or spoken number words. Prior to the calculation system, the input stimuli are thought to be converted to an abstract semantic representation which may represent the characteristics of the numeral formats. It is suggested that these representations may take the form of semantic representations, phonological representations of number words, or graphemic representations of digits that highlight a digit's abstract identity. The calculation system includes memory for basic number facts and rules and incorporates procedures for more complex arithmetic, for example, multi-digit addition or multiplication. Finally a response system is considered to recode the abstract output from the comprehension and calculation systems into Arabic, or written or spoken verbal number responses.

12.3.2 Clark & Campbell's (1991) Encoding complex approach

The encoding complex approach is in contrast to the abstract modular theory. This associative network model assumes that arithmetic facts are stored in multiple representational formats and modality specific codes interconnected in a complex network. It is thought that number processing is based on visual, visuo-spatial and phonological codes which are primarily modality-specific processes. Different input numerical forms, for example, digits written as words, digits written as digits and auditory input of numerical information, can influence the codes or strategies that are necessary for the completion of a task. A single numerical function may involve multiple associations within the network.

12.3.3 Dehaene's (1992) Triple-code theory

This theory suggests that number processing entails an analogue magnitude representation which determines approximate calculation, numerical size comparisons and may assist in subitizing tasks. Subitizing is the ability to recognise small numbers of items (from 1 up to 4 items) automatically without consciously counting. The visual Arabic number form allows for digital input and output, parity judgements and multi-digit operations. Written and spoken input and output, together with counting processes, are processed by the auditory verbal code system. Within this model each code type is assumed to link to a specific set of numerical operations. This model assumes that the three different types of codes activate one another directly and are not mediated by an abstract representational system. However, there is considered to be a visual image of the arrangement of the digits to be processed.

12.3.4 Noel & Seron's (1992) Preferred entry code model

This model assumes that performance is based on modality-specific codes, for example, phonological and visual Arabic. Numerals are recoded from Arabic to verbal form by converting digit representations directly into word representations without the intervention of a semantic representational format. It is suggested that each individual has a preferred entry code. For example, input numerals may be converted to a verbal representation before the mental calculation is performed or they may remain in the Arabic notation format. This theory appears to highlight individual differences.

12.4 Similarities between the models of numerical cognition

The basic principle associated with the abstract-modular model proposed by McCloskey, Caramazza & Basili (1985) and the triple-code theory Dehaene (1992) is modularity. The abstract-modular model and the triple-code theory assume three independent number processing systems. The abstract-code model assumes a comprehension system for encoding number input, a separate calculation system and an output production system. Dehaene's (1992) alternative model suggested that number processing involves an analogue magnitude representation supporting approximate calculations, numerical-size comparisons and subitizing tasks. The Arabic form mediates digital input and output, parity judgements and multi-digit operations. The auditory verbal system controls written and spoken input, output, counting processes and simple addition and multiplication facts. Both of these models suggest that number fact retrieval processes are the same irrespective of the different format of the input stimuli and they are based on the assumption that the initial representation of the problem is converted to an internal abstract code prior to calculation and response. For example, '2 x 3 =?' and 'two x three' are assumed to employ the same retrieval process. However, Campbell & Clark (1992) suggested that tasks involving different imput stimuli tasks might use different representations of number which may vary from individual to individual. This appears to be in agreement with the preferred entry code model proposed by Noel & Seron (1993). In both the abstract modular theory and the encoding complex approach calculation and arithmetic fact retrieval take the format of an abstract representational system.

12.5 Differences between the models of numerical cognition

One area open to discussion is whether the numerical processing system is modular or calculations are performed through an associative network. Campbell

(1994) argued that digits and words elicit different degrees or strengths of activation in a single hypothetical associative network. However, McCloskey et al.'s (1985) and Dehaene's (1992) models predicted independent memory processes for digits and words.

The encoding complex theory of Clark & Campbell (1991) was in complete contrast to the modular theories and assumed an integrated network. This implied that different forms of number input stimuli differ in their ability to activate the specific codes, for example 2 x 3 = ? would activate different specific codes as compared with two x three = ?. This model does not have a separate abstract representation system and or a separate calculation system. It appears to be in its entirety a network system. This interactive model does not specify the different number representations and the way in which they interact. It seems to be restricted to basic fact retrieval and automatic retrieval of previously learnt arithmetic knowledge, and no attempt is made to identify the methods used in processing multi-digit problems, size comparison, subitizing and parity. In contrast the triple-code theory of Dehaene (1992) offered an account of multi-digit calculations, size comparison, subitizing and parity operations. It was suggested that the representation of the numerical input used in calculation is not abstract but is comparable to a visual image of the arrangements of digits to be processed. It was suggested that numerical quantities are represented in an analogical number line. In contrast to the other models mentioned Dehaene's triple-code theory appears to account for more complex numerical processing.

12.6 Working memory and numerical cognition

Short and long term memory processes are fundamental to the manipulation and calculation of numerical information. As there are associations between numerical cognition and memory processes, the working memory model of Baddeley (1986) and its interpretation in relation to the study of numerical cognition was considered.

The working memory model assumes that temporary maintenance and manipulation of information are necessary for the execution of cognitive tasks. This tripartite model is comprised of a central executive a phonological loop and a visuo-spatial sketchpad. The central executive is thought to be responsible for reasoning, decision making, complex processing and problem solving and is also concerned with the attentional control of information. The phonological loop maintains speech based information and the visuo-spatial sketchpad specialises in the visual and spatial coding necessary for recalling information relating to the spatial relationships between items.

The relationship between Baddeley's (1986) working memory model and the structure of numerical processing has only recently begun to be studied. The literature suggests that working memory plays an important role in the performance of arithmetic operations, in conjunction with stored knowledge in long-term memory. However, as there has been very little empirical evidence available, it is not possible to provide a detailed assessment of the role of the various components. Ashcraft (1995) integrated the working memory model and provided predictions as to the possible numerical calculation processes in relation to the three components of working memory. He suggested that for solving problems requiring the carrying or borrowing of figures both the central executive and the visuo-spatial sketchpad are involved. A further suggestion was that the central executive requires the visual spatial component to hold a visual image of the problems to be calculated.

12.7 Rationale for the factor analytic study

The starting point for this research was the factorial studies of numerical ability by Coombs (1941), Geary, Widaman & Little (1986) and Geary & Widaman (1987, 1992) reviewed in Chapter 4. These studies used a broad range of measures. However the studies were not designed to assess directly the structure of numerical cognition or how the components of working memory reflect numerical processing. The investigators used the traditional individual differences strategy of including in their test batteries generally unclear "general ability" measures that might measure several aspects of cognition. A more specific approach is used in this thesis by employing more narrowly focused measures that can be clearly and convincingly related to cognitive theories.

The factor analysis method was implemented to investigate the cognitive processes that are involved in the solution of arithmetic problems of differing kinds, for example simple and complex addition and multiplication problems. The extensive factor analytic study was specifically designed to cover a spectrum of arithmetic problem solving abilities in conjunction with specific memory processes, particularly those hypothesised in Baddeley's working memory model. The earlier studies discussed in Chapter 4 had investigated arithmetic problem solving and memory but had not integrated them in the specific way proposed in this thesis and had not related so closely to the predictions of published models and thus had not offered clear c onclusions about the competing theories in this field.

This factor analytic study was a preliminary investigation into the relationship between aspects of numerical cognition and more general cognitive processes involved in working memory and long-term memory and was designed to:

- Elucidate the factor structure of the processes that underlie numerical cognition.
- Investigate the various components of the working memory model in relation to arithmetic and include long-term memory processes.
- Examine the extent to which findings from the factor analysis are compatible with key features arising from the models of numerical cognition developed on the basis of experimental studies.

12.8 Objectives of Study 1

As noted in Chapters 1-3, existing models of numerical cognition emphasize the representation of numerical information. McCloskey et al. (1985) suggested the involvement of an abstract semantic internal representation mechanism. McCloskey (1993) argued that research had not adequately considered the relationship between processes for numerical and non-numerical processing. This issue concerns whether numerical processing is separate from or integrated within the cognitive language processing system. McCloskey further suggested that research was required in the area of general cognitive processing, and specifically the areas of working memory and spatial processing. The objectives for the present study included addressing some of the issues raised by McCloskey by using a battery of tests that covered a broad sample of mental processes.

Dehaene's (1992) triple code model made provision for format specific representations and seemed to accommodate the abilities to compare and to approximate numerical quantities by way of the analogue magnitude representation component. Dehaene considered the analogue magnitude representation to be important in understanding the quantity that a numeral represents and in checking the accuracy of calculations. This component was considered to provide a semantic representation of numbers. Thus the properties of McCloskey's abstract semantic representation system and Dehaene's analogue representational system appear to accommodate very different calculation procedures. The processing of arithmetic problems was investigated using different tests for example, mental arithmetic problems, addition and subtraction, subitizing and magnitude comparison.

It was proposed that, if the analysis produced a factor largely based on the representation of information, this would lend support to McCloskey's theory. However, if separate factors emerged for number tasks, for example subitizing and magnitude judgement tasks, then this would lend support to Dehaene's model. Clark & Campbell (1991) and Geary & Widaman (1992) had suggested that information is accessed through an integrated associative network that would result in a number of different tasks appearing in the same factor. Within Dehaene's model multi digit calculations may utilize the visual spatial component of working memory together with central executive control. Simple arithmetic, however, may not require working memory operations but retrieval of previously learnt information from long-term memory, therefore complying with Campbell's associated network theory. In line with previous work Heathcote (1994) working memory tasks may form a factor alongside more complex problems for example, multi-digit addition as previously discussed in Chapter 3. The way in which numerical information is processed may be linked to the way in which information is represented within the components of the working memory model. More complex problems may also require specific working memory procedures. These issues are addressed in an extensive factor analytic study.

12.9 Study 1 – A Factor Analytic Study

The first study in the research programme reported in this thesis was a factor analytic study that aimed to investigate the possible structure of numerical cognition through an examination of the underlying relationships between the responses received from each of the test variables used. It was further intended to explore the field of numerical cognition to discover the main dimensions using the framework provided by the theories of numerical cognition and the working memory model.

The intention was to map the field of numerical cognition by sampling a large range of variables. Where suitable tests were available in published material they were selected to cover the domains that were targeted. In some cases it was necessary to construct new tests in order to achieve the very broad sampling that was required. The aim, using the battery of tests, was to investigate numerical processing in conjunction with other cognitive processes that were known (or hypothesized) to be relevant.

The tests for Study 1 were selected against the following criteria:

1. The tests should explore a wide range of cognitive processes including all the key processes that had been hypothesized to contribute to numerical cognition in the theoretical work reviewed in Chapters 1 – 2.

2. The test battery should have balanced responses. For example, computer tests required equal numbers of 'yes' and 'no' keyed responses.
3. Methods of responding to the tests should be varied. Both computer and pencil and paper tests employed different methods, for example, the production of an answer, verification tasks and multiple-choice tasks.
4. Arithmetic tests should cover both short and long-term memory processes.
5. The battery should include tests designed to investigate the hypothesized components of working memory that were highlighted in the review in Chapter 3.

12.10 Summary of findings from the factor analytic study

The correlation matrix showed a large number of interesting associations that might be linked to underlying phenomena. To gain a fuller insight into the relationships between the variables a factor analysis of the data was carried out to uncover the underlying factor structure. The method of factor analysis used to analyse the data set was principal component analysis with Varimax rotation. This method is exploratory and is used as a tool to reduce a large set of variables to smaller sets in order to identify groups of variables that are inter-related. The first principal component extracted accounts for the most variance and the further components are ordered in size as they are extracted.

Factor 1, access to representations, accounted for 21.45% of the variance with six variables found to load onto this factor. The tasks loading onto this factor were the English and French lexical decision tasks, the magnitude judgement of numbers and animals, rotation of letters and subitizing circles. To explain the diversity of the tasks included in this factor a number of concepts were examined in greater depth in experimental work that was reported in subsequent chapters.

The two tests of magnitude judgement loaded onto Factor 1. Dehaene's (1992) notion of a number line was proposed as a means of accommodating numerical comparison but not the comparison of any other stimuli. However, the fact that the two comparison tests loaded onto the same factor in this study suggested that both types of stimuli might access similar internal representations. It was this suggestion that was investigated in the experiment reported in Chapter 8.

Chapter 9 built on the conclusions drawn in Chapter 8 and reported two experiments carried out to explore the nature of magitude representations using the dual task methodology. Experiment 1 investigated the effect of interference tasks at the lexical level of processing whereas Experiment 2 considered possible interference at the semantic level. Accuracy and speed of making judgements relating to numerical magnitude were only found to be impaired in Experiment 2 where the interference task involved making judgements with regard to a range of objects. The conclusion arrived at was that magnitude judgements may be represented at the level of semantic processing and not necessarily specific to numbers.

The subitizing circles task loaded on Factor 1 as well. The reason for this was investigated in the experimental work reported in Chapter 10 using the dual task methodology. It was speculated that subitizing circles might depend on speed of access to stored representations and the automatic recognition of the number of items that are clustered together to form canonical configurations of visual items. Consideration was also given to the hypothesis that subitizing circles is not necessarily closely related to the processing of arithmetic information but more closely related to the analysis of shape involving pre-lexical processing. This issue is discussed in more detail below.

It appears from the results of the factor analytic study that the structure of numerical cognition reflected in Factor 1, access to representations, comprise both language and numerically based tests. From the results of the data analysis in Chapter 8 the common element linking the magnitude judgement of numbers and animals is that the analogue scale suggested by Dehaene (1992) is not exclusively appropriate for use only for the judgement of numerical information but can be utilised for the judgement of objects. It appears from the experimental data reported in Chapter 9 that language processes play a significant part in the magnitude judgement of numbers and objects. The experimental work using subitizing circles reported in Chapter 10 investigated speed of access to stored representations taking into consideration the cognitive processes required for the completion of the task. It would seem to be reasonable to speculate that the tests found to load onto Factor 1 utilise a number of separate yet linked cognitive processes for the completion of the tasks. From the results of the experimental work in Chapters 8, 9, and 10 there seems to be clear justification as to why these particular tests load onto Factor 1.

Table 12.1 Summary of findings from the factor analytic studys

Factor	Findings	Experimental work
Factor 1 - access to representations	Association between the language and numerically based tests	**Chapter 8** The nature of magnitude comparison, Dehaene's (1992) analogue scale. **Chapter 9** The magnitude judgement of numbers related to lexical and semantic procession of numbers and words. **Chapter 10** Subitizing, the relationship between subitizing and pre-lexical processing
Factor 2 – working memory	Complex addition and multiplication in conjunction with tests traditionally associated with working memory	**Chapter 11** Visual spatial involvement in the addition of multi-digit problems.

12.11 The role of working memory in numerical cognition

Seven variables loaded onto Factor 2, working memory, accounting for 13.2% of the variance. The seven variables were Stroop, Trails B, forward and backward digit span, the story taken from the Rivermead Behavioural Memory Test and complex addition and multiplication. The most interesting aspect of this factor is the inclusion of complex addition and multiplication. Aschraft (1995) speculated that the central executive is involved in carrying and borrowing procedures with the phonological loop holding intermediate values and the visuo-spatial sketchpad responsible for the visual characteristics of the problems. The relationship of complex arithmetic to working memory was investigated in greater depth in

Chapter 11 with particular emphasis on the involvement of the visuo-spatial sketchpad component.

12.12 Factors accounting for under 10% of the variance

Factors 3 – 7 accounted for 5 – 9 % of the variance each. They were not the subject of further experimental work in this thesis but provide interesting areas for future research.

Factor 3, basic number facts, seems to be clearly a simple arithmetic factor indicating independent processes from those tests identified in Factor 1 and Factor 2. The arithmetic operations included in the test battery were addition and multiplication with both simple and complex problems to solve. To include subtraction and division problems would have completed the range of arithmetic operations included in test battery. However, it was not practically possible to do so due to the length of time that each participant would take to complete the battery of tests. This is a research direction worth exploring in the future as until now much research has focused on the role of working memory in addition and multiplication while subtraction and division have received little attention.

It would be interesting to study whether simple subtraction and division would load on the same factor as simple addition and multiplication and whether the retrieval strategies for addition and multiplication are similar to those used for subtraction and multiplication. Campbell & Xue (2001) suggested that there are important distinctions between the basic arithmetic operations and that participants seem to rely more heavily on retrieval strategies for addition and multiplication than for subtraction and division.

Factor 7, visual spatial short-term memory, accounted for 5% of the variance. The abstract pictures test was expected to correlate with the complex arithmetic tests. However this result was not found. A possible explanation for this is that the abstract visual pictures task relied upon the visual encoding of shape and colour rather than the visual spatial processes that may be required for solving the complex arithmetic problems discussed in Chapter 11. This would be an interesting avenue to explore as the abstract pictures test seems to be a relatively pure measure of visual spatial short-term memory.

12.13 Future Research arising from the factor analytic study

12.13.1 The contribution of factor analytic studies to research on the structure of numerical cognition

The contribution of factor analytic studies has been underplayed in recent years particularly within the area of cognitive psychology. This research programme begins with a study that was designed to demonstrate the value of this technique in this field.

The participants who took part in the factor analytic research reported in this thesis were drawn from a student population. Although their age ranges did vary it was predominantly a population of young adults within an educational environment. It seems that more work could be undertaken to investigate whether the structure of numerical cognition undergoes developmental changes as individuals pass through adulthood. More research could be conducted in this area to assess developmental changes. If developmental changes were observed further experimental work on participants within specific age ranges would assist in clarifying the nature of these changes and so providing a further contribution to understanding the structure of numerical cognition.

For the present factor analytic study participants were from an educational environment and may represent a vocationally narrow population. Informal observation of a small number of participants' performance during the test battery indicated that participants who had employment involving the handling of money appeared to produce more accurate responses to simple and complex arithmetic problems than those participants who did not. An interesting outcome from the factor analytic study is that not only could developmental changes be taken into account in any future research but also participants' employment.

12.13.2 Speed of processing

The six tests loading onto Factor 1 (access to representations) reflect speed of processing. It would be valuable to test the generality of this factor using similar test but not necessarily analyzing reaction time data. The question arises as to whether this factor is an artifact or whether a wider range of responses to similar tests would load onto the same factor. It is interesting to note that subitizing numbers (analysed using reaction time data) was found to load onto Factor 4 along with the test 13 basic arithmetic facts.

12.14 Conclusion from the factor analytic study

From this extensive factor analytic study clear and interesting patterns as to the structure of numerical cognition have emerged. The following section gives an overview and draws conclusions from the experimental work conducted in Chapters 8 - 11.

12.15 Experimental studies

The results of the factor analytic study provided a solid foundation from which to explore in more depth specific aspects of numerical cognition. The further investigations involved reanalysis of some data reported in the factor analytic study (Chapter 8) and four new pieces of experimental work (Chapters 9-11). The experimental work, using the dual task methodology, explored the main themes that had emerged from the factor rotated matrix.

12.16 The nature of magnitude comparison

It was noted in Chapter 8 that the two tests of magnitude judgement, animals and numbers, loaded convincingly onto Factor 1. Dehaene's notion of a number line is considered to accommodate numerical comparison and not the comparison of any other stimuli. However the fact that the two comparison tests loaded onto the same factor suggested that both types of stimuli may access similar internal representations. To investigate this further a detailed analysis was performed comparing the two magnitude judgement tests and the effects of size of comparison targets and distance between targets in terms of their size.

Data analysis using a three way repeated measures Anova indicated that the classic number effect was found with small numbers faster to judge than large numbers and a greater distance between the numbers producing increased reaction times than for smaller distances. The patterns found in numbers were replicated for animals. From these results it does appear that size may be encoded as a property of any object and the strength of this encoding is greater for highly familiar items, for example, low numbers and common animals. The overall reaction times for animals was found to be slower perhaps due to the unfamiliar nature of the task in that the comparison of animal size information was not as frequent in everyday life as the comparison of numerical size information.

To explain the results of the significant three-way interaction a number of perspectives were considered. The emphasis of the modular model proposed by McCloskey et al. (1986) was on the semantic representation of numbers. However, if it is assumed that there is a conversion of numerical input into a semantic representation, then it could be considered that for the comparison of animals a semantic processing element is present. The question arises from this model as to whether semantic processing is required for the comparison of numbers, or is it based on a shallower level of processing that simply takes into account previously learnt, familiar information relating to quantity retrieved from long-term memory? With respect to the animal stimuli judgements may be based upon the physical characteristic of size but not processed in relation of quantity.

Dehaene (1992) suggested that approximate representation of magnitude could be characterised as a mental number line that becomes increasingly compressed as magnitudes increase. The number line is accessed by a process, which repeatedly activates small areas of the number line in approximately the correct location. As the two magnitude judgement tests have produced a significant interaction it remains unclear what processes support the representation of size in terms of the number line. It is possible that the number line is exclusive to number comparisons, but the results of the factor analytic study suggest that both tests support a common element. The result may reflect the notion that numerical size comparison is underpinned by semantic encoding and this issue was investigated further in the study reported in Chapter 9.

In contrast Campbell (1995) suggested that arithmetic facts are represented by activating physical codes in every form, for example, visually written words, imaginary number lines and colours. The view is that during arithmetic problem solving not only is there activation of physical codes but activation of a magnitude code. Throughout the network physical codes and magnitude codes are connected through a series of nodes. Analysis of the data relating to the performance of the two magnitude tests reflect this model and suggest that the retrieval of magnitude comparison of both types of stimuli is dependent upon the strength of the association between nodes.

12.16.1 Future research on magnitude comparison

It appears that the results cannot be reconciled within the framework of the modular theory proposed by McCloskey et al. (1986) and the triple code theory Dehaene (1992). If no interaction had been found then it would be possible to assume that numerical and animal size judgements were processed using a similar information processing mechanism. It appears from the results this is not the case and that the interactive model discussed by Campbell based on representations of varying degrees of strength and familiarity may provide an alternative explanation for the results. The results may reflect a lack of size range in the animal stimuli in comparison to the numerical stimuli. Numerical information is precise in relation to size and quantity but size information about animals is less well defined and not necessarily processed in terms of size and quantity. Furthermore the range of numerical size information, for example from 1 to 10,000, may be considered a larger range than observed in animals, for example, the range from a mouse to an elephant. The abstract representation system, proposed by McCloskey et.al. (1986), and the number line proposed by Dehaene (1992) may be used but only when particular types of stimuli are processed. To continue the investigation into the processes involved in magnitude judgements a suggestion for future research would be to replicate this study but substitute the judgements of size between objects, for example pebble and house, for the animal stimuli. It is possible that the animals chosen for this study do not represent a sufficiently wide scale. As discussed above, numerical stimuli are precise whereas animal stimuli tend to represent less well defined categories, therefore, making it more difficult to manipulate the small vs. large difference. Object sizes may provide a wider range.

The results of the factor analytic study revealed a highly significant correlation between two separate tests, the magnitude comparisons of numbers and magnitude comparisons of animals. The fact that the two comparison tasks loaded onto the same factor, Factor 1, access to representations, suggested that both types of stimuli might access similar internal representations. Detailed analysis of the reaction time results from the two magnitude tests was conducted and reported in Chapter 8. The key outcomes of the analysis of the data was that the number specific analogue scale proposed by Dehaene may not be purely exclusive to the magnitude judgement of numbers and that the magnitude judgement of animals appeared to follow a similar pattern to that of numbers. The two experiments reported in Chapter 9 build on the results from the two magnitude comparison tests reported in the factor analytic study and the detailed analyses of the reaction time results reported in Chapter 8. The development of the argument reported in Chapter 9 was to consider the view that to judge the relative size of two numbers requires semantic processing and that the number-specific analogue scale proposed by Dehaene (1992) may not be specific to numbers.

12.17 Magnitude judgement of numbers – Experimental studies

The next two experimental studies in the research programme, which were described in Chapter 9, investigated the notion that numerical size comparison is underpinned by semantic encoding. The dual task method was used to investigate whether numerical processing is linked to a long-term semantic system. The data in both experiments was analysed using repeated measures 2 factor mixed Anovas. The assumption was made that, if magnitude comparison of numbers involves accessing a long-term semantic store, then requiring participants to process aurally presented words (which requires semantic processing) will interfere with the magnitude comparison of visually presented numerical stimuli.

Experiment 1 (lexical processing) focused on the effects of the auditory presentation of lexical processing tasks and the visual presentation of the magnitude comparison of numbers task. It was anticipated that no significant interaction would be found. In Experiment 2 (semantic processing) it was expected that the auditory presentation of the semantic processing tasks would produce a significant interaction with the magnitude comparison of numbers task.

The results of Experiment 1 did not reveal an interference effect. This suggested that the two tasks were utilizing different processing systems. To build on the outcome of this experiment the second experiment was conducted to investigate the effects of semantic processing on magnitude comparison.

The results of Experiment 2 were in the predicted direction showing e ffects on reaction time and accuracy data from the interference of a size comparison tasks. This experiment investigated semantic interference with two conditions either a decision process directly related to size in the experimental condition or a decision process made between living and man-made objects in the control group. The experimental group showed considerable decrement with a reduction in accuracy and increased reaction time in the dual task condition. The control group showed a sight difference in performance of the magnitude judgement task but very little change was evident in the performance of the lexical decision task.

The results of the two experiments indicate that firstly lexical tasks do not interfere with numerical size judgement and secondly the magnitude judgement of numbers can be disrupted by performing a concurrent semantic magnitude judgement task. This disruption appears to be at the semantic level of processing.

McCloskey (1993) suggested that the relationship between numerical and non-numerical processing mechanisms had not been researched in sufficient detail. He raised the question whether numerical processing systems were separate or were incorporated within the cognitive language processing system. From the results of the experiments reported here there seemed to be some support for the view that the magnitude judgement of numbers may utilize the language processing system. Furthermore Warrington & Shallice, (1984) and Caramazza & Shelton (1998) report findings from patients suffering from brain injury for a distinction in the representation of knowledge between the semantic and lexical components of the language system.

The analysis of the data from these studies has provided interesting results in relation to the understanding of the common element that may link the magnitude comparison of numbers to Factor 1 of the factor analytic study, 'access to representations.' It seems possible that the results of Experiment 2 lend support to the view that the common element is associated with the semantic processing system required for the successful performance of the two tests. In discussing the results in Chapter 9 I developed the argument that to judge the relative size of two numbers requires semantic processing and that the number specific analogue scale proposed by Dehaene (1992) may not be specific to numbers.

12.17.1 Future research using dual task methodology to study magnitude comparison

It appeared from their results that the verbal tasks used in experiment 1 were seen by participants as more difficult than participants found in Experiment 2. The difference in task difficulty could account for the differences in the reation time data between the two experiments as participants may have allocated their resources differently depending upon the task demands. Participants in Experiment 1 appeared to find the verbal task difficult and may have concentrated on maintaining reaction times on the magnitude comparison of numbers task at the expense of the verbal tasks. In Experiment 2 participants seemed to answer the verbal tasks with relatively little difficulty. However the reaction times suffered to a greater extent than in Experiment 1. Further work is need in this area and in particular to replicate the experiment using lexical and semantic tasks which are of equivalent difficulty. Furthermore neuroimaging techniques have enabled the investigation of neural mechanisms involved in specific mathematical processes (Zago, Pesenti, Mellet, Crivello, Mazoyer & Tzourio-Mazoyer 2001; Dehaene, Piazza, Pinel & Cohen 2003). Future work combining experimental and neuroimaging techniques would contribute to resolving the debate surrounding the lexical and semantic processing of words and numbers.

12.18 Subitizing

From the results of the factor analysis study discussed in Chapter 6 the subitizing circles task was found to load onto Factor 1, 'access to representations'. The focus of this chapter was to explore the subitizing process in terms of shape and representations. The experiment reported in Chapter 10 focused on the effects of lexical and pre-lexical processing on a visually presented subitizing task using the dual task method. In general the pattern of the results showed that there was disruption when participants were required to undertake a subitizing task in conjunction with simultaneously presented pre-lexical and lexical processing tasks. It appears that both the aurally presented word tasks produced an interference effect on the visually presented subitizing task. This lends support for the view that the auditory word tasks and the visually presented subitizing tasks use the same cognitive processing systems.

This study conducted on a normal population of participants emphasises an interesting link between linguistic and subitizing processes. Consideration was given to the view that there may be activation of representations in the visual image system with subitizing involving the recognition of shape and size which utilises early perceptual processes. In general the pattern of the results showed that there was disruption when participants were required to undertake a subitizing task in conjunction with simultaneously presented pre-lexical and lexical processing tasks. The between groups results in single and dual task conditions were very similar with slightly less correct responses recorded in the dual task conditions than found in the single task condition. Both the control and experimental conditions produced approximately equal levels of interference suggesting that early perceptual processing may be involved in a subitizing task.

12.18.1 Future research on subitizing

In general the pattern of the results showed that there was disruption when participants were required to undertake a subitizing task in conjunction with simultaneously presented pre-lexical and lexical processing tasks. There is a great amount of speculation and debate on the processes involved in subitizing with many questions still unanswered. The results of the present study support the view that early pre-lexical processes play a part in subitizing tasks. To investigate further the process involved in subitizing it would be beneficial to elaborate on this study to include a verbally presented semantic task with the visual presentation of arrays of circles displayed consistently in pattern arrangements, for example, a square, a triangle, or a diamond. This would then allow for a comparison study between lexical and semantic processing with possible findings suggesting a lack of interference from a semantic task given that subitizing is seen as an early perceptual process.

12.19 Visual spatial involvement in the addition of multi-digit problems

Chapter 11 considered a specific aspect of Factor 2 – (working memory). An interesting aspect of this factor was the inclusion of complex addition and multiplication. Aschraft (1995) speculated that the central executive is involved in carrying and borrowing procedures with the phonological loop holding intermediate values and the visuo-spatial sketchpad responsible for the visual characteristics of the problems. The first key objective of the study reported in this chapter was to consider the relationship between complex arithmetic and working memory with particular emphasis on the involvement of the visuo-spatial sketchpad component. Aschraft (1995) suggested that there would be disruption of arithmetic processing that required carrying operations or problems where digits are maintained in columns when this processing is accompanied by a secondary task utilizing visuo-spatial processing. The second objective of the study was to lend support for the distinction between visual spatial memory and visual memory within the visuo-spatial sketchpad component of working memory suggested by Logie & Marchetti (1991).

The results of the experiment provided evidence to suggest the involvement of the visual spatial component of working memory in the mental addition of multi-digit addition problems. There was a very significant dual task effect for the experimental group on the matrix scores indicating that the ability to remember 4 x 4 matrix patterns is significantly impaired by the concurrent addition of two digit arithmetic problems. It is interesting to note that participants appeared to have attempted to preserve their performance on the addition problems across the single and dual conditions at the expense of their performance on the matrices task.

The results of this experiment are also in line with the results obtained by Logie & Marchetti (1991) on the systems involved in retaining visual and spatial information. The control group in the present experiment was required to retain visual information with stimuli based on the memory for a particular shade of colour while the experimental group was required to memorize the spatial layout of matrix patterns while solving concurrent addition problems. The results suggested a relatively consistent performance from the control group in both the single and dual task conditions. In the experimental group, however, there is a significant difference between the single and dual task conditions with a decline in performance in the dual task condition. The analysis of the data obtained from the experiment indicates that memory for the visual spatial layout of matrix patterns was disrupted by the concurrent arithmetic task. The result lends tentative support to the view that retention of visual and spatial material is a function of separate systems, one responsible for temporary retention of visual material and one responsible for spatial material.

There does appear to be support for the prediction that visual spatial memory is involved in the calculation of 2 digit addition problems. More specifically the results lend support to the modified version of the working memory model proposed by Logie (1995) where visual spatial memory comprises a visual temporary store and a spatial temporary store. Furthermore the results provide support for the view held by Aschraft (1995) of the involvement of the visual spatial component of working memory in the calculation of multi-digit problems.

12.19.1 Future research combining working memory and arithmetic

Research into the contribution of the visuo-spatial sketchpad remains limited with experimental work producing differing results. Seitz & Schumann-Hengsteler (2000) argued that the visuo-spatial tapping did not interfere with the solving of complex problems and Logie et al. (1994) found that solving complex addition problems was not affected by the presentation of irrelevant pictures nor by spatial tapping when the numerals were presented orally. However, Logie et al. found that spatial tapping did hamper complex addition when the numerals were presented visually. The role of the visuo-spatial sketchpad in complex problem solving seems as yet uncertain. Further experimental work manipulating the quantity and type of information presented to participants in each array and the introduction of subtraction and division problems could provide an interesting outcome. As complex problem solving involves the retrieval of intermediate results further work examining the role of the central executive and phonological loop could be undertaken. From the work included in this thesis and future research a model that incorporatesboth the working memory and the long-term memory processes involved in complex arithmetic could be produced.

12.20 Conclusion

This thesis began with an account of four models of numerical cognition, the abstract modular theory, McCloskey, Caramazza & Basili (1985), the encoding complex approach, Clark & Campbell 1991), the triple-code theory, Dehaene (1992) and the preferred entry code, Noël & Seron (1992). The relationship between numerical cognition and working memory was also discussed with particular emphasis on Ashcraft's (1995) adaptation of the working memory model. From the results of the factor analytic study there emerged an interesting factor structure from which further experimental work was conducted. It was found that the data analysis reported in Chapter 8 could not be explained within the framework of the modular theory proposed by McCloskey et al. (1986) or the triple code theory Dehaene (1992). A more feasible explanation was that provided by Clark & Campbell's (1991) interactive model based on representations of varying degrees of strength and familiarity. The key outcome of the analysis of the data was that the number specific analogue scale proposed by Dehaene (1992) may not be exclusive to the magnitude judgement of numbers. The two experiments conducted in Chapter 9 were carried out to explore the nature of magnitude representations and from the findings it was concluded that the number specific analogue scale may not be purely exclusive to the magnitude judgement of numbers.

Chapter 10 focused on the effects of lexical and pre-lexical processing on a visually presented subitizing task. This study considered the interesting link between linguistic and subitizing processes. According to Dehaene & Cohen (1995) the triple-code theory provides a direct route that links the Arabic and verbal codes without an intermediate representation stage as envisaged in the abstract modular theory (McCloskey et al., 1985). According to Dehaene et al. (1995), the triple-code model may be similar to models of word recognition and reading in that to have an understanding of numbers it is not necessary to process the information through a semantic representation of the quantities. The results of this study support the triple-code theory in that the subitizing processes may not utilise semantic processing systems but represent early lexical processes.

The work in this thesis also focused on the relationship between the working memory model (Baddeley, 1986) and how the representation and manipulation of spatial information affects the impact of dual-task performance on verbally presented addition problems. This research also aimed to relate the findings to the speculative view held by Aschraft (1995) that the visuo-spatial sketchpad may play a role in complex arithmetic problem solving. The results did provide support for the view held by Aschraft of the involvement of the visuo-spatial component of working memory in the calculation of multi-digit problems.

Thus the research reported in this thesis has used a range of investigative techniques and data analysis with the aim of clarifying the scope and the limitations of major recent models of numerical cognition and the role of wQorking memory in numerical processing. The results of the research programme supported those models which link numerical cognition with other forms of mental processing by identifying specific ways in which diverse numerical processes such as magnitude comparison, subitizing and the calculation of multi-digit problems draw on forms of processing associated with other types of stimuli.

Appendix 1

Chapter 9 Magnitude judgement of numbers

Experiment 1 – Visual Stimuli (single task)

Visual stimuli used in the single task condition of the experimental and control groups.

30 stimuli in total

Right hand responses	
2	3
5	6
53	54
78	79
92	93
1	3
3	6
6	9
61	63
82	84
1	5
3	8
52	57
93	98
3	9
51	57
73	79
2	9
91	98
61	69

Left hand responses (distractors)	
4	3
88	87
6	4
57	55
5	1
8	3
98	93
68	62
8	1
9	1

Appendix 2

Chapter 9 Magnitude judgement of numbers

Experiment 1 – Visual Stimuli (dual task)

Visual stimuli used in the dual task condition of the experimental and control groups.

30 stimuli in total

20 right hand responses

Right hand responses	
3	4
7	8
8	9
65	66
87	88
2	4
5	7
55	57
77	79
94	96
71	75
2	7
4	9
84	89
2	8
1	7
62	68
1	8
82	89
63	67

Left hand responses (distractors)	
6	5
66	65
4	2
63	61
75	71
7	2
89	84
7	1
9	2
69	61

Appendix 3

Chapter 9 Magnitude judgement of numbers

Experiment 1 – Auditory Stimuli (single task)

Auditory stimuli used in the single task condition of the experimental and control groups.

Words beginning with 'p'	**Words beginning with 'c'**	**Non-words beginning with 'p'**	**Non-words beginning with 'c'**	**Words**	**Non-words**
Pot	Cat	Plub	Clore	Flip	Dlip
Pint	Catch	Pib	Clug	Splash	Blosh
Pond	Coil	Plish	Clig	Squid	Quib
Pin	Cramp	Plint	Climp	Thimble	Mimble
Picnic	Crane	Pob	Crump	Dent	Fant
Ponder	Church	Prid	Compy	Funny	Ooly
Pram	Concert	Priddle	Conder	Bread	Slodder
Perish	Cottage	Pid	Crappel	Abbey	Drif
				House	Verd
				Milk	Twint
				Train	Noblong
				Door	Blumper
				Tyre	Throbbel
				Smile	Flomber
				Spank	Dompy
				Rent	Toffel

Appendix 4

Chapter 9 Magnitude judgement of numbers

Experiment 1 – Auditory Stimuli (dual task)

Auditory stimuli used in the dual task condition of the experimental and control groups.

Words beginning with 'p'	Words beginning with 'c'	Non-words beginning with 'p'	Non-words beginning with 'c'	Words	Non-words
Plod	Cram	Plit	Clos	Drip	Girdy
Pile	Crack	Pont	Crint	Guard	Offling
Pick	Cliff	Plig	Crage	Herd	Gickel
Pigeon	Card	Poggle	Clom	Splint	Rumplate
Preach	Charge	Plock	Cromp	Rattle	Lozard
Paper	Common	Prad	Clum	Bowl	Spattle
Park	Case	Prip	Clupper	Arms	Hoffle
Priest	Cruise	Plim	Clud	Ticket	Daggle
				Town	Dilty
				Skirt	Blit
				Rain	Muthy
				Dust	Scrapple
				Flower	Trid
				Mail	Vomp
				Decade	Yamp
				Tribe	Niddle

Appendix 5

Chapter 9 Magnitude judgement of numbers

Experiment 2 – Auditory Stimuli (single task)

Auditory stimuli used in the single task condition of the experimental and control groups.

Manmade Object Larger than a 'sheep'	Manmade Object Smaller than a 'sheep'	Living Object Larger than a 'sheep'	Living Object Smaller than a 'sheep'
Bus	Radio	Lady	Rat
Stepladder	Kettle	Reindeer	Squirrel
Wardrobe	Spoon	Ostrich	Goldfish
Cottage	Jug	Hippopotamus	Rabbit
Trailer	Plate	Kangaroo	Sparrow
Fence	Shoes	Volcano	Fox
Boat	Football	Llama	Goose
Barge	Telephone	Ox	Chicken
Tram	Iron	Lion	Pumpkin
Pyramid	Fork	Zebra	mouse
Escalator	Vase	Lake	Cat
Bed	Pen	Dolphin	Crow

Appendix 6

Chapter 9 Magnitude judgement of numbers

Experiment 2 – Auditory Stimuli (dual task)

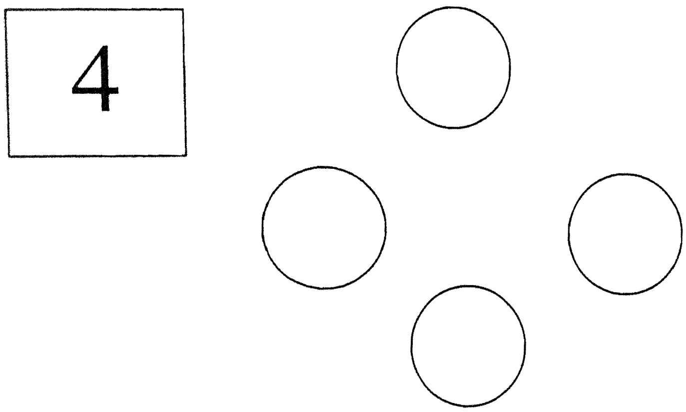
Auditory stimuli used in the dual task condition of the experimental and control groups.

Manmade Object Larger than a 'cow'	Manmade Object smaller than a 'cow'	Living Object Larger than a 'cow'	Living Object Smaller than a 'cow'
Bridge	Watch	Horse	Fox
Taxi	Book	Giraffe	Alligator
Church	Chair	Whale	Sheep
Train	Cup	Camel	Butterfly
Lamppost	Violin	Elephant	Dog
House	Umbrella	Buffalo	Tiger
Aeroplane	Bath	Oak tree	Frog
Tractor	pillow	Rhinoceros	Ant
Yacht	Knife	Iceberg	Penguin
Shed	Bicycle	Mountain	Gerbil
Car	Spoon	Palm tree	Leopard
Garage	Guitar	River	Wolf

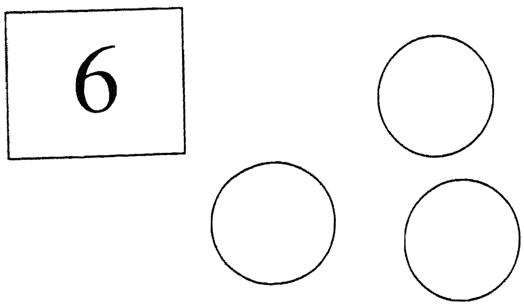
Appendix 7

Chapter 10 Subitizing Visual Stimuli

Example of the visual stimuli requiring a correct response used in the single task and dual task conditions of the experimental and control groups.



Example of visual stimuli requiring an incorrect response used in the single task and dual task conditions of the experimental and control groups.

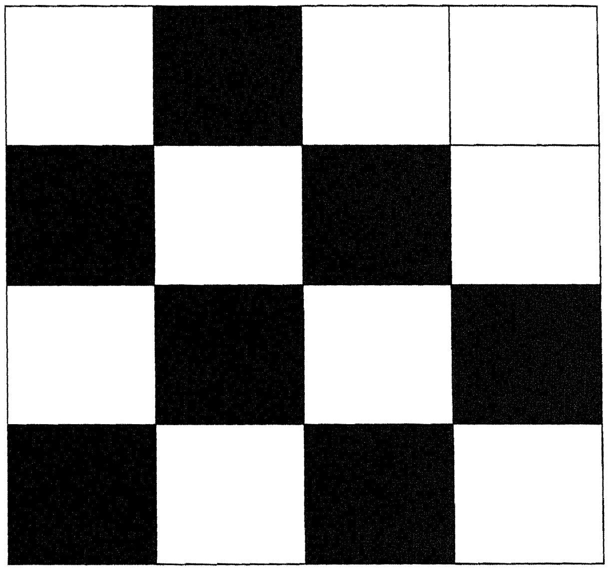


Appendix 8

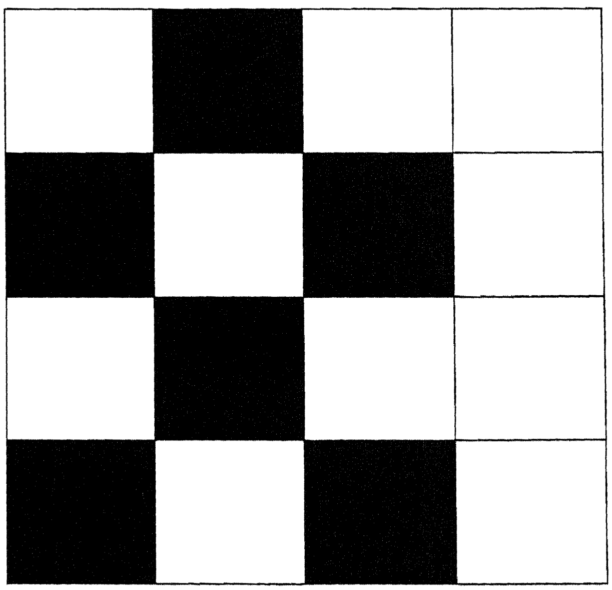
Chapter 11 Visual spatial involvement in the addition of multi-digit problems.

Experimental group – Example of 4 x 4 matrix patterns used in the single and dual task conditions.

Target stimuli to be remembered



Comparison stimuli requiring an incorrect response



Appendix 9

Chapter 11 Visual spatial involvement in the addition of multi-digit problems.

Addition problems used in the single condition of the experimental and control groups.

Addition problems with no carrying operations	Addition problems with carrying operations
27 + 31 = 58	36 + 49 = 85
23 + 16 = 39	47 + 59 = 106
13 + 56 = 69	26 + 67 = 93
63 + 24 = 87	86 + 93 = 179
13 + 36 = 49	42 + 89 = 131
24 + 73 = 97	63 + 87 = 150
32 + 11 = 43	68 + 33 = 101
24 + 35 = 59	93 + 74 = 167
34 + 12 = 46	76 + 38 = 114
17 + 12 = 29	57 + 49 = 106

Appendix 10

Chapter 11 Visual spatial involvement in the addition of multi-digit problems.

Addition problems used in the dual condition of the experimental and control groups.

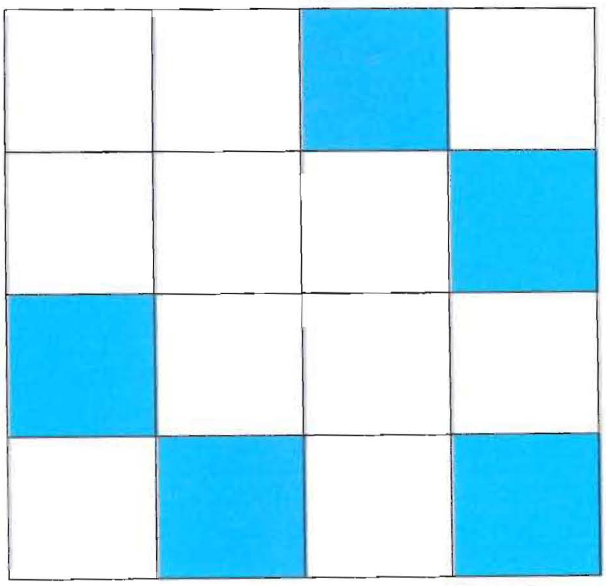
Addition problems with no carrying operations	Addition problems with carrying operations
23 + 35 = 58	29 + 13 = 42
22 + 37 = 59	24 + 56 = 80
15 + 13 = 28	19 + 38 = 57
24 + 73 = 97	29 + 15 = 44
33 + 54 = 87	41 + 68 = 109
23 + 66 = 89	56 + 83 = 139
64 + 35 = 99	69 + 54 = 123
43 + 32 = 75	37 + 75 = 112
15 + 14 = 29	85 + 19 = 104
21 + 12 = 33	57 + 74 = 131

Appendix 11

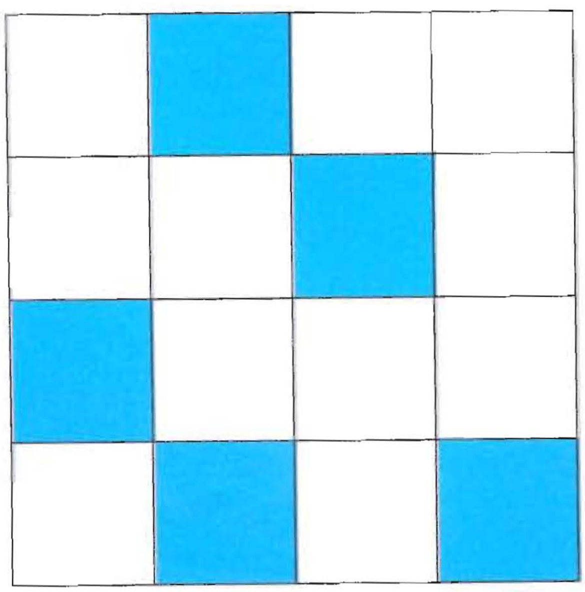
Chapter 11 Visual spatial involvement in the addition of multi-digit problems.

Control group – An example of a 4 x 4 coloured matrix pattern used in the single and dual task conditions.

Target stimuli to be remembered



Comparison stimuli requiring an correct response



References

Akin, O. & Chase, W. (1978). Quantification of three-dimensional structures. *Journal of Experimental Psychology: Human Perception & Performance*, 4(3), 397-410.

Anderson, S.W., Damasio, A.R., & Damasio, H. (1990). Troubled letters but not numbers. Domain specific cognitive impairments following focal damage in frontal cortex. *Brain*, 113(3), 749-766.

Arnett, J. A. & Labovitz, S. S. (1995). Effect of physical layout in performance of the Trail Making Test. *Psychological Assessment*, 7(2), 220-221.

Ashcraft, M. H. (1982). The development of mental arithmetic: a chronometric approach. *Developmental Review*, 2(3), 213-236.

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, 44(1-2), 75-106.

Ashcraft, M. H. (1995). Cognitive Psychology and simple arithmetic: A review and summary of new directions. Mathem*atical Cognition*, 1(1), 3-34.

Aschraft M. H. & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory*, 4, 527-538.

Ashcraft, M. H., Fierman, B. A., & Bartolotta, R. (1984). The production and verification tasks in mental addition: An empirical comparison. *Developmental Review*, 4(2), 157-170.

Baddeley, A.D. (1986). *Working memory*. Oxford: Clarendon.

Baddeley, A.D. (1992). Is working memory working? The fifteenth Bartlett lecture. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 44A(1), 1-31.

Baddeley, A.D. (1993). *Working memory and conscious awareness*. In A. F. Collins & S. E. Gathercole (Eds.), Theories of memory. (pp. 11-28). Hove: Erlbaum.

Baddeley, A.D. (1996). Exploring the central executive. Quarterly Journal of *Experimental Psychology: Human Experimental Psychology*, 49A(1), 5-28.

Baddeley, A.D. (1997). *Human memory*: Theory and proctice. Hove, UK: Psychology Press.

Baddeley, A.D. (2000). The episodic buffer: A new component of working memory? *Trends in Cognitive Science*, 4, 417-423.

Baddeley, A.D. (2002), Is working memory still working. *European Psychologist*, 7(2), 85-97.

Baddeley, A. D., Emslie, H., & Nimmo-Smith, I. (1992). *The Speed and Capacity of Language Processing (SCOLP) Test*. Bury St Edmunds: Thames Valley Test Company.

Baddeley, A. D., Emslie, H., & Nimmo-Smith, I. (1994). *Doors and People: A test of visual and verbal recall and recognition*. Bury St Edmunds: Thames Valley Test Company.

Baddeley, A. D. & Hitch, G. (1974). *Working memory*. In G. A. Bower (Ed.), *The Psychology of Learning and Motivation*. (pp. 47-90). New York: Academic Press.

Baddeley, A. D., Lewis, V., & Vallar, G. (1984). Exploring the articulatory loop. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 36A(2), 233-252.

Baddeley, A. & Logie, R. (1992). Auditory imagery and working memory. In D. Reisberg (Ed.), *Auditory Imagery*. (pp. 179-197). Hillsdale: Lawrence Erlbaum.

Baddeley, A. D., Thomson, N., & Buchanan, M. (1975). Word length and the structure of short-term memory. *Journal of Verbal Learning & Verbal Behavior*, 14(6), 575-589.

Balakrishnan, J. D. & Ashby, F. G. (1992). Subitizing: Magical numbers or mere superstition? *Psychological Research/Psychologische Forschung*, 54(2), 80-90.

Banks, W. P., Fujii, M., & Kayra-Stuart, F. (1976). Semantic congruity effects in comparative judgments of magnitudes of digits. *Journal of Experimental Psychology: Human Perception & Performance*, 2(3), 435-447.

Baroody, A. J. (1983). The development of procedural knowledge: An alternative explanation for chronometric trends of mental arithmetic. *Developmental Review, 3(2),* 225-230.

Bertillon, J., (1881). La vision et la mémoire des nombres. *La Nature*, 408, 202-203.

Bijeljac-Babic, R., Bertoncini, J., & Mehler, J. (1991). How do four-day-old infants categorize multisyllabic utterances? *Developmental Psychology*, 29(4), 711-721.

Browne, C. E. (1906). The psychology of the simple arithmetical processes: A study of certain habits of attention and association. *American Journal of Psychology*, 17(1), 2-37.

Buckley, P. B., & Gillman, C. B. (1974). Comparison of digits and dot patterns. *Journal of Experimental Psychology*, 103(6), 1131-1136.

Butterworth, B. (1999). *The Mathematical Brain*. London: Macmillan.

Butterworth, B., Cipolotti, L., & Warrington, E. K. (1996). Short-term memory impairment and arithmetical ability. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 49A(1), 251-262.

Butterworth, B., Zori, M., Girelli, L., & Jonckheere, A. R. (2001). Storage and retrieval of addition facts: The role of number comparison. *The Quarterly Journal of Experimental Psychology*, 54A(4), 1005-1029.

Campbell, J. I. D. (1987). Production, verification, and priming of multiplication facts. *Memory & Cognition*, 15(4), 349-364.

Campbell, J. I. D. (1994). Numerical cognition: Evidence for hyperspecific, interactive operations. *Cahiers de Psychologie Cognitive/Current Psychology of Cognition*, 13(3), 297-320.

Campbell, J. I. D. (1995). Mechanisms of simple addition and multiplication: A modified network-interference theory and simulation. *Mathematical Cognition*, 1, 121-164.

Campbell, J. I. D. & Clark, J. M. (1988). An encoding-complex view of cognitive number processing: Comment on McCloskey, Sokol, and Goodman (1986). *Journal of Experimental Psychology: General*, 117(2), 204-214.

Campbell, J. I. D. & Clark, J. M. (1992). *Cognitive number processing: An encoding-complex perspective.* In: J. I. D. Campbell (Ed.) 1992. The Nature and Origin of Mathematical Skills, Advances in Psychology. (pp.457-491). London: North Holland.

Campbell, J. I. & Graham, D. J. (1985). Mental multiplication skill: Structure, process, and acquisition. *Canadian Journal of Psychology*, 39(2), 338-366.

Campbell, J. I. D., Kanz, C. L., & Xue, Q. (1999). Number processing in Chinese-English bilinguals. *Cognition*, 5(1), 1-39.

Campbell, J. I. D., & Tarling, B., (1996). Production, verification and error priming in cognitive arithmetic. *Memory and Cognition*. 24(2), 156-172.

Campbell, J. I. D. & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*. 130(2), 299-315.

Caramazza, A. & Hillis, A. (1990). Where do semantic errors come from? *Cortex*, 16, 95-122.

Caramazza, A. & Shelton, J. R. (1998). Domain-specific knowledge systems in the brain: The animate-inanimate distinction. *Journal of Cognitive Neuroscience*, 10(1), 1-34.

Carroll, J. B. (1976). *Psychometric tests as cognitive tasks: A new "structure of intellect"*. In L. B. Resnick (Ed.) The Nature of Intelligence. (pp. 74-116). Hillsdale: Laurence Erlbaum.

Cattell, R. B. (1963). Theory of fluid and crystallized intelligence: A critical experiment. *Journal of Educational Psychology*, 54(1), 1-22.

Cattell, R. B. (1978). *The Scientific Use of Factor Analysis*. New York: Plenum Press.

Cipolotti, L., Butterworth, B., & Denes, G. A. (1991). *A specific deficit for numbers in a case of dense acalculia*. Brain, 114, 2619-2637.

Clark, J. M. & Campbell, J. I. D. (1991). Integrated versus modular theories of number skills and acalculia. *Brain & Cognition*, 17(2), 204-239.

Cohen, L. & Dehaene, S. (1995). Number processing in pure alexia: The effect of hemispheric asymmetries and task demands. *NeuroCase*, 1, 121-137.

Cohen, L. & Dehaene, S. (2000). Calculating without reading: Unsuspected residual abilities in pure alexia. *Cognitive Neuropsychology*, 17(6), 563-583.

Coombs, C. H. (1941). A factorial study of number ability. *Psychometrika*, 6(3), 161-189.

Cooper, L. A. & Shepard, R. N. (1973). The time required to prepare for a rotated stimulus. *Memory & Cognition.* 1(3), 246-250.

Cornet, J. A., Seron, X., Deloche, G., & Lories, G. (1988). Cognitive models of simple mental arithmetic: A critical review. *Cahiers de Psychologie/Current Psychology of Cognition*, 8(6), 551-571.

Dagenbach, D. & McCloskey, M. (1992). The organization of arithmetic facts in memory: Evidence from a brain-damaged patient. *Brain & Cognition*, 20(2), 345-366.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44(1-2), 1-42.

Dehaene, S. (1996). The organization of brain activations in number comparison: Event-related potentials and the additive-factors method. *Journal of Cognitive Neuroscience*, 8(1), 47-68.

Dehaene, S. (1997). *The Number Sense: How the Mind Created Mathematics*. New York: Oxford University Press.

Dehaene, S. & Akhavein, R. (1995). Attention, automaticy, and levels of representation in number processing. *Journal of Experimental Psychology: Learning, Memory & Cognition*, 21(2), 314-326.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, 122(3), 371-396.

Dehaene, S. & Cohen, L. (1991). Two mental calculation systems: A case study of severe acalculia with preserved approximation. *Neuropsychologia*, 29(11), 1045-1074.

Dehaene, S. & Cohen, L. (1994). Dissociable mechanisms of subitizing and counting: Neuropsychological evidence from simultanagnosis patients. *Journal of Experimental Psychology: Human Perception & Performance*, 20(5), 958-975.

Dehaene, S. & Cohen, L. (1995). Toward an anatomical and functional model of number processing. *Mathematical Cognition*, 1, 83-120.

Dehaene, S. & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociations between Gerstmann's acalculia and subcortical acalculia. *Cortex*, 33, 219-250.

Dehaene, S. Piazza, M. Pinel, P. & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20(3/4/5/6), 487-506.

Delazer, M. & Butterworth, B. (1997). A dissociation of number meanings. *Cognitive Neuropsychology*, 14(4), 613-636.

Deloche, G. & Seron, X. (1987). Numerical transcoding: A general production model. In: G. Deloche & X. Seron (Eds.) *Mathematical Disabilities: A cognitive neuropsychological perspective,* (pp.137-170). Hillsdale: Erlbaum.

De Rammelaere, S., Stuyren, E. & Vandierendonck, A. (1999). The contribution of working memory resources in the verification of simple mental arithmetic sums. *Psychological Research*, 62, 72-77.

De Rammelaere, S., Stuyren, E. & Vandierendonck, A. (2001). Verifying simple sums and products: Are the phonological loop and central executive involved? *Memory and Cognition*. 29, 267-273.

Duncan, J. (1986). Disorganisation of behaviour after frontal lobe damage. *Cognitive Neuropsychology*, 3, 271-290.

Ebbinghaus, H. (1864). *Memory: A Contribution to Experimental Psychology*. New York: Dover Publications.

Ekstrom, R. B., French, J. W., & Harman, H. H. (1976) *Manual for kit of factor-referenced cognitive tests*. Princeton, NJ: Educational Testing Service.

Ellis, A. W. & Young, A. W. (1996 Eds.). *Human Cognitive Neuropsychology. A text book with readings*. Psychology Press Ltd..

Fias, W., Brysbaert, M., Geypens, F., & d'Ydewalle, G. (1996). The importance of magnitude information in numerical processing: Evidence from the SNARC effect. *Mathematical Cognition*, 2, 95-110.

Fink, G. R., Marshall, J. C., Gurd, J., Weiss, P.H., Zafiris, O., Shah, N. H., & Zilles, K. (2000). Deriving numerosity and shape from identical visual displays. *NeuroImage*, 13(1)46-55.

Fodor, J. A. (1985). Precis of the modularity of mind. *Behavioral & Brain Sciences*, 8(1), 1-42.

Foltz, G. S. (1982). *The representation and processing of number.* Unpublished doctoral dissertation, University of Denver.

Foltz, G. S., Poltrock, S. E., & Potts, G. R. (1984). Mental comparison of size and magnitude: Size congruity effects. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 10(3), 442-453.

Fürst, A. J., & Hitch, G., J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memeory and Cognition*, 28, 774-782.

Gallistel, C. R. & Gelman, R. (1991). Subitizing: the preverbal counting process. In W. Kessen & A. Ortony (Eds.), *Memories, Thoughts, and Emotions: Essays in Honor of George Mandler*. Hillsdale, N. J.: Lawrence Erlbaum.

Gallistel, C. R. & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, 44(1-2), 43-74.

Galton, F. (1880). Visualised Numerals. *Nature*. 21, 252-256.

Geary, D. C. & Widaman, K. F. (1987). Individual differences in cognitive arithmetic. *Journal of Experimental Psychology: General*, 116(2), 154-171.

Geary, D. C. & Widaman, K. F. (1992). Numerical cognition: On the convergence of componential and psychometric methods. *Intelligence*, 16(1), 47-80.

Geary, D. C., Widaman, K. F., & Little, T. D. (1986). Cognitive addition and multiplication: Evidence for a single memory network. *Memory & Cognition,* 14(6), 478-487.

Groen, G. J. & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review*, 79(4), 329-343.

Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi addends. *Cahiers de Psychologie Cognitive/Current Psychology of Cognition*, 13(2), 207-245.

Hecaen, H., Angelergues, R., & Houillier, S. (1961). Les varietes cliniques des acalculies au cours lesions retrolandique: Approche statistique du probleme (Types of acalculia resulting from retrolandic lesion: A statistical approach to the problem). *Revue Neurologique*, 105, 85-103.

Herdman, C. M. & LeFevre, J. (1992). Individual differences in the efficiency of word recognition. *Journal of Educational Psychology*, 84(1), 95-102.

Hillis, A. E. & Caramazza, A. (1991). Deficit to stimulus-centered, letter shape representations in a case of "unilateral neglect." *Neuropsychologia*, 29(12), 1223-1240.

Hillis, A.E., & Caramazza, A. (1995). Cognitive and neural mechanisms underlying visual and semantic processing. *Journal of Cognitive Neuroscience,* 7, 457-478.

Hines, T. M. (1990). An odd effect: Lengthened reaction times for judgments about odd digits. *Memory & Cognition*, 18(1), 40-46.

Hinrichs, J., V., (1981). Two digit number comparison. *Journal of Experimental Psychology Human Perception and Performance*, 7(4), 890-901.

Hitch, G. J. (1978). The role of short-term working memory in mental arithmetic. *Cognitive Psychology*, 10(3), 302-323.

Hittmair-Delazer, M., Semenza, D., & Denes, G. (1994). Concepts and facts in calculation. *Brain*, 117, 715-728.

James, W. (1890). *The Principles of Psychology*. New York: Dover Publications.

Jerons, W. S., (1871). The power of numerical discrimination. *Nature* Vol. III, 281-282.

Joreskog, K. & Sorbom, D. (1984). *LISREL VI User's Guide*. Indiana: Scientific Software.

Kaiser, H. F. (1960). The application of electronic computers to factor analysis. *Educational and Psychological Measurement,* 20, 141-151.

Kaufmann, L. (2002). More evidence for the role of the central executive in retrieving arithmetic facts – A case study of severe developmental dyscalculia. *Journal of Clinical and Experimental Neuropsychology*, 24(3), 302-310.

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology*, 62, 498-525.

Kay, J. & Ellis, A. W. (1987). A cognitive neuropsychological case study of anomia: Implications for psychological models of word retrieval. *Brain*, 110, 613-629.

Klahr, D. & Wallace, J. G. (1976). *Cognitive Development: An Information-processing View*. Hillsdale: Erlbaum.

Kohs, S. C. (1923). *Intelligence Measurement: A Psychological and Statistical Study Based Upon the Block-Design Test*. New York: Macmillan.

Kosslyn, S. M. (1980). *Image and Mind*. Cambridge, Mass: Harvard University Press.

Kosslyn, S. M. (1991). A cognitive neuroscience of visual cognition: Further developments. In: R. H. Logie & M. Denis (Eds.), *Mental Images in Human Cognition: Advances in Psychology*, 80. (pp. 95351-381). Oxford: North-Holland.

Lamberty, G. J., Putnam, S. H., Chatel, D. M., Bieliauskas, L. A., & Adams, K. A. (1994). Derived trail making test indices. *Neuropsychiatry, Neuropsychology and Behavioural Neurology*. 7(3), 230-234.

LeFevre J. A., Bisanz, J., & Mrkonjic, L. (1988). *Cognitive arithmetic: Evidence for obligatory activation of arithmetic facts. Memory & Cognition*, 16(1), 45-53.

LeFevre, J. A., Sadesky, G. S., & Bisartz (1996). Selection of procedures in mental addition: Reassessing the problem. *Memory and Cognition,*_22(1), 216-230.

Lehto, J. (1995). Working memory and school achievement in the Ninth Form. *Educational Psychology*, 15(3), 271-281.

Lemaire, P., Abdi, H. & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology*, 8, 73-103.

Logie, R.H. (1991). Visuo-spatial short-term memory: visual working memory or visual buffer? In C. Cornoldi & M. McDanial (Eds), *Imagery and Cognition*, 77-102. Berlin: Springer-Verlag.

Logie, R.H. (1995). *Visuo-spatial working memory: Essays in Cognitive Psychologie*. Hove: Lawrence Erlbaum.

Logie, R. H. & Baddeley, A. D. (1987). Cognitive processes in counting. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 13(2), 310-326.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition*, 22(4), 395-410.

Logie, R. H. & Marchetti, C. (1991). Visuo-spatial working memory: Visual, spatial, or central executive? In: R. H. Logie & M. Denis (Eds.) *Mental Images in Human Cognition: Advances in Psychology*, 80, (pp.105-115). Oxford: North-Holland.

Logie, R. H., Zucco, G. M., & Baddeley, A. D. (1990). Interference with visual short-term memory. *Acta Psychologica*, 75(1), 55-74.

Luria, A. R. (1966). *Higher Cortical Functions in Man*. London: Tavistock.

Macaruso, P., McCloskey, M., & Aliminosa, D. (1993). The functional architecture of the cognitive numerical-processing system: Evidence from a patient with multiple impairments. *Cognitive Neuropsychology*, 10(4), 341-376.

Mandler, G. & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, 111(1), 1-22.

Martin, G. N., (1998). *Human Neuropsychology*. Prentice Hall.

McCarthy, R. & Warrington, E. K. (1985). Category specificity in an agrammatic patient: The relative impairment of verb retrieval and comprehension. *Neuropsychologia*, 23(6), 709-727.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalulia. *Cognition*, 44(1-2), 107-157.

McCloskey, M. (1993). Theory and Evidence in cognitive neuropsychology: A "radical" response to Robertosn, Knight, Rafal and Shimamura (1993). *Journal of Experimental Psychology: Learning, Memory and Cognition*, 19(3), 718-734.

McCloskey, M, Caramazza, A., & Basili, A. (1985). Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain & Cognition*, 4(2), 171-196.

McCloskey, M. & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist*, 50(5), 351-363.

McNeil, J. & Warrington, E. K. (1994). A dissociation between addition and subtraction and written calculation. *Neuropsychologia,* 32(6), 717-728.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, 63, 81-97.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex frontal lobe tasks: A latent variable analysis. *Cognitive Psychology*, 41, 49-100.

Miyake, A. & Shah, P. (1999) Eds. *Models of working memory: Mechanisms of active maintenance and executive control.* New York: Cambridge University Press.

Morris, N. & Jones, D. M. (1990). Memory updating in working memory: The role of the central executive. *British Journal of Psychology*, 81(2), 111-121.

Moyer, R. S. (1973). Comparing objects in memory: Evidence suggesting an internal psychophysics. *Perception & Psychophysics*, 13(2), 180-184.

Moyer, R. S. & Landauer, T. K. (1967). Time required for judgements of numerical inequality. Nature, 215(5109), 1519-1520.

Nairne, J. S. (1983). Associative processing during rote rehearsal. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 9, 3-20.

Noël, M.-P. & Seron, X. (1992). Notational constraints and number processing: A reappraisal of the Gonzales and Kolers (1982) study. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 45A(3), 451-478.

Noël, M.-P. & Seron, X. (1993). Arabic number reading deficit: A single-case study of when 236 is read (2306) and judged superior to 1258. *Cognitive Neuropsychology*, 10 317-339.

Noël, M. P. & Seron, X. (1995). Lexicalization errors in writing Arabic numerals: A single-case study. *Brain & Cognition*, 29(2), 151-179.

Noël, M. P., Fias, W. & Brysbaert, M. (1997), About the influence of the presentation format on arithmetic-fact retrieval processes. *Cognition*, 63, 335-374.

Noël, M., P., Robert, A. & Brysbaert, M (1998). Does language really matter when doing arithmetic/ Reply to Campbell (1998). *Cognition*, 67, 365-373.

Noël, M., P., Désert, M., Aubrun, A & Seron, X. (2001). Involvement of short-term m emory i n c omplex m ental c alculation. *Memory and Cognition*, 29, 34-42.

Norman, D. A. & Shallice, T. (1986). Attention to action: Willed and automatic control of behavior. In R. J. Davidson, G. E. Schwartz, & D. Shapiro (Eds.) *Consciousness and Self Regulation* (Vol. 4). New York: Plenum.

Paivio, A. (1971). *Imagery and Verbal Processes*. New York: Holt, Rinehart & Winston.

Paivio, A. (1975). Perceptual comparisons through the mind's eye. *Memory & Cognition*, 3(6), 635-647.

Paivio, A. (1986). *Mental Represenations: A Dual Coding Approach, Oxford Psychology Series, 9*. New York: Oxford University Press.

Parkin, A. J. (1996). *Memory and Amnesia: An Introduction*. (2nd Ed.). Oxford: Blackwell.

Parkman, J. M. (1971). Temporal aspects of digit and letter inequality judgments. *Journal of Experimental Psychology*, 91(2), 191-205.

Parkman, J. M. (1972). Temporal aspects of simple multiplication and comparison. *Journal of Experimental Psychology*, 95(2), 437-444.

Parkman, J. M., & Groen, G, J., (1971). Temporal aspects of simple addition and comparison. *Journal of Experimental Psychology*. 89(2), 335-342.

Pavese, A. & Umiltà, C. (1998). Symbolic distance between numerosity & identity modulates Stroop interference. *Journal of Experimental Psychology: Human Perception and Performance*, 24, 1535-1545.

Pellegrino, J. W. & Glaser, R (1979). Cognitive correlates and components in the analysis of individual differences. *Intelligence*, 3(3), 187-214.

Pesenti, M. Thioux, M. Seron, X. & De Volder, A. (2000). Neuroanatomical substrates of Arabic number processing, numerical comparison, and simple addition: A PET study. *Journal of Cognitive Neuroscience*, 12(3), 461-479.

Phillips, W. A. & Christie, D. F. (1997a). Components of visual memory. *Quarterly Journal of Experimental Psychology*, 29(1), 117-133.

Phillips, W. A. & Christie, D. F. (1997b). Interference with visualization. *Quarterly Journal of Experimental Psychology*, 29(4), 637-650.

Piazza, M., Mechelli, A., Butterworth, B., & Price, C. J. (2002). Are subitizing and counting implemented as separate or functionally overlapping processes? *Neuroimage*, 15, 435-446.

Pillon, A. & Pesenti, M. (2001). Calculating without reading? Comments on Cohen & Dehaene (2000). *Cognitive Neuropsychology*, 18(3), 275-284.

Pylyshyn, Z. (1989). The role of location indexes in spatial perception: A sketch of the FINST spatial-index model. *Cognition*, 32(1), 65-97.

Pylyshyn, Z. (1993). Metaphorical imprecision and the "top-down" research strategy. In A. Ortony (Ed.), *Metaphor and Thought.* (pp. 543-561). Cambridge: Cambridge University Press.

Quinn, J. G. (1991). Encoding and maintenance of information in visual working memory. In: R. H. Logie & M. Denis (Eds.), *Mental Images in Human Cognition: Advances in Psychology*, **80.** (pp. 95-104). Oxford: North-Holland.

Quinn, J. G. & McConnell, J. (1996). Indications of the functional distinction between the components of visual working memory. *Psychologische Beiträge*, 38(3-4), 355-367.

Reitan, R. M. (1958). Validity of the Trail Making Test as an indicator of organic brain damage. *Perceptual & Motor Skills*, 8, 271-276.

Restle, F. (1970). Speed of adding and comparing numbers. *Journal of Experimental Psychology*, 83(2), 274-278.

Roberts, P. & Ward, A (1998a). Adult mathematical ability as a distinct skill: A factor analysis study. Paper presented at British Psychological Society London Conference, The Institute of Education University of London. *Proceedings of The British Psychological Society* 6(1), 78.

Roberts, P. & Ward, A. (1998b). Mouse to an elephant is more than 2 to 0. The case for size comparison based on semantics. Paper presented at British Psychological Society Annual Conference, The Brighton Centre. *Proceedings of The British Psychological Society,* 7(2).

Roberts, P. & Ward, A. (1999). Spatial abilities involved in the addition of multi-digit problems. Poster presented at British Psychological Society Cognitive

Section Annual Conference, University of York. *Proceedings of The British Psychological Society,* 8(1).

Roberts, P. & Ward, A. (2003). The nature of magnitude representations. Poster presented at British Psychological Society Cognitive Section, University of Reading. In press.

Rossor, N. M., Warrington, E. K., & Cipolotti, L. (1995). The isolation of calculation skills. *Journal of Neurology*, 242, 78-81.

Royer, F. L. (1977). Information processing in the Block Design Task. *Intelligence*, 1(1), 32-50.

Rudel, R. G. & Denckla, M. B. (1974). Relation of forward and backward digit repetition to neurological impairment in children with learning disabilities. *Neuropsychologia*, 12(1), 109-118.

Saltzman, I. J. & Garner, W. R. (1948). Reaction time as measure of span attention. *Journal of Psychology*, 25, 227-241.

Sathian, K., Simon, T. J., Peterson, S., Patel, G. A., Hoffman, J. M., & Grafton, S. T. (1999). Neural evidence linking visual object enumeration and attention. *Journal of Cognitive Neuroscience*, 11(1), 36-51.

Scholnick, E. K. & Friedman, S. L. (1993). Planning in context: Developmental and situational considerations. *International Journal of Behavioral Development*, 16(2), 145-167.

Seitz, K. & Schumann-Hengsteler, R. (2000). Mental multiplication and working memory. *European Journal of Cognitive Psychology*, 12, 552-570.

Seron, X. & Noël, M.-P. (1992). Language and numerical disorders: A neuropsychological approach. In J. Alegria & D. Holender (Eds.), *Analytic Approaches To Human Cognition*. (pp. 291-309). Amsterdam: North-Holland.

Seron, X. & Noël, M.-P. (1995). Transcoding numbers from the Arabic code to the verbal one or vice vbersa: How many routes? *Mathematical Cognition*, 1(2), 215-243.

Seron, X., Pesenti, N., & Noël, M.-P. (1992). Images of numbers: Or when 98 is upper left and 6 is sky blue. *Cognition*, 44(1-2), 159-196.

Seymour, S. E., Reuter-Lorenz, P. A. & Gazzaniga, M. S. (1994). The disconnection syndrome:Basic findings reaffirmed. *Brain*, 117, 105-115.

Shallice, T. (1982). Specific impairment of planning. *Philosophical Transactions of the Royal Society (London)* B298, 199-209. (Reprinted in D. E. Broadbent & L. Weiskrantz (Eds), *The neuropsychology of cognitive function*. London: The Royal Society.

Shepard, R. N., Kilpatric, D. W., & Cunningham, J. P. (1975). The internal representation of numbers. *Cognitive Psychology*, 7(1), 82-138.

Shepard, R. N. & Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science*, 171(3972), 701-703.

Smyth, M. M., Morris, P. E., Levy, P., & Ellis, A. W. (1987). *Cognition in Action*. London: Erlbaum.

Sokol, S. M., McCloskey, M., Cohen, N. J., & Aliminosa, D. (1991). Cognitive representations and processes in arithmetic: Inferences from the performance of brain-damaged subjects. *Journal of Experimental Psychology: Learning, Memory & Cognition*, 17(3), 355-376.

Starkey, P. & Cooper, R. G. (1980). Perception of numbers by human infants. *Science*, 210(4473), 1033-1035.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, 36(2), 97-127.

Sternberg, R. J. (1980). Sketch of a componential subtheory of human intelligence. *Behavioral & Brain Sciences*, 3(4), 573-614.

Stevens, J. (1996). *Applied Multivariate Statistics for the Social Sciences*. (3rd ed.). Mathwah: Laurence Erlbaum.

Stroop, J. R. (1935). Studies of interference in serial verbal reactions. *Journal of Experimental Psychology*, 18, 643-662.

Svenson, O. (1975). Analysis of time required by children for simple additions. Acta Psychologica, 39(4), 289-301.

Taves, E. H. (1941). Two mechanisms for the perception of visual numerousness. *Archives of Psychology*, 265, 47.

Temple, C. M. (1989). Digit dyslexia: A category-specific disorder in development dyscalculia. *Cognitive Neuropsychology*, 6(1), 93-116.

Thioux, M., Pillon, A., Samson, D., de Partz, M.-P., & Noël, M.-P. (1998). The isolation of numerals at the semantic level. *Neurocase*, 4(4-5), 371-389.

Thurstone, L. L. (1938). *Primary mental abilities*. Chicago: The University of Chicago Press.

Thurstone, L. L. (1941). *Factoral studies of intelligence*. Chicago: The University of Chicago Press.

Tolman, E. C. (1932). *Purposive Behavior in Animals And Men*. New York: The Century Co.

Trabasso, T., & Riley, C. A. (1975). The construction and use of representations involving linear order. In R. L. Solso (Ed.), *Information Processing and Cognition: The Layola Symposium*. Hillsdale, N. J.: Erlbaum.

Trick, L. M. (1992). A theory of enumeration that grows out of a general theory of vision: Subitizing, counting, and FINSTs. In J. I. D. Campbell (Ed.), *The Nature and Origins of Mathematical Skills: Advances in Psychology*, 91. 257-299.

Trick, L. M. & Pylyshyn, Z. W. (1993). What enumeration studies can show us about spatial attention: Evidence for limited capacity preattentive processing. *Journal of Experimental Psychology: Human Perception & Performance*, 19(2), 331-351.

Trick, L. M. & Pylyshyn, Z. W. (1994a). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review*, 101(1), 80-102.

Trick, L. M. & Pylyshyn, Z. W. (1994b). Cueing and counting: Does the position of the attentional focus affect enumeration? *Visual Cognition*, 1(1), 67-100.

Ullmann, S. (1984). Visual routines. *Cognition*, 18(1-3), 97-159.

Ungerleider, L. G., & Minshkin, M. (1982). Two cortical visual systems. In J. Jingle, M. A. Goodale, & R. J. W. Mansfield (Eds.), *Analysis of Visual Behavior*. Cambridge, Mass.: MIT Press.

van Loosbroek, E., & Smitsman, A.,W. (1990). Visual perception of numerosity in infancy. *Developmental Psychology*,26,916-922.

Walen, J. McCloskey, M. Lindemann, M. & Bouton, G. (2002). Representing arithmetic table facts in memory: Evidence from acquired impairments. *Cognitive Neuropsychology*, 19(6), 505-522.

Warren, H. C. (1897). The reaction time of counting. *The Psychological Review*, 4(6), 569-590.

Warrington, E. K. (1982). The fractionation of arithmetical skills: A single case study. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 34A(1), 31-51.

Warrington, E. K. & McCarthy, R. (1983). Category specific access dysphasia. *Brain*, 106(4), 859-878.

Warrington, E. K. & Shallice, T. (1984). Category-specific semantic impairment. *Brain*, 107(4), 829-853.

Wechsler Adult Intelligence Scale – Third Edition (1997). *The Psychological Corporation*, Harcourt Brace & Company.

Werner, H. & Strauss, A. (1939). Problems and methods of functional analysis in mentally deficient children. *Journal of Abnormal & Social Psychology*, 34, 37-62.

Widaman, K. F., Geary, D. C., Cormier, P., (1986). A componential model for mental addition. *Journal of Experimental Psychology: Learning, Memory & Cognition*, 15(5), 898-919.

Wilson, B. A., Cockburn, J., & Baddeley, A. D. (1991). *The Rivermead Behavioural Memory Test*. Bury St Edmunds: Thames Valley Test Company.

Wolfe, J. M., Cave, K. R., & Franzel, S. L. (1989). Guided search: An alternative to the feature integration model for visual search. *Journal of Experimental Psychology: Human Perception & Performance*, 15(3), 419-433.

Woltz, D. J. (1988). An investigation of the role of working memory in procedural skill acquisition. *Journal of Experimental Psychology: General,* 117(3), 319-331.

Zago, L., Pesenti, M., Mellet, E., Crivello, F., Mazoyer, B. & Tzourio-Mazoyer, N. (2001). Neural correlates of simple and complex mental calculation. *NeuroImage,* 13, 314-327.

Zbrodoff, N J. & Logan, G. D. (1990). On the relation between production and verification tasks in the psychology of simple arithmetic. *Journal of Experimental Psychology*, 16(1), 83-97.

Zbrodoff, N J. & Logan, G. D. (2000). When it hurts to be misled: A Stroop-like effect on simple addition production task. *Memory and Cognition*, 28, 1-7.

Zhang, J. & Norman, D. A. (1994). Representations in distributed cognitive tasks. *Cognitive Science*, 18(1), 87-122.

Zingeser, L. B., & Berndt, R. S. (1988). Grammatical class and context effects in a case of pure anomia: Implications for models of language production. *Cognitive Neuropsychology*, 5, 473-516.